# Multilateral Comparisons of Productivity, Terms-of-Trade and Factor Accumulation

**by**

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## Abstract

This paper combines two recently-proposed innovative techniques to produce a new approach for multilateral comparisons using index numbers. The application of this new technique to international comparisons allows the examination of differences between the economies at various levels. The Minimum Spanning Tree algorithm, based on the idea of minimizing substitution bias of bilateral comparisons, provides a possible ordering for panel data. Making use of the suggested ordering, bilateral Tornqvist price and quantity indexes are calculated and multilateral indexes are obtained by chaining. An index-number based approach is then used to decompose the differences in GDP at the bilateral level. Different sources that contribute to the differences in GDP are considered: Productivity differences, terms of trade differences, factor endowments differences and domestic output price differences. Countries in the study include China, Japan, Canada, the US and Australia. Issues of the construction of purchasing power parities (PPPs) for data in index format are discussed.

## I. *Introduction*

Many different methods have been proposed for making multilateral comparisons. Generally, these can be done in two ways. The first is to construct bilateral indexes[1] which are intransitive and then link them by chaining to obtain transitivity,[2] whereas the second way is to use multilateral indexes which are transitive by construction. There is rapid growth in interest in the methodological problems relating to both ways (e.g. Caves, Christensen and Diewert (1982), Selvanathan and Rao (1992), Pilat and Rao (1996), Dowrick and Quiggin (1994,1997), Nuxoll (1994), Hill (1997) and Neary (1997)). In this paper, we are interested in dealing with the first way of multilateral comparisons. Some basic problems for this way of construction of multilateral comparisons concern the ordering of observations, the linking of bilateral comparisons and the conversion of national currency values to a common value. These problems have been constantly discussed in the literature. However, no perfect method has been generally recognized to deal with these.

Regarding the ordering of observations, in a time-series context, it is common to chain bilateral indexes chronologically as adjacent periods tend to have similar patterns of price and consumption.[3] It is the ordering of cross-section data that requires some thought. Some criterion needs to be set. At present, there appears to be an absence of methods

[1] A bilateral index is an index number that compares the price or quantity between two countries only. Examples include: Paasche price and quantity indexes (compare each period with the base period using the current period's basket (or price) as weights), Laspeyres price and quantity indexes (compare each country with the base country, using the base country's basket (or price) as weights), Fisher price and quantity index and Tornqvist price and quantity indexes. The latter two indexes are superlative indexes, which are exact and superlative for the homogenous quadratic and translog flexible functional forms, respectively. This means that an index number formula can be derived from a particular aggregator function and it is thereby exact for the particular aggregator function which it corresponds to.

[2] Transitivity is the property by which the price and quantity relationship among any three countries is the same, no matter the binary comparison is conducted with the original two countries or from the comparison of each country with any third country, e.g. With 3 countries, A, B and C, the index B/A multiplied by index C/B should equal index C/A.

suggested for application to the problem of ordering in the context of panel data. By making use of the Paasche-Laspeyres spread (PLS),[4] this paper uses the basic idea of the Kruskal's MST algorithm to provide a possible criterion for ordering panel data, see Hill (1999b). This provides a new way for linking the initially calculated bilateral indexes. The Tornqvist bilateral indexes are used for bilateral comparisons in this paper. A wide range of technical issues are also examined in this paper, which include the decomposition of gross domestic product (GDP) differences between countries over time and the conversion of national currency values to a common unit.

Generally, the present paper creates a new way for performing international comparisons using index numbers by combining two innovative approaches. These two approaches are backed up by strong theoretical foundations. There are two main aims of the paper. The first aim is to provide a possible criterion for ordering panel data, based on Kruskal's Minimum Spanning Tree (MST) (see Hill (1999a,b)) which utilizes the idea of minimizing substitution bias. This has not been done in previous literature. The second aim is to look at Fox's (1999) relative GDP decomposition technique in a panel data context in combination with the MST algorithm.

The link between the first and second aims can be formed by first using Kruskal's MST algorithm to obtain a favourable ordering on how the panel data should be linked, then multilateral indexes can be obtained by chaining the bilateral indexes across the MST. Fox's decomposition technique, (which is based on a technique proposed by Diewert and

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[3] See Diewert (1996) for arguments for chronological chaining of time series data and see Hill (1999b) for the justification of this using Kruskal's Minimum Spanning Tree (MST) algorithm.

[4] The Paasche-Laspeyres spread is the numerical difference of these two indexes, which provides a measure of the substitution bias. It reflects the variability in relative quantities and prices, and the correlation between, for example, two countries.

Morrison (1986); see also Kohli (1990) and Fox and Kohli (1998)), is applied at each of the bilateral comparison levels. Contributions of domestic price, terms of trade (TOT), labor and capital quantity to GDP can then be obtained.

An application using data on five countries is considered. The countries in the sample are China, Japan, US, Canada and Australia. The sample period covers 1986 to 1992. National currencies of the sampled countries are converted to a common unit using both exchange rates and PPPs. The results from using these alternative methods are compared.

The paper is structured as follows. Section II describes the method used to calculate purchasing power parities (PPPs), the methodology of Kruskal's Spanning Tree Method and Fox's decomposition technique for panel data. Section III describes the data set. Section IV discusses the results, and Section V concludes.

## *II. Methodology*

This section provides a brief summary of the methodology used in this paper. Before the introduction of the new approach for multilateral comparison, technical issues regarding the conversion of national currency values are discussed.

### *Calculation of PPPs Using the Geary-Khamis Method*

To facilitate the comparison of GDP between countries, we have to convert the national currencies of each country in each year to a common unit. There are at least two ways we can do this. Academic economists favour the use of PPPs while exchange rates are also commonly used. As is widely known, exchange rates may not adequately reflect real price differences between countries, so, besides using exchange rates, we also convert the national currency values using PPPs, and make a comparison of the results from the two different methods.[5] Regarding the calculation of PPPs, we make use of a variation of the Geary-Khamis (GK) method. The GK method[6] originated from Geary (1958), and was modified by Khamis (1967, 1969, 1970, 1972).

In this paper, due to the fact that our price data are expressed in index form, with the quantity series represented by constant price series (1985 level) and prices calculated as current price series/quantity, rather than in prices and physical quantities for sets of goods and services as originally propounded by the GK method, a slight variation of the GK method had to be made. See equations (3) and (4).[7]

$$\bar{P}_i = \sum_{j=1}^{M} \frac{P_{ij}}{\overline{PPP}_j}\left[\frac{Q_{ij}}{\sum_{j=1}^{M} Q_{ij}}\right] \quad (3)$$

$$\widehat{PPP}_j = \frac{\sum_{i=1}^{N} P_{ij} Q_{ij}}{\sum_{i=1}^{N} \bar{P}_i Q_{ij}} \quad (4)$$

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[5] The exchange rates for the OECD countries are taken from the dx database and those for China are taken from the China Statistical Yearbook 1997.

[6] The original purpose of the GK method was " to obtain unique global exchange rates, or PPPs and average prices in a uniform currency to enable the calculation of different types of international indices, and to aggregate over different commodities and countries for any meaningful economic flow…", (Khamis (1984)). The following shows the two linear equations which form the basis of the GK system:

$$P_i = \sum_{j=1}^{M} \frac{P_{ij}}{PPP_j}\left[\frac{Q_{ij}}{\sum_{j=1}^{M} Q_{ij}}\right] \text{ ------ } (1) \qquad PPP_j = \frac{\sum_{i=1}^{N} P_{ij} Q_{ij}}{\sum_{i=1}^{N} P_i Q_{ij}} \text{ ------ } (2)$$

where $i$ represents different categories of goods and $j$ represents different countries The first equation calculates the international prices for each category of goods, which reflects relative category values, whereas the second equation calculates the country PPPs, which depict relative country price levels. These equations are estimated simultaneously. The GK method in this present form is applicable to data pertaining to one single year.

where $i$ represents different categories of goods and $j$ represents different observations (a particular country at a particular year)

Instead of the actual values of PPPs, $\widehat{PPP}_j$ represents the change of PPPs (relative to US 1985 level). Similarly, $\overline{P}_i$ represents an average rate of international price change. There are four categories that we have to consider, namely, consumption, investment, export and imports. In this paper, the numerator of equation (3) is GDP, which is calculated using the expenditure approach, i.e. (consumption + investment + export – imports).[8]

In this paper, equations (3) and (4) are applied to panel data and the resulting PPPs are those which are relative to that of US 1985. Exchange rates are shown in Table 1 and PPPs are shown in Table 2.[9] Both of them are used to convert our price data respectively.

Once the conversion has been done, the MST algorithm can be used to generate the ordering of observations for comparison.

***The Spanning Tree Method***

As mentioned in the previous section, we make use of the MST algorithm to get an ordering for the panel data. By using the idea of minimizing the substitution bias,[10] we ensure that similar observations are linked together. A spanning tree is a connected graph

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[7] See Khamis (1984) for other possible variants of the Geary equations.

[8] Note that the conditions for the GK method to lead to unique and positive PPPs and average prices require that the quantity data entering the two equations are non-negative. However, this precludes the application of the GK method to the value added approach or other approaches involving subtracting one aggregate (e.g. intermediate inputs) from another aggregate.

[9] See Section 4 for a detailed discussion of these tables.

[10] Substitution bias arises when we construct an index using quantity or price vector of one particular observation as weights. In this case, the fact that there are corresponding relative changes in price or

which links a set of vertices (periods or countries) together in such a way that there is only one path between any pair of vertices. A spanning tree with K vertices should have K-1 edges. By connecting the edge of two vertices, a bilateral index number comparison can be obtained. Multilateral indexes can then be obtained through chaining of these bilateral indexes.

One problem associated with chaining is to decide which spanning tree to use since underlying any method of linking bilateral indexes to obtain multilateral indexes is itself a spanning tree. There are $K^{K-2}$ different possible spanning trees in the case of K vertices. Since there are five countries in our data set, each covering the period 1986-1992, there are thirty-five vertices and thirty-four edges for each of the $35^{35-2}$ spanning trees.

***The Minimum Spanning Tree***

The MST is the one which represents the "best possible" way of linking the vertices. Kruskal's algorithm uses the criterion that the linked vertices should have the smallest summed K-1 PLS bilateral indexes.[11] Since Paasche and Laspeyres indexes provide the lower and upper bounds on the true indexes, the MST indirectly implies that the generated multilateral indexes are the closest on average to the true indexes.

To summarize, the MST specifies the order of bilateral comparisons, which gives the smallest sum of PLS bilateral indexes, and the multilateral indexes can be obtained by chaining these bilateral indexes. One justification for the use of the spanning tree method is that it lets the data decides the spanning tree structure, instead of imposing it arbitrarily.

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quantity is not taken into account. The size of the substitution bias depends on the size of substitution elasticities and the magnitude of relative price changes. The PLS provides a measure of substitution bias.

[11] For the logic behind this criterion, see Hill (1999a).

It provides a contribution to the index number literature in the sense that it provides the ordering of observations, by which the resulting multilateral comparisons are the least sensitive to the bilateral index formula used.

***Decomposition of GDP in a Panel Context***

Once an ordering of the observations (each observation is for a country in a particular year) has been decided by the MST, bilateral comparisons can be calculated and the decomposition of GDP can then be carried out.

The technique originated from Diewert and Morrison (1986) and was subsequently developed by Kohli (1990) and Fox (1999). The former two papers consider the decomposition of the GDP growth of a country in a time-series context, whereas the latter considers the decomposition in a panel context. In this paper, we extend the technique to decomposing differences in GDPs for panel data. This approach has several advantages: (1) It does not ignore the fact that many countries are open economies, rather then closed economy. Effects of TOT movements on economic growth are explicitly included; (2) It makes it possible to work with relatively short time periods; (3) It allows the assessment of the contribution of increases in domestic output prices to nominal GDP growth; and (4) With the assumption that the technology can be approximated by a translog GDP function, the decomposition of GDP can be obtained by using the data alone, without the knowledge on the technology.

The approach to decomposing GDP differences has a solid theoretical foundation. Following theorem 1 in Diewert and Morrison (1986), the geometric mean of the Laspeyres and Paasche-type productivity indexes is set up to measure the impact on GDP that has occurred solely due to the change in technology. The Laspeyres-type productivity index, $R^{L}_{t,t-1}$ is shown in equation (5) and the Paasche-type productivity index is shown in equation (6).

$$R^{L}_{t,t-1} = \frac{\pi(p_{t-1}, x_{t-1}, t)}{\pi(p_{t-1}, x_{t-1}, t-1)} \qquad (5)$$

$$R^{P}_{t,t-1} = \frac{\pi(p_{t}, x_{t}, t)}{\pi(p_{t}, x_{t}, t-1)} \qquad (6)$$

$\pi$ represents the GDP function, $\pi = \pi(p_t, x_t, t) = \max_{y_t} \{p_t' y_t : (y_t , x_t) \in T_t\}$ and $T_t$ is the production possibility set at time t. P and x stand for prices and quantities respectively.

According to Diewert and Morrison (1986), if the GDP function has a translog form as shown in equation (7), the geometric mean of the Paasche and Laspeyres-type productivity indexes is exactly equal to a Tornqvist output index divided by a Tornqvist input index, which is shown in equation (8). In this case, a complete decomposition of GDP growth can be obtained. See equation (9).

$$\ln \pi = \alpha_0 + \Sigma\, \alpha_i \ln p_i + \Sigma\, \beta_j \ln x_j + ½\, \Sigma\Sigma\, \gamma_{ih} \ln p_i \ln p_h + ½\, \Sigma\Sigma\, \phi_{jk} \ln x_j \ln x_k$$
$$+ \Sigma\Sigma\, \delta_{ij} \ln p_i \ln x_j + \Sigma\, \delta_{iT} \ln p_i t + \Sigma\, \phi_{jT} \ln x_j t + \beta_T t + 1/2 \phi_{TT} t^2 \qquad (7)$$

$i , h \in \{ X, M, I, G, C \}$ ; $j , k \in \{ L, K \}$

$\Sigma\ \alpha_i = 1$, $\Sigma\ \beta_j = 1$, $\gamma_{ih} = \gamma_{hi}$, $\phi_{jk} = \phi_{kj}$, $\Sigma\ \gamma_{ih} = 0$, $\Sigma\ \phi_{jk} = 0$, $\Sigma_i\ \delta_{ij} = 0$, $\Sigma_j\ \delta_{ij} = 0$, $\Sigma\ \delta_{iT} = 0$ and $\Sigma\ \phi_{jT} = 0$. X, M, I, G, C, L, K are export, import, investment, government purchases, consumption goods, labor and capital respectively. Labor and capital are exogenously given inputs. Imports are treated as a negative output according to the standard definition of GDP (Burgess (1974), Kohli (1978)).

$$R_{t,t-1} = \frac{\Gamma_{t,t-1}}{P_{t,t-1} \bullet X_{t,t-1}} \qquad (8)$$

where

$\Gamma_{t,t-1}$ is (1 plus) the rate of increase in nominal GDP between times t-1 and t

$P_{t,t-1}$ is Tornqvist output price index

$X_{t,t-1}$ is Tornqvist fixed input quantity index

R measures the effect of change in technology (i.e. TFP change) on GDP between t and t-1. Rearrange equation (8):

$$\Gamma_{t,t-1} = R_{t,t-1} \bullet P_{t,t-1} \bullet X_{t,t-1} \qquad (9)$$

Equations (5) to (9) are specifically designed for decomposing GDP growth for time series data, however, in this paper, besides decomposing GDP growth within countries, we are also interested in decomposing differences in GDP between countries over time. In other words, we want to apply the technique to panel data. To achieve this aim, an extension of the Diewert and Morrison (1986) decomposing technique is needed and it is discussed as follows:

*Modification of the Diewert and Morrison (1986) technique*

In the case of two observations (a and b, each represents a particular country at a particular time), the geometric mean of the Paasche and Laspeyres-type productivity indexes of them, denoted by $R^{P}_{a,b}$ and $R^{L}_{a,b}$ is equal to R = $(R^{P}_{a,b}R^{L}_{a,b})^{1/2} = \frac{\Gamma}{P \bullet X}$.

Differences in GDP between these two observations can be decomposed into differences in productivity, prices and quantities, i.e. $\Gamma = R \bullet P \bullet X$, where $\Gamma$ is the ratio of GDP between the two observations in comparison. It is equal to one if GDP of the two observations in comparison is the same. If it is greater than 1, then the GDP of the observation is greater than that of the base observation. Similarly, if it is smaller than 1, the GDP of the observation is smaller than that of the base observation. R is the productivity index and P is the price index and X is the quantity index.

$$\Gamma = \frac{P_b X_b}{P_a X_a} \tag{10}$$

$$P = \exp \left[ \sum_{n=1}^{N} \tfrac{1}{2} (S_{na} + S_{nb}) \ln \frac{P_{nb}}{P_{na}} \right] \tag{11}$$

$$X = \exp \left[ \sum_{m=1}^{M} \tfrac{1}{2} (S_{ma} + S_{mb}) \ln \frac{X_{mb}}{X_{ma}} \right] \tag{12}$$

where $S_{na}$, $S_{ma}$ and $S_{nb}$, $S_{mb}$ are the shares of price and quantity in GDP for observations a and b.

The price index can be further decomposed into TOT adjustment indexes, i.e. $A_{a,b}$ and the non-trade good price indexes, i.e. $P_{Na,b}$. Similarly, the input quantity indexes can be furthered decomposed into specific indexes for labor and capital respectively, i.e. $X_{ja,b}$.

The productivity index is treated as a residual. Each of these effects is calculated as follows:

$$A_{a,b} = \exp\{½\,(S_{Ma}+S_{Mb})\ln\frac{P_{mb}}{P_{ma}} + ½\,(S_{Xa}+S_{Xb})\ln\frac{P_{xb}}{P_{xa}}\} \qquad (13)$$

where $S_{Ma}$ and $S_{Mb}$ are the shares of imports in GDP for observations a and b and $S_{Xa}$ and $S_{Xb}$ are the shares of exports in GDP for observations a and b. Since import is treated as a negative output, $S_{Ma}$ and $S_{Mb}$ have negative magnitudes. Note that each observation represents a particular country at a particular time period.

$$X_{ja,b} = \exp\{½\,(S_{ja} + S_{jb})\ln\frac{X_{jb}}{X_{ja}}\},\ \ j \in \{L,K\} \qquad (14)$$

where $S_{ja}$ and $S_{jb}$ are the shares of factors (i.e. labor and capital) in GDP for observations a and b. Note that for the calculation of quantity indexes, there is a need to convert the quantity between the two observations in comparison into common units.[12]

$$P_{Na,b} = \exp\{½\,(S_{1a} + S_{1b})\ln\frac{P_{1b}}{P_{1a}} + ½\,(S_{Ga} + S_{Gb})\ln\frac{P_{Gb}}{P_{Ga}}$$

$$+ ½(S_{Ca}+S_{Cb})\ln\frac{P_{cb}}{P_{ca}}\} \qquad (15)$$

where $S_{Ia}$, $S_{Ib}$, $S_{Ga}$, $S_{Gb}$, $S_{Ca}$ and $S_{Cb}$ are the shares of investment, government and consumption in GDP for observations a and b respectively.

$A_{a,b}$ measures the deviation in TOT of observation b from that of observation a

[12] See Appendix 1 for the justification of this adjustment.

$X_{ja,b}$ measures the deviation in factor j of observation b from that of observation a

$P_{Na,b}$ measures the deviation in domestic prices of observation b from that of observation a

Following Kohli (1990), by using equations (10) to (15), a complete decomposition of differences in GDP between two observations in the Translog case can be obtained (note that it is the observed GDP differences based on the data directly):

$$\Gamma_{a,b} = R_{a,b} \bullet A_{a,b} \bullet X_{La,b} \bullet X_{Ka,b} \bullet P_{Na,b} \tag{16}$$

$\Gamma_{a,b}$ represents the difference in nominal GDP between observations a and b and $R_{a,b}$ represents the productivity deviation between observations a and b and it is treated as a residual.

The above formulae are applied to each of the (K-1) direct bilateral comparison between two observations indicated by the MST. Any decomposition for indirect comparison between two observations can be calculated as follows:

$$\begin{aligned} \Gamma_{a,b} \bullet \Gamma_{b,c} &= (R_{a,b} \bullet P_{a,b} \bullet X_{a,b})\ (R_{b,c} \bullet P_{b,c} \bullet X_{b,c}) \\ &= (R_{a,b} \bullet R_{b,c})\ (P_{a,b} \bullet P_{b,c})\ (X_{a,b} \bullet X_{b,c}) \end{aligned} \tag{17}$$

$\Gamma_{a,b}$ represents the ratio of GDP between observations a and b and $\Gamma_{b,c}$ represents the ratio of GDP between observations c and b and the multiplication of the two ratios gives the ratio of GDP between observations a and c, i.e. $\Gamma_{a,c}$. R, P and X represent the bilateral Tornqvist productivity, price and quantity indexes between two observations. $(R_{a,b} \bullet R_{b,c})$, $(P_{a,b} \bullet P_{b,c})$ and $(X_{a,b} \bullet X_{b,c})$ measure the deviations in productivity, prices and quantity of observation c to observation a respectively. Note that ($R_{a,b} \bullet R_{b,c}$), ($P_{a,b} \bullet P_{b,c}$) and ($X_{a,b} \bullet X_{b,c}$) are not equal to the direct comparisons, i.e. $R_{a,c}$, $P_{a,c}$ and $X_{a,c}$ between observations a and c.

The above algorithm can be generalised to include more than one intermediate observation. The number of intermediate observations between the comparison of two observations are determined by the structure of the MST.

To obtain multilateral comparison for all observations, each observation's GDP is compared to a common base observation's GDP, and the decomposition procedure is carried out using the above steps. An advantage of this method is that the ratio of GDP among all observations are the same, regardless of the base observation chosen. Similarly, the ratios of the resulting decomposition components among observations are base invariant as well. This means that the multilateral indexes being calculated are unaffected by the choice of base observation. With all the components of the base observation equalling one, the resulting decomposition components can then be compared relatively across observations.

Note that the resulting decomposition indexes may change if new observations are added or old observations are removed, as the number of intermediate comparisons for some indirect comparisons may increase or decrease.

## *III. Data*

The methods described in Section 2 are applied to annual data for 4 OECD countries, (US, Japan, Canada and Australia), and China. The data set includes aggregate prices and quantities for consumption (government and non-government), investment, exports, imports, labor and capital for the years 1986-1992. The sum of the value of the first four variables equals aggregate GDP (where imports are taken to be a negative output).

Annual data of current and constant prices for the OECD countries are taken from Fox (1997), which provides a description of the method used for constructing the OCED data set. Most of the raw data in Fox (1997) are taken from the OECD National Accounts. For annual data of China, current and constant prices of the expenditure components are taken from Table 1 and Table 2 of World Bank (1997). Current prices for indirect taxes and subsidies are drawn from Table 1 of World Bank (1997b) as well.[13] Regarding the data for labor, total employment and total labor income are used. These two series are taken and calculated respectively from the Appendix table of Hu and Khan (1997).[14] The capital income series can be constructed by multiplying (1-labor income share) by the national income series. To obtain the quantity of capital services, we use the procedure of Kohli (1982).

Certain adjustments have been made to the two data set, so that they are as comparable as possible.[15] The current-price series were treated as values and the constant-price series were treated as quantities. The price series were then derived by taking value/quantity. All of the original variables are expressed in national currencies.

[13] The consumption price series was multiplied by (1-tax rate) and the import price series was multiplied by (1+tax rate), where the tax rate was calculated as Indirect Taxes Less Subsidies divided by Government and Private Consumption plus Imports, so that the figures of nominal GDP are expressed in producer prices, rather than at market prices. We do this because we are more interested in goods and services that are produced rather than those that are bought by consumers.
[14] The labor income is calculated by multiplying column 1 (national income) and column 6 (labor income share) from the Appendix table.
[15] See Appendix 2 for details of adjustments on the data set.

Note that since the price series are in index format, with the price of each variable for each country in 1985 set to 1, a problem arises when constructing the MST, regardless of the use of exchange rate or PPPs as the conversion factor.[16] Due to this, we start off with the second observation, i.e. use 1986 as the first year of our sample.

## *IV. Results*

### *Exchange Rates vs PPPs*

The exchange rates for each country from 1985-1992 are shown in Table 1. They are expressed in terms of USD. PPPs expressed in terms of USD are presented in Table 2. For each country in the sample, the PPPs have been increasing over time. This indicates that there is a need to pay more to buy the same amount of products than in the past. Note that Table 1 and Table 2 cannot be directly comparable because for Table 1, each year's exchange rates of all the countries are compared to the US at that particular year, whereas for Table 2, all the PPPs are relative to US 1985.

From Table 2, we can see that the PPPs for the US has actually been increasing as well. Comparing 1986 and 1992, the rate of increase is about 27 per cent. Since the US is used as the numeraire for each year's comparison of exchange rates across countries, the change in the value of the US dollar over time can only be detected by looking at the PPPs. However, we are still interested to look at the comparison of the results using prices adjusted by exchange rates and PPPs respectively.

The exchange rates and PPPs are used respectively as the conversion factors for the price data, so that every country's currencies are now expressed in common units. The two sets

[16] See Appendix 4 for explanation of possible problems for the construction of the MST when data are

of price data are applied to the construction of the MST and the decomposition part of this paper respectively. Comparisons of the results are shown later in this paper.

**TABLE 1**
**Exchange Rate**

| | Australia | Canada | Japan | US | China |
|---|---|---|---|---|---|
| 1985 | 0.6985 | 0.7322 | 0.0042 | 1.0000 | 0.3405 |
| 1986 | 0.6685 | 0.7197 | 0.0059 | 1.0000 | 0.2896 |
| 1987 | 0.6999 | 0.7540 | 0.0069 | 1.0000 | 0.2687 |
| 1988 | 0.7805 | 0.8120 | 0.0078 | 1.0000 | 0.2687 |
| 1989 | 0.7906 | 0.8446 | 0.0072 | 1.0000 | 0.2655 |
| 1990 | 0.7802 | 0.8569 | 0.0069 | 1.0000 | 0.2090 |
| 1991 | 0.7788 | 0.8725 | 0.0074 | 1.0000 | 0.1879 |
| 1992 | 0.7341 | 0.8273 | 0.0079 | 1.0000 | 0.1813 |

**TABLE 2**
**Pseudo-PPPs**

| | Australia | Canada | Japan | US | China |
|---|---|---|---|---|---|
| 1985 | 0.6866 | 0.7200 | 0.0042 | 1.0000 | 0.3411 |
| 1986 | 0.7376 | 0.7298 | 0.0044 | 1.0265 | 0.3646 |
| 1987 | 0.7882 | 0.7613 | 0.0043 | 1.0587 | 0.3883 |
| 1988 | 0.8556 | 0.7882 | 0.0043 | 1.1027 | 0.4336 |
| 1989 | 0.9073 | 0.8224 | 0.0044 | 1.1500 | 0.4714 |
| 1990 | 0.9427 | 0.8521 | 0.0044 | 1.1981 | 0.5054 |
| 1991 | 0.9617 | 0.8666 | 0.0046 | 1.2400 | 0.5389 |
| 1992 | 0.9727 | 0.8714 | 0.0046 | 1.2732 | 0.5774 |

expressed as index format in this case.

***Structure of the Minimum Spanning Tree***

The MSTs for China, Japan, Australia, Canada and US, with price data adjusted by exchange rate and PPPs, respectively, from 1986-1992 are depicted in Figures 1A and 1B in Appendix 5. Each of them have 34 edges. Generally, each country forms a cluster in both MSTs. This is logical as it implies that each country is more similar to itself in different years than it is to other countries in any year of the sample.[17] For Japan, China and US, most of the adjacent years are connected together, like the setting of "string" spanning trees.[18] The symptom of star spanning trees[19] can also be found. For example, Australia 91 acts like the center of a star spanning tree, which is connected to several observations for Canada in both figures. US 86 and US 89 are also connected with more than 3 other observations in Figure 1A, whereas US87 is connected with five observations in Figure 1B.

***Sensitivity Analysis on the Structure of the Minimum Spanning Tree***

In order to assess the sensitivity of the MST method, all observations for each year are dropped in turn, in both MSTs. Additionally, the effects from dropping observations from more than one year at a time are also examined. It is found that the structures of the MST of the remaining observations are robust in both cases, i.e. the remaining observations are still connected to the same observations as when the full sample was used. Those which were originally connected to one or more of the deleted observations, are now connected to new observations.[20] Hence, when viewed from the perspective of clusters of countries, both MSTs are stable when data are deleted or added.

[17] Hill (1999b) obtains similar results in time series comparisons.
[18] See Figure 2A in Appendix 6 for the graph of a string spanning tree.
[19] See Figure 2B in Appendix 6 for the graph of a star spanning tree.
[20] By constructing two MSTs using two cross-sections of data for 1980 and 1985 drawn from the ICP, Hill (1999a) found that with the perspective of clusters of countries, the MST is reasonably stable over time. For

Similarly, if the observations/time series for a particular country are removed, the structures of both MSTs for the remaining observations will be mostly the same as if the removal had not been done, because each country nearly forms a cluster by itself. Hence, the MSTs are also stable when a new country's time series is added.

Note that the results of the above sensitivity analyses are empirical observations specifically for the current sample only, similar research on other samples would be worthwhile in the future.

***Results of Decomposition***

The results from decomposing the differences in GDP between two observations (based on the direct bilateral comparisons indicated by Figures 1A and 1B are shown in Table 3 and Table 4. Table 3 and Table 4 show the results calculated using the exchange rate-adjusted and PPP–adjusted price data respectively. Since there is a slight difference in the structures of both MSTs, there will be a difference in some bilateral comparisons in both tables as well.

In both Table 3 and Table 4, the last column shows the ratio of GDP between the two observations.[21] Column 1 shows the difference in productivity between two observations. Column 2 and column 3 show the differences in labor and capital utilization respectively, calculated using equation (14). Column 4 shows the difference in TOT,[22] which is

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just one cross-section of data, deletion of a country will or will not affect the structure of the original MST, depends crucially on which country is deleted. If the country is one of those which is connected to only one other country in the original MST, then the original structure of the MST will be preserved. However, if the country is connected to more than 2 other countries before, then deleting it will change the structure of the MST.

[21] Note that each of the bilateral comparisons are decided by the structure of the MST.

[22] Diewert and Morrison (1986) argued that in an open economy, an improvement in the TOT will lead to an increase in domestic production for any given amount of inputs, so that it has the effect similar to an

calculated using equation (13) and column 5 shows the difference in domestic prices, calculated by using equation (15). Each of these components is calculated using the Tornqvist bilateral indexes.

If the nominal GDP ratio is greater than one, any component that has a magnitude greater than one indicates that it contributes positively to the larger GDP in comparison. On the other hand, if the component has a magnitude smaller than one, it indicates that it is shaving off GDP from the observation with larger GDP. For example, the nominal GDP ratio of China 91 to China 90 in Table 3 indicates that GDP in 1991 is larger and this is the result of positive contribution of productivity. Labor, capital, TOT and domestic prices have negative effects on the nominal GDP of China 91 instead.

On the other hand, if the nominal GDP ratio is smaller than one, any component that has a magnitude greater than one means that the component is actually narrowing down the difference between the nominal GDPs in comparison. Similarly, if the component has a magnitude smaller than one, this indicates that it is the main factor which has led to the smaller GDP for the observation with a smaller GDP. Consider the comparison between China 87 and China 91 in Table 3, the nominal GDP ratio is smaller than one which indicates that China 87 has a lower nominal GDP. The reason is that China 87 has weaker contributions by productivity. Although capital, labor, TOT and domestic price indexes seem to be higher in China 87 than in China 91, they do not add to higher GDP.

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increase in TFP, whereas a deterioration in the TOT will decrease domestic production, having an effect similar to a decrease in TFP.

Regarding the comparison between Table 3 and Table 4, most of the results for the same bilateral comparisons for price effects are similar in their magnitudes,[23] except for some comparisons between different observations for China and different observations for Japan. For example, consider the direct comparison between China 91 and China 90 in Table 3. Both the TOT and domestic price indexes have magnitudes smaller than one, whereas in Table 4, the magnitudes of these two indexes are greater than one. The same thing happens for comparisons between China 87 and China 91 and Japan 88 and Japan 87.

Take the comparison between Japan 88 and Japan 87 as an illustration, Table 3 and Table 4 imply that the exchange rate-adjusted and PPP-adjusted GDP of Japan 88 is greater than that for Japan 87. In Table 3, TOT and domestic price effects are both contributing positively to higher nominal GDP for Japan 88, whereas in Table 4, these two factors do not have significant effects for larger GDP.

Regarding the results of quantity indexes for same bilateral comparisons in Table 3 and Table 4, most of them have similar magnitudes, with only a few exceptions. For example, the comparisons between China 91 and China 90, China 87 and China 91 and China 92 and China 88, have different magnitudes in both tables. Nevertheless, the bilateral comparisons between China 86 and US 86 have significantly larger magnitudes in labor and capital than other comparisons in both tables.

For productivity indexes, most of the bilateral comparisons in both tables have different magnitudes, except some comparisons between Australia and Canada, which have

[23] Similarity here means that the two ratios in comparison are both greater than one or both smaller than

magnitudes greater than one in both tables, e.g. the comparisons between Canada 89 and Australia 91 and Canada 88 and Australia 91.

**TABLE 3**
**Results from Direct Comparisons indicated by the Minimum Spanning Tree (with price data adjusted by exchange rates)**

| Direct Bilateral Comparisons | Productivity Indexes | Labor quantity indexes | Capital quantity indexes | TOT indexes | Domestic price indexes | Nominal GDP ratio |
|---|---|---|---|---|---|---|
| 1. China91/China90 | 1.1600 | 0.9581 | 0.9826 | 0.9989 | 0.9603 | 1.0475 |
| 2. China87/China91 | 0.6720 | 1.1300 | 1.0018 | 1.0006 | 1.0397 | 0.7913 |
| 3. China88/China87 | 1.0326 | 1.0121 | 1.0649 | 0.9964 | 1.1255 | 1.2480 |
| 4. China92/China88 | 1.6050 | 0.8697 | 0.9644 | 1.0004 | 0.8941 | 1.2042 |
| 5. China89/China88 | 0.9877 | 1.0031 | 1.0508 | 0.9991 | 1.0764 | 1.1195 |
| 6. China86/China89 | 0.8670 | 1.0053 | 0.8889 | 1.0066 | 0.8438 | 0.6580 |
| 7. Us86/China86 | 0.8281 | 6.0586 | 2.3141 | 1.0192 | 1.1189 | 13.2400 |
| 8. Us88/Us86 | 1.0174 | 1.0421 | 1.0110 | 0.9946 | 1.0789 | 1.1503 |
| 9. Us87/Us86 | 1.0038 | 1.0222 | 1.0056 | 0.9950 | 1.0367 | 1.0643 |
| 10. Us89/Us87 | 1.0220 | 1.0329 | 1.0109 | 0.9993 | 1.0853 | 1.1574 |
| 11. Us90/Us89 | 0.9995 | 1.0018 | 1.0044 | 0.9976 | 1.0442 | 1.0476 |
| 12. Us92/Us86 | 1.0368 | 1.0567 | 1.0259 | 0.9968 | 1.2439 | 1.3936 |
| 13. Us91/Us92 | 0.9886 | 0.9890 | 0.9972 | 0.9988 | 0.9728 | 0.9474 |
| 14. Jap87/Us89 | 0.5971 | 0.6444 | 0.8202 | 1.0283 | 1.4302 | 0.4641 |
| 15. Jap88/Jap87 | 0.9139 | 1.1149 | 1.0511 | 1.0017 | 1.1185 | 1.1999 |
| 16. Jap86/Jap87 | 1.1390 | 0.8864 | 0.9453 | 0.9972 | 0.8728 | 0.8306 |
| 17. Jap89/Jap86 | 0.8786 | 1.2027 | 1.1085 | 0.9993 | 1.2237 | 1.4323 |
| 18. Jap91/Jap89 | 1.0242 | 1.0258 | 1.0416 | 0.9958 | 1.0763 | 1.1728 |
| 19. Jap92/Jap91 | 0.9534 | 1.0292 | 1.0321 | 1.0036 | 1.0784 | 1.0960 |
| 20. Jap90/Jap89 | 1.0861 | 0.9726 | 1.0034 | 0.9918 | 0.9728 | 1.0225 |
| 21. Can92/Us92 | 0.8556 | 0.1800 | 0.5373 | 0.9984 | 1.1014 | 0.0910 |
| 22. Can90/Can92 | 0.9678 | 1.0432 | 0.9947 | 1.0066 | 1.0059 | 1.0170 |
| 23. Can91/Can90 | 0.9807 | 0.9933 | 1.0147 | 0.9954 | 1.0388 | 1.0220 |
| 24. Aus92/Can90 | 1.1099 | 0.5913 | 0.7389 | 0.9968 | 1.0645 | 0.5145 |
| 25. Aus86/Us89 | 1.0051 | 0.1037 | 0.3477 | 0.9943 | 0.8973 | 0.0323 |
| 26. Can87/Us86 | 0.9718 | 0.1884 | 0.4781 | 1.0002 | 1.0763 | 0.0942 |
| 27. Aus87/Can87 | 1.0218 | 0.6119 | 0.7539 | 0.9975 | 1.0515 | 0.4944 |
| 28. Aus91/Can87 | 0.8607 | 0.6924 | 0.8099 | 1.0086 | 1.4781 | 0.7195 |
| 29. Can86/Aus91 | 1.2557 | 1.3678 | 1.1997 | 0.9808 | 0.6022 | 1.2170 |
| 30. Aus90/Aus91 | 0.9669 | 1.0231 | 0.9972 | 1.0048 | 0.9776 | 0.9689 |
| 31. Aus89/Aus90 | 1.0126 | 0.9936 | 0.9991 | 1.0096 | 0.9693 | 0.9837 |
| 32. Aus88/Aus89 | 1.0246 | 0.9636 | 0.9855 | 1.0038 | 0.9268 | 0.9050 |
| 33. Can89/Aus91 | 1.0007 | 1.6154 | 1.3164 | 1.0013 | 0.8500 | 1.8112 |
| 34. Can88/Aus91 | 1.0534 | 1.5571 | 1.2818 | 0.9970 | 0.7758 | 1.6262 |

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**TABLE 4**
**Results from Direct Comparisons indicated by the Minimum Spanning Tree (with price data adjusted by PPPs)**

| | Direct Bilateral Comparisons | Productivity Indexes | Labor quantity indexes | Capital quantity indexes | TOT indexes | Domestic price indexes | Nominal GDP ratio |
|---|---|---|---|---|---|---|---|
| 1. | China91/China90 | 0.9778 | 1.0516 | 1.0620 | 1.0037 | 1.1339 | 1.2427 |
| 2. | China87/China91 | 1.3337 | 0.8159 | 0.6990 | 0.9905 | 0.5292 | 0.3987 |
| 3. | China88/China87 | 0.9247 | 1.0595 | 1.1359 | 0.9959 | 1.2573 | 1.3935 |
| 4. | China92/China88 | 0.8133 | 1.1889 | 1.3920 | 1.0005 | 1.7643 | 2.3762 |
| 5. | China89/China88 | 0.8979 | 1.0495 | 1.1047 | 0.9981 | 1.1852 | 1.2315 |
| 6. | China86/China89 | 1.2227 | 0.8509 | 0.7446 | 1.0129 | 0.5946 | 0.4666 |
| 7. | Us86/China86 | 1.0137 | 5.4023 | 2.1164 | 1.0264 | 0.9076 | 10.7972 |
| 8. | Us88/Us87 | 0.9730 | 1.0488 | 1.0179 | 0.9978 | 1.0859 | 1.1257 |
| 9. | Us87/Us86 | 0.9732 | 1.0444 | 1.0151 | 0.9936 | 1.0707 | 1.0977 |
| 10. | Us89/Us87 | 0.9409 | 1.0936 | 1.0371 | 0.9963 | 1.1824 | 1.2572 |
| 11. | Us90/Us89 | 0.9593 | 1.0304 | 1.0174 | 0.9965 | 1.0890 | 1.0914 |
| 12. | Us92/Us91 | 0.9852 | 1.0301 | 1.0107 | 1.0007 | 1.0559 | 1.0838 |
| 13. | Us91/Us87 | 0.8714 | 1.1424 | 1.0663 | 0.9956 | 1.3748 | 1.4529 |
| 14. | Jap86/Us89 | 1.0886 | 0.4190 | 0.6751 | 1.0275 | 0.7833 | 0.2478 |
| 15. | Jap88/Jap87 | 1.0332 | 1.0179 | 1.0142 | 0.9990 | 0.9879 | 1.0526 |
| 16. | Jap87/Jap86 | 1.0317 | 1.0011 | 1.0099 | 0.9977 | 0.9748 | 1.0145 |
| 17. | Jap89/Jap86 | 1.0672 | 1.0441 | 1.0480 | 0.9954 | 1.0085 | 1.1723 |
| 18. | Jap91/Jap89 | 1.0010 | 1.0389 | 1.0469 | 0.9960 | 1.0955 | 1.1878 |
| 19. | Jap92/Jap91 | 0.9923 | 0.9993 | 1.0202 | 1.0030 | 1.0355 | 1.0507 |
| 20. | Jap90/Jap89 | 1.0126 | 1.0156 | 1.0210 | 0.9919 | 1.0334 | 1.0763 |
| 21. | Can86/Us87 | 1.0388 | 0.1682 | 0.4591 | 1.0040 | 0.9221 | 0.0742 |
| 22. | Can92/Can90 | 0.9723 | 0.9976 | 1.0232 | 0.9922 | 1.0577 | 1.0416 |
| 23. | Can91/Can90 | 0.9782 | 0.9926 | 1.0143 | 0.9961 | 1.0406 | 1.0209 |
| 24. | Can90/Aus92 | 1.2023 | 1.3966 | 1.2299 | 1.0123 | 0.6977 | 1.4586 |
| 25. | Aus86/Aus87 | 1.0529 | 0.9390 | 0.9698 | 0.9889 | 0.8838 | 0.8379 |
| 26. | Aus92/Aus90 | 1.0188 | 1.0018 | 1.0183 | 0.9865 | 1.0803 | 1.1077 |
| 27. | Aus87/Can87 | 0.9207 | 0.6568 | 0.7833 | 0.9949 | 1.1702 | 0.5514 |
| 28. | Can87/Aus91 | 1.3596 | 1.2652 | 1.1524 | 0.9970 | 0.5750 | 1.1364 |
| 29. | Aus91/Can86 | 0.7091 | 0.8324 | 0.8916 | 1.0103 | 1.8821 | 1.0007 |
| 30. | Aus90/Aus91 | 0.9881 | 1.0085 | 0.9898 | 1.0053 | 0.9561 | 0.9481 |
| 31. | Aus89/Aus92 | 1.0463 | 0.9591 | 0.9633 | 1.0267 | 0.8499 | 0.8435 |
| 32. | Aus88/Aus89 | 1.0726 | 0.9355 | 0.9696 | 1.0061 | 0.8833 | 0.8645 |
| 33. | Can89/Aus91 | 1.2666 | 1.3804 | 1.2149 | 1.0075 | 0.6675 | 1.4284 |
| 34. | Can88/Aus91 | 1.3214 | 1.3282 | 1.1811 | 1.0026 | 0.6150 | 1.2782 |

We are more interested in looking at the results in Table 5 and Table 6, which show the relative comparisons among all observations, with China 90 acting as the base for all comparisons. Each component is calculated using the algorithm indicated by equation (17). All the interpretations explained previously apply to the results in these two tables as well.

Comparing the results between Table 5 and Table 6, one significant difference is found in the domestic price effects. In Table 5, almost all the domestic price indexes have magnitudes greater than one, whereas in Table 6, the opposite happens. Regarding productivity indexes, the magnitudes are opposite in both tables as well. Most of the productivity indexes have magnitudes smaller than one in Table 5, whereas in Table 6, most of them have magnitudes greater than one.

Same as the explanation given in previous paragraphs, whether a component adds to GDP of a particular observation or not depends on whether the nominal GDP ratios are greater than or smaller than one. Therefore, although the domestic price effects are opposite in magnitudes in both tables, given the differences in GDP ratios in both tables, these domestic price effects are actually found to be significant factors for GDPs in China, Canada and Australia in both tables. For productivity effects, they are generally not contributing positively to GDP when prices are adjusted by exchange rates. However, they are affecting GDPs of Australia, the US and Japan when prices are adjusted by PPPs.

Looking specifically at the quantity differences, the US and Japan have more labor and capital than other countries, which act as the main factors contributing to their relatively larger GDPs compared to the base observation. As shown in both Table 5 and Table 6, the magnitudes of both labor and capital quantity indexes for US and Japan are greater than those of other countries. Labor and capital have found to be significant factors of GDPs for Australia and Canada in both tables, regardless of the differences in their magnitudes.

TOT effects are similar in both Table 5 and Table 6 in their magnitudes, which are mostly greater than one. Again, as discussed in previous paragraphs, whether a component contributes to GDP positively or not depends not just on the magnitudes of the indexes, it also depends of the ratios of the GDPs. Although the TOT contributions are relatively smaller compared to labor and capital effects, they both show positive contributions to larger GDPs in the US and Japan. However, for Canada and Australia, the results for Table 5 and Table 6 are opposite. When prices are adjusted by exchange rates, TOT effects are contributing to higher GDPs for both countries, whereas when prices are adjusted by PPPs, higher TOT effects do not contribute to higher GDPs.

Looking specifically at China, it has relatively high productivity indexes as compared to the OECD countries in Table 5. Especially for China observations in later years (with nominal GDP compared to that of China 1990 greater than one), the magnitudes of productivity indexes are greater than one, which indicate that productivity is the main factor that has driven the higher GDP. Labor effects in 86-87 and 91-92 are found not to be the main factors for GDP in Table 5. Capital effects, on the other hand, are significant components for GDP in China.

In Table 6, labor and capital are both found to be the main driving force for GDP for China. These effects are found to be the main factors for smaller GDP and larger GDP (compared to that for China 1990) for 1986-1989 and 1991-1992, respectively. Productivity, on the other hand, is found to be an insignificant factor to GDP.

TOT has relatively smaller magnitudes in China in both Table 5 and Table 6, which is different from that in the OECD countries. Generally, it is found to be an insignificant factor in Table 5, but a significant factor in Table 6.

Domestic price effects are relatively small in China in Table 5 while relatively large in Table 6. Generally, these are found to have contributed positively to larger GDP in later years in China when the price data is adjusted by the exchange rates and PPPs, respectively.

**TABLE 5**
**Multilateral Comparisons, with China 1990 as the base – (Calculated using Price data adjusted by Exchange Rates)**

| | Productivity | Labor Quantity | Capital Quantity | TOT | Domestic Price | Nominal GDP ratio |
|---|---|---|---|---|---|---|
| China1986 | 0.6892 | 1.1048 | 0.9789 | 1.0016 | 1.0207 | 0.7621 |
| China1987 | 0.7796 | 1.0826 | 0.9843 | 0.9994 | 0.9985 | 0.8289 |
| China1988 | 0.8048 | 1.0957 | 1.0482 | 0.9959 | 1.1238 | 1.0344 |
| China1989 | 0.7949 | 1.0990 | 1.1014 | 0.9950 | 1.2096 | 1.1580 |
| China1990 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| China1991 | 1.1601 | 0.9581 | 0.9826 | 0.9989 | 0.9603 | 1.0475 |
| China1992 | 1.2920 | 0.9529 | 1.0108 | 0.9963 | 1.0048 | 1.2458 |
| USA1986 | 0.5708 | 6.6938 | 2.2654 | 1.0208 | 1.1420 | 10.0900 |
| USA1987 | 0.5729 | 6.8425 | 2.2780 | 1.0158 | 1.1839 | 10.7386 |
| USA1988 | 0.5807 | 6.9756 | 2.2904 | 1.0153 | 1.2321 | 11.6059 |
| USA1989 | 0.5855 | 7.0676 | 2.3028 | 1.0151 | 1.2848 | 12.4286 |
| USA1990 | 0.5852 | 7.0805 | 2.3130 | 1.0127 | 1.3416 | 13.0207 |
| USA1991 | 0.5850 | 6.9954 | 2.3176 | 1.0164 | 1.3819 | 13.3215 |
| USA1992 | 0.5917 | 7.0733 | 2.3240 | 1.0176 | 1.4205 | 14.0605 |
| Japan1986 | 0.3982 | 4.0370 | 1.7855 | 1.0408 | 1.6039 | 4.7915 |
| Japan1987 | 0.3496 | 4.5544 | 1.8888 | 1.0438 | 1.8376 | 5.7686 |
| Japan1988 | 0.3195 | 5.0777 | 1.9852 | 1.0456 | 2.0554 | 6.9219 |
| Japan1989 | 0.3499 | 4.8551 | 1.9792 | 1.0401 | 1.9627 | 6.8629 |
| Japan1990 | 0.3800 | 4.7221 | 1.9859 | 1.0315 | 1.9092 | 7.0176 |
| Japan1991 | 0.3583 | 4.9804 | 2.0615 | 1.0357 | 2.1124 | 8.0490 |
| Japan1992 | 0.3416 | 5.1256 | 2.1276 | 1.0395 | 2.2781 | 8.8220 |
| Canada1986 | 0.5994 | 1.1942 | 1.0523 | 1.0101 | 1.0940 | 0.8324 |
| Canada1987 | 0.5546 | 1.2610 | 1.0831 | 1.0210 | 1.2291 | 0.9505 |
| Canada1988 | 0.5028 | 1.3595 | 1.1243 | 1.0267 | 1.4094 | 1.1122 |
| Canada1989 | 0.4777 | 1.4104 | 1.1547 | 1.0312 | 1.5442 | 1.2388 |
| Canada1990 | 0.4900 | 1.3281 | 1.2421 | 1.0227 | 1.5739 | 1.3010 |
| Canada1991 | 0.4805 | 1.3192 | 1.2603 | 1.0180 | 1.6349 | 1.3296 |
| Canada1992 | 0.5063 | 1.2730 | 1.2487 | 1.0160 | 1.5646 | 1.2793 |
| Australia1986 | 0.5885 | 0.7329 | 0.8007 | 1.0093 | 1.1529 | 0.4019 |
| Australia1987 | 0.5546 | 1.2610 | 1.0831 | 1.0210 | 1.2291 | 0.9505 |
| Australia1988 | 0.5028 | 1.3595 | 1.1243 | 1.0267 | 1.4094 | 1.1122 |
| Australia1989 | 0.4777 | 1.4104 | 1.1547 | 1.0312 | 1.5442 | 1.2388 |
| Australia1990 | 0.4900 | 1.3281 | 1.2421 | 1.0227 | 1.5739 | 1.3010 |
| Australia1991 | 0.4805 | 1.3192 | 1.2603 | 1.0180 | 1.6349 | 1.3296 |
| Australia1992 | 0.5063 | 1.2730 | 1.2487 | 1.0160 | 1.5646 | 1.2793 |

**TABLE 6**
**Multilateral Comparisons, with China 1990 as the base – (Calculated using Price data adjusted by Pseudo-PPPs)**

| | Productivity | Labor Quantity | Capital Quantity | TOT | Domestic Price | Nominal GDP ratio |
|---|---|---|---|---|---|---|
| China1986 | 1.3239 | 0.8118 | 0.6936 | 1.0009 | 0.5317 | 0.3967 |
| China1987 | 1.3041 | 0.8580 | 0.7423 | 0.9941 | 0.6000 | 0.4955 |
| China1988 | 1.2060 | 0.9091 | 0.8432 | 0.9900 | 0.7544 | 0.6905 |
| China1989 | 1.0828 | 0.9541 | 0.9314 | 0.9882 | 0.8942 | 0.8503 |
| China1990 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| China1991 | 0.9778 | 1.0516 | 1.0620 | 1.0037 | 1.1339 | 1.2427 |
| China1992 | 0.9809 | 1.0809 | 1.1737 | 0.9906 | 1.3310 | 1.6407 |
| USA1986 | 1.3420 | 4.3858 | 1.4679 | 1.0274 | 0.4826 | 4.2836 |
| USA1987 | 1.3061 | 4.5806 | 1.4900 | 1.0208 | 0.5167 | 4.7022 |
| USA1988 | 1.2709 | 4.8043 | 1.5167 | 1.0186 | 0.5611 | 5.2931 |
| USA1989 | 1.2289 | 5.0094 | 1.5453 | 1.0171 | 0.6110 | 5.9114 |
| USA1990 | 1.1789 | 5.1617 | 1.5722 | 1.0135 | 0.6654 | 6.4519 |
| USA1991 | 1.1381 | 5.2328 | 1.5888 | 1.0163 | 0.7104 | 6.8318 |
| USA1992 | 1.1212 | 5.3905 | 1.6057 | 1.0171 | 0.7501 | 7.4043 |
| Japan1986 | 1.3377 | 2.0988 | 1.0433 | 1.0450 | 0.4786 | 1.4650 |
| Japan1987 | 1.3802 | 2.1012 | 1.0536 | 1.0427 | 0.4665 | 1.4861 |
| Japan1988 | 1.4260 | 2.1388 | 1.0685 | 1.0416 | 0.4609 | 1.5644 |
| Japan1989 | 1.4277 | 2.1913 | 1.0934 | 1.0402 | 0.4827 | 1.7173 |
| Japan1990 | 1.4456 | 2.2255 | 1.1163 | 1.0318 | 0.4988 | 1.8483 |
| Japan1991 | 1.4291 | 2.2764 | 1.1446 | 1.0360 | 0.5287 | 2.0398 |
| Japan1992 | 1.4181 | 2.2748 | 1.1677 | 1.0392 | 0.5475 | 2.1433 |
| Canada1986 | 1.3567 | 0.7702 | 0.6840 | 1.0250 | 0.4765 | 0.3491 |
| Canada1987 | 1.3080 | 0.8112 | 0.7028 | 1.0324 | 0.5156 | 0.3969 |
| Canada1988 | 1.2713 | 0.8516 | 0.7203 | 1.0381 | 0.5515 | 0.4465 |
| Canada1989 | 1.2186 | 0.8851 | 0.7409 | 1.0432 | 0.5985 | 0.4989 |
| Canada1990 | 1.1644 | 0.9047 | 0.7561 | 1.0396 | 0.6462 | 0.5351 |
| Canada1991 | 1.1391 | 0.8980 | 0.7669 | 1.0355 | 0.6725 | 0.5462 |
| Canada1992 | 1.1322 | 0.9025 | 0.7736 | 1.0315 | 0.6835 | 0.5573 |
| Australia1986 | 1.2680 | 0.5003 | 0.5339 | 1.0156 | 0.5332 | 0.1834 |
| Australia1987 | 1.2043 | 0.5328 | 0.5505 | 1.0271 | 0.6033 | 0.2189 |
| Australia1988 | 1.0054 | 0.7940 | 0.6921 | 1.0776 | 0.4493 | 0.2675 |
| Australia1989 | 0.9374 | 0.8488 | 0.7137 | 1.0710 | 0.5087 | 0.3094 |
| Australia1990 | 0.9506 | 0.6466 | 0.6037 | 1.0409 | 0.8574 | 0.3312 |
| Australia1991 | 0.9621 | 0.6412 | 0.6099 | 1.0355 | 0.8967 | 0.3493 |
| Australia1992 | 0.8959 | 0.8851 | 0.7409 | 1.0432 | 0.5985 | 0.3668 |

## V. *Conclusion*

This paper uses the combination of two different newly developed approaches to deal with the usual problems arising in multilateral comparisons. Alternative uses and aspects of the application of these approaches have been proposed and discussed. Special attempts have been made to take account of the nature of the available data.

The combination of Hill's (1999a,b) spanning tree methodology and the Fox (1999) decomposing technique forms a new technique for multilateral comparisons. The former uses the Paasche-Laspeyres spread (PLS) index to provide a possible way of linking, whereas the latter decomposes GDP differences among countries over time. Panel data of five countries, including China, the US, Canada, Australia and Japan has been used for numerical illustration.

This paper extends Hill's MST approach to provide a way of ordering panel data. The structure of the resulting MST shows that there is a general tendency for observations of the same countries to form a cluster by themselves. Fox (1999) decomposing technique is used to decompose the differences of GDP among observations. It allows the assessment of the impacts of TOT, factors and prices in contributing to differences in GDP across countries and over time. In this paper, the combination of these two methods are applied to panel data and the properties of this newly formulated method in a panel context has been evaluated.

The resulting multilateral indexes, which are formed using the linking up of direct bilateral indexes indicated by the structure of the MST are base-invariant. This provides strong support for using the newly formulated method for multilateral comparisons as it can give a cardinal scaling of observations (In this paper, they are particular countries at particular time periods) that is independent of the choice of the base observation.

Nevertheless, the resulting decomposition indexes may change if new observations are added or old observations are removed when constructing the MST, as the number of intermediate comparisons for some indirect comparisons may increase or decrease, depending on the structure of MST.

Certain problems on the construction of MST and multilateral indexes with data expressed in index format have been discussed in this paper as well. Exchange rates and pseudo-PPPs are used as the two conversion factors for national data. Calculation of the pseudo-PPPs has carried out using the basics of the Geary-Khamis (GK) method. To take account of the nature and type of the available data, variants of the GK equations have been proposed. Such an alternative may have different economic implications. Indeed, the accuracy and usefulness of the application of this GK variation needs furthered research.

Regarding the decomposition results, some differences are found from the two sets of results (price being adjusted by exchange rate and PPPs respectively). Almost all the domestic price indexes have magnitudes greater than one when price are adjusted by exchange rates, whereas when prices are adjusted by PPPs, the opposite happens. With productivity indexes, it has been found that the magnitudes are mostly greater than one when prices are adjusted by PPPs whereas mostly smaller than one when prices are adjusted by exchange rate. Nevertheless, some similarities of the two sets of results are found as well. TOT effects are similar in their magnitudes, which are mostly greater than one in both sets of results.

Different factors are found to have contributed to different countries' GDP. Productivity is generally found to be a major component of China's GDP, and its magnitude is relatively large compared to those in other countries over time when prices are adjusted by exchange rates. However when prices are adjusted by PPPs, labor, capital, TOT and domestic price effects are instead found to have direct influences on GDP.

Labor and capital are found to have contributed significantly to the GDP in Japan and the US, regardless of the conversion factors. For Japan and the US, although the contributions of TOT and domestic price are relatively smaller compared to the labor and capital effects, they all show positive contributions to larger GDP when prices are adjusted by exchange rates. However, when prices are adjusted by PPPs, TOT and productivity are found to have contributed positively to the higher GDP instead.

For Australia, TOT is found to be an insignificant factor affecting GDP, when prices are adjusted by PPPs. However, it is a significant factor when prices are adjusted by exchange rates. When prices are adjusted by exchange rates, labor, capital, TOT and domestic prices are found to have positively to higher GDP for Australia. For prices adjusted by PPPs, productivity, labor, capital and domestic prices are the main factors for affecting GDP instead.

For Canada, productivity effects are found to be the only insignificant contributor to GDP when prices are adjusted by exchange rates. When prices are adjusted by PPPs, both productivity and TOT are not significant contributors to GDP.

Regardless of the conversion factors, Australia and Canada are found to be similar in the main factors affecting GDP. Japan and the U.S., are found to be similar in this respect as well. China, on the other hand, has no particular similarities to any of these OECD countries.

## *Appendix 1. Justification of Adjustment of Quantity in the Calculation of Translog Quantity Indexes*

When calculating the quantity indexes for the decomposition of GDP differences between observations, there is a need to convert the quantities of each observation into a common unit. This can be done by multiplying the quantity of each observation, i.e. $X_{ja}$ and $X_{jb}$ in equation (14) by the relevant conversion factors (E).

$$X_{ja,b} = \exp \{ ½ (S_{la} + S_{lb}) \ln [\frac{X_{lb}}{X_{la}} \bullet E] + ½ (S_{ka} + S_{kb}) \ln [\frac{X_{kb}}{X_{ka}} \bullet E]\},\ \ j \in \{ L,K \}$$

$$= \exp \{ ½ (S_{la} + S_{lb}) \{\ln [\frac{X_{lb}}{X_{la}}] + \ln E\}\} \bullet \exp \{ ½ (S_{ka} + S_{kb}) \{\ln [\frac{X_{kb}}{X_{ka}}] + \ln E\}\}$$

$$= \frac{X_{lb}}{X_{la}}^{½ (Sla + Slb)} \bullet E^{½ (Sla + Slb)} \bullet \frac{X_{kb}}{X_{ka}}^{½ (Ska + Skb)} \bullet E^{½ (Ska + Skb)}$$

$$= \frac{X_{lb}}{X_{la}}^{½ (Sla + Slb)} \bullet \frac{X_{kb}}{X_{ka}}^{½ (Ska + Skb)} \bullet E^{½ (Ska + Skb + Sla + Slb)}$$

$$= \frac{X_{lb}}{X_{la}}^{½ (Sla + Slb)} \bullet \frac{X_{kb}}{X_{ka}}^{½ (Ska + Skb)} \bullet E^{1} \qquad \text{(A1)}$$

$S_{la} + S_{lb} + S_{ka} + S_{kb} = 2$ as $S_{la} + S_{ka}$ and $S_{lb} + S_{kb}$ are both equal to one.

Similar to the quantity index, the price index in equation (11) can also be rewritten as:

$$P_{a,b} = \frac{P_{zb}}{P_{za}}^{½ (Sza + Szb)} \bullet E^{1} \qquad z \in \{ c, i, g, x, m \} \qquad \text{(A2)}$$

c, i, g, x and m stand for private consumption, investment, government consumption, export and import, respectively.

Recall that before conversion to a common unit, $\Gamma = \frac{GDP_a}{GDP_b} = R \bullet P \bullet X$ (A3)

where Γ is the GDP ratio of two countries, with each GDP expressed in national currencies. R represents the productivity index, P represents the price index and X represents the quantity index.

Then, we use the conversion factor, E to convert the GDP of both countries into a common currency. Correspondingly, there is a need to adjust each component in the RHS of equation (A3) using the conversion factor:

$$\Gamma_1 = \frac{GDP_b}{GDP_a} \bullet E = (R \bullet P \bullet X) \bullet E \qquad \text{(A4)}$$

According to equations (A1) and (A2), equation (A4) can be re-written as:

$$\Gamma_1 = \frac{GDP_b}{GDP_a} \bullet E = (R/E*E) \bullet (P \bullet E) \bullet (X \bullet E) \qquad \text{(A5)}$$

where $\Gamma_1$ is the adjusted GDP ratio, (P • E ) and (X • E) are the price index and quantity index adjusted by conversion factor respectively. We have shown in equations (A1) and (A2) that after conversion, the price index and quantity index become (P*E) and (X*E) respectively. (R/E*E) acts as the residual, which counteracts the effects of the conversion factors in (P • E ) and (X • E), so that $\frac{GDP_b}{GDP_a}$ is equal to **(**R/E*E) • (P • E) • (X•E). So it is clear that besides converting the prices of countries in comparison when calculating the price index, there is also a need to convert the quantities of countries when calculating the quantity indexes as well.

### *Appendix 2. Adjustment to enhance the Consistency of the Two Data Sets*

Certain adjustments have been done to make the OECD and China data sets comparable. First, both data sets are normalised as that quantities are in 1985 domestic currency. Second, due to the method of derivation of the capital series,[24] the share of labor and share of capital of GDP add up to one for each country in the OECD data set. To be consistent with the OECD data set, the labor share and capital share of national income for China is taken as proxy for the labor and capital shares of GDP for China. The values of labor and capital are calculated as the product of GDP and the respective labor and capital shares. Quantity of capital is calculated using the procedure of Kohli (1982). A description of the method is provided in Appendix 3 below. Since there is the absence of independent estimates of depreciation, an assumed depreciation rate of 5% is used. The initial 1985 capital stock in 1985 constant price is estimated to be equal to 1701.44 billion yuan.

***Appendix 3. Calculation of the Capital Series***

To obtain the quantity of capital services, we used the method used by Kohli (1982) and Fox and Kohli (1998):

Two assumptions are made:

(a) The flow of capital services is proportional to the beginning-of-period capital stock and this stock can be obtained by accumulating real gross investment subject to a constant depreciation rate, δ.

$$X_{kt} = (1\text{-}\delta)X_{kt\text{-}1} + Y_{1t\text{-}1} \qquad t = 1985\text{-}1992 \qquad \text{(A6)}$$

Which can be written as $X_{kt} = Y_{it-1} + (1\text{-}\delta)Y_{it-2} + (1\text{-}\delta)^2 Y_{it-3} + \ldots\ldots, \quad t = -\infty, \ldots.., 1992$

(A7)

(b) Prior to some period T ≤ 1985, real investment grew at a constant rate, γ

$$Y_{it} = Y_{it\text{-}1}(1+\gamma) \qquad t = -\infty, \ldots\ldots, T \qquad \text{(A8)}$$

By making use of equations (A7) and (A8) valued at time T, $X_{kt}$ at 1985 can be obtained by:

$$X_{kt} = \frac{Yit}{1+g}\left[1 + \frac{1-d}{1+g} + \left(\frac{1-d}{1+g}\right)^2 + \ldots\ldots.. \right]$$

[24] See p.8 in Fox (1997) for the calculation of the capital series for the OECD data set.

$$= \frac{Yit}{1+g} \left[ 1 / 1 - \frac{1-d}{1+g} \right]$$

$$= \frac{Yit}{d+g} \qquad \text{(A8)}$$

Since there is no corresponding official data on real investment before the year 1985, γ is calculated as the within sample average rate of growth of real investment from 1985 to 1992, which is equal to 10.52%. δ is set to 5%. The resulting 1985 real capital stock is found to be 1701.44 billions of 1990 yuan.

## *Appendix 4. Problem Associated with Data Expressed in Index Format when Constructing the Minimum Spanning Tree*

If the data are expressed in index format, with one year acting as the base year, i.e. the price for each variable in that year is equal to 1, then there is a problem when constructing the MSTs. For example, if 1985 is treated as the base year, when more than one country is considered, the 1985 entry for one country against another in the PLS indexes will be equal to 1. Taking the log of the PLS matrix, the 1985 entry of one country against another country will be equal to 0. This has the effect of avoiding comparisons between these two countries in 1985 when using the Spanning Tree algorithm, i.e. countries in 1985 will never be linked together in a spanning tree if their price series are indexes based in 1985. This is same as setting a restriction to avoid comparison between the countries in 1985, which is not what we intend to do. This problem exists as long as the data are expressed in index format.

***Appendix 5. Minimum Spanning Trees***

**FIGURE 1A**
**Minimum Spanning Tree (Prices adjusted by exchange rates)**

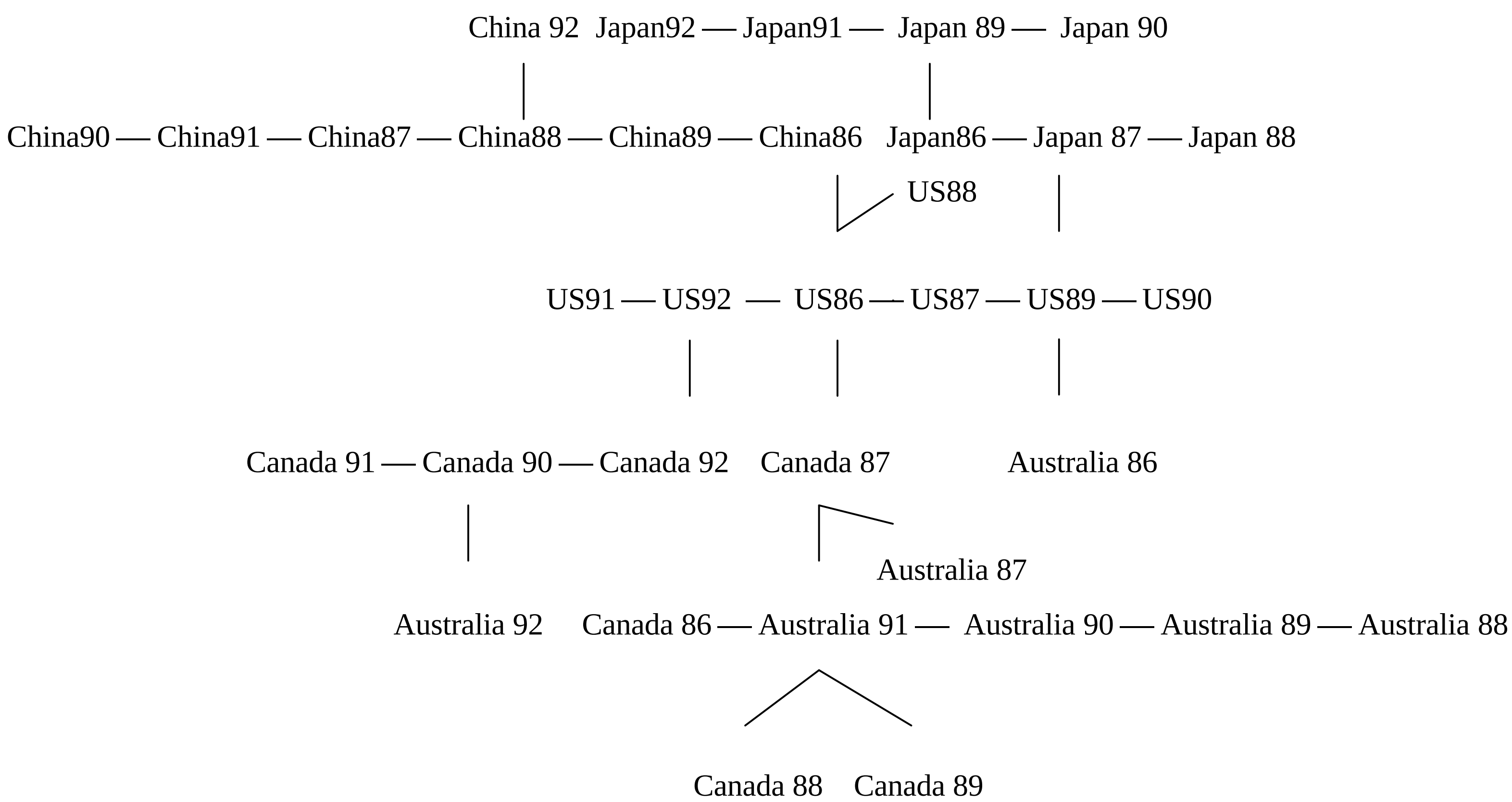
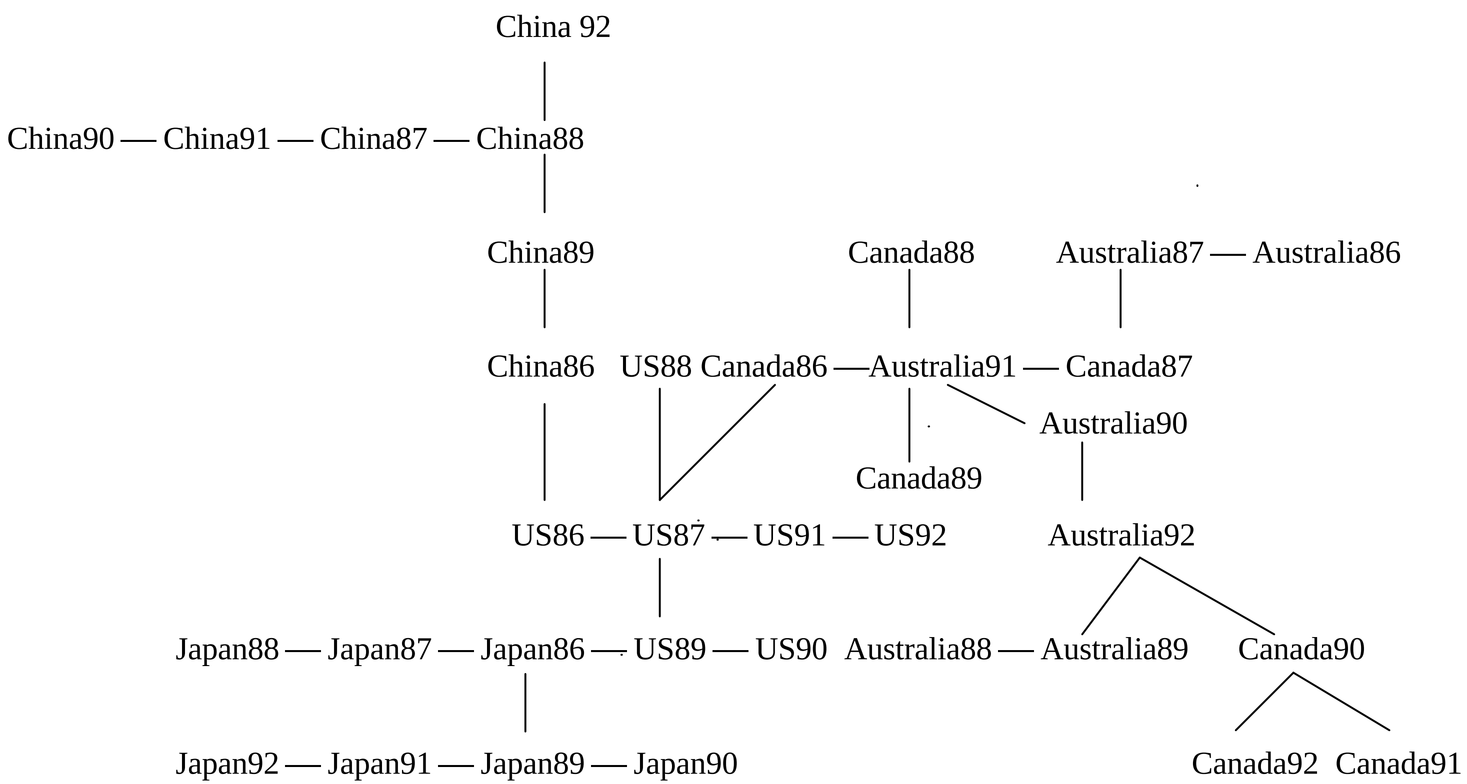


**FIGURE 1B**
**Minimum Spanning Tree (Prices adjusted by PPPs)**



*Appendix 6. Different Types of Spanning Trees*

**FIGURE 2A**
**A String Spanning Tree**

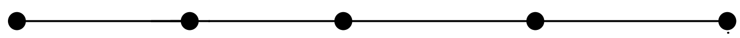
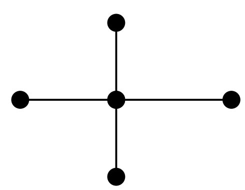


**FIGURE 2B**
**A Star Spanning Tree**



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