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On Capital and Productivity: Harrodian and Keynesian Measures

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by

René Durand and Thomas K. Rymes

Introduction

When measuring total factor productivity, it is often implicitly if not explicitly assumed that one is in a Classical or Marshallian economy where, aside from temporary equilibria aberrations, the inputs of labour [working], capital [waiting] and land [natural agents] are fully employed. Here, we more deeply explore the concepts of capital employed in Harrodian productivity measurement, look at problems of measurement in temporary equilibria and begin an approach to the problem of how should total factor productivity be measured in a Keynesian world where unemployment of working and waiting is the more general case, where monetary policy has an effect on both productivity growth and its measurement.

The standard Hicks-Solow-Jorgenson measure treats capital as if it were a primary non-produced input. It is an endogenous intermediate input. The capital input in its primary form is the flow of waiting. Part 1 of the paper reviews alternative measures of productivity growth which consider capital as endogenous. These measures provide a more comprehensive account of the impact of technical progress on output growth than the standard neoclassical approach which neglects to take into account the productivity gains associated with the production of capital goods. Regardless of what assumption is made about the exogeneity or endogeneity of technical progress, it remains the case that in measures of such technical change, the more theoretically acceptable, comprehensive in a complete accounting sense, requires that the measures of total factor productivity are Harrodian[1].

1. See Rymes, "On the Cambridge Correction to the Measurement of Productivity in Manufacturing", a paper presented to the CSLS Conference on the Canada-US Manufacturing Productivity Gap, Ottawa, 12-22 January, forthcoming in A. Sharpe, J. Berstein and R. Harris, **THE CANADA-US MANUFACTURING PRODUCTIVITY GAP** (Montreal: McGill-Queen's University Press) in which arguments, criticizing the standard or Hicks-Solow-Jorgenson measures of total factor productivity, advanced by Barro and Sala-I-Martin and Edward Denison are reported. Support for the Harrodian measures of total factor productivity is also found in Charles Hulten and Sylaja Srinivasan, "Indian manufacturing industry: Elephant or tiger? New evidence on the Asian miracle, NBER Working Paper 7441, December 1999 and Charles R. Hulten, "Total factor productivity: A short biography", NBER Working Paper 7471, January 2000.

In measures of total factor productivity, technology and its advance is either exogenous as in the early Solow growth model or endogenous in that the measured rate of productivity advance may well be a function of the rate of capital accumulation, either reproducible or human capital embodying the latest in information technology.

Growth accounting must deal with general problems. First, the Hicks-Solow-Jorgenson measures of total factor productivity or sources of growth must be replaced by the Harrod measures simply because the capital input in the traditional measures is endogenous, is determined by techniques, technical progress and the fundamentals of working and waiting and therefore cannot be a separate source of growth. Of course, if new techniques are embodied in the new capital goods, then investment is required for growth but that merely gives rise to (i) problems involved in the measurement of quality change in the new capital goods and (ii) problems involved in the measurement of depreciation or obsolescence. The measurement and the concept of technical change taking place remains Harrodian, the increasing efficiency of working and waiting. We argue this problem has been resolved in favour of the Harrod measures.

As Hicks himself said (Hicks, 1973, 182-3), if saving or waiting is allowed, if reinvestment is not treated as an independent (exogenous) variable, his own classification of (neutral) technical change

> "…can have nothing to do with an equilibrium that is reached by saving… If there is any sense in it, it must refer to a much shorter run".

Hicks also said (120):

> "When, however, we come to history – or to applied economies – should we not go further? I have so far been telling the story in the conventional terms, of shifts in technology and switches within the technology; but, at the point we have reached, do not the 'technology' and the 'technological frontier' themselves become suspect? They are essential tools of static analysis' but in dynamic analysis, such as this, do we need them? We should be much happier without them. The notion of a 'technology', as a collection of techniques, laid up in a library (or museum) to be taken down from their shelves as required, has been deservedly criticized; in itself it is a caricature of the inventive process. Let us try to get rid of it.
>
> Why should we not say that every change in technique is an *invention*, which may be large or small? It certainly partakes, to some degree, of the character of an invention; for it requires, for its application, some new knowledge, or some new expertise. There is no firm line, on the score of novelty, between shifts that change the technology and shifts that do not."

Hicks's critique of his own concept of the distinction between shifts and movements along production functions is fundamental. It means that our productivity measures through time must be Harrodian.[2] Everything from temporary equilibria to steady states as reflected in the System of National Accounts has to be accounted for in Harrodian terms where waiting is the primary input associated with the returns to capital.

In Part 1 of the paper (mostly written by Durand), we are concerned with measures of total factor productivity, where the flow of waiting services is the primary input associated with returns to capital while in Part 2 (largely written by Rymes), we turn our attention to Keynesian problems where the concept of the flow of liquidity services as a part of the flow of the services of waiting is examined.

Introduction to Part 1

Read and Rymes (1968) introduced a measure of technical progress based on Harrod's [and Robinson's] theory which takes into account that, in an economy experiencing advance in technical knowledge, the growing stock of capital goods are produced with an ever increasing efficiency. Capital goods appear as produced (endogenous) inputs rather than as non-produced (exogenous) *primary* inputs. The production of capital as with all other goods requires the application of basically two primary inputs: labour and waiting. (We ignore natural agents. See Rymes, 1993)

Waiting as an input corresponds to the fact that private individuals by saving, are postponing present consumption in exchange for future consumption. Their non-consumption permits the maintenance and the augmentation of the stock of capital. Capital goods, because they are produced, are directly subject to the impact of technical progress. Technical progress changes both the quality and the quantity of capital goods that may be produced through time with the same quantity of labour and waiting inputs.

The impact of technical progress on capital accumulation has also been recognized by Hulten. Trying to reconcile Rymes with the neoclassicals, Hulten (1979, 1992, 2000) has reformulated Rymes' model of productivity growth in a dynamic framework in which technical progress is seen as both shifting the production function as would the neoclassicals would have it and fostering the growth of the stock of capital as Rymes has suggested - seen by Hulten as a move along the production function rather than as a shift in the production function (see also Hulten 1993).

Hulten (1979, 1992), however, states that there exists only one primary input, namely labour, and that capital goods are intermediate inputs that can be transformed dynamically into labour inputs. Hence, for Hulten, multifactor productivity (in his Harrodian case) reduces to what we may call *total* labour productivity, as opposed to the partial measure of labour productivity in which capital appears as a separate primary input as in the neoclassical (Hicksian) model[3].

The latter school of thought has always had the view that the stock of capital goods is a distinct input treated symmetrically to labour and intermediate inputs. The industry production function is assumed to be perfectly represented by a function having gross

2. It will also be noted in relation to the reswitching component of the Cambridge Capital Controversy that Hicks says as well (120) that "…on this plane of discourse there can, *by definition,* be no re-switching. Every technique to which a switch is made is a new technique. In practice, isn't it?" Thus, one need not necessarily be concerned with the relationship, if any, between rates of return to capital and static overall measures of capital intensity but rather should be concerned with the relationships between rates of return and rates of growth of capital accumulation associated with rates of growth of Harrodian technical progress. See Rymes 1989.

output on one side of the equation and the intermediate and primary inputs, all treated symmetrically and without distinction, on the other side of the equation. All inputs can all be taken as exogenously given in the computation of productivity growth, including capital services. Yet the neoclassical school of thought considers that the aggregate production process for the entire economy or its business sector is more adequately represented by a production function having real value added on the output side and only capital and labour on the input side. How that transformation of the production process occurs from the disaggregated to the aggregated level has never been satisfactorily explained by the tenants of that school.

In Part 1 of the paper, Rymes' and Durand's (1996) alternative formulation of the waiting model are discussed first. Hulten's dynamic residual is scrutinized next. Problems are encountered in the relationship between the dynamic index and the maximization of welfare, namely the fact that Hulten (1992, section III) does not use the same concept of net output and value-added when discussing the welfare maximization problem when deriving his dynamic index (in section IV). The connection he attempts to establish between welfare maximization and his dynamic residual is not tenable. Finally, that connection is re-established using a new formulation of the dynamic residual. We conclude that the new productivity residual, defined on net income (wage and interest income), is also a welfare residual so that net domestic product is the most appropriate measure of output for both productivity and welfare analysis. Part 1 of the paper also introduces an alternative model of capital (waiting) services that extend the Berndt-Fuss (1986) approach in order to take into account potential under utilization of capital over the short-run so that the productivity residual accounts for technical progress (change in the production possibility frontier) exclusive of efficiency change (move to or away from the frontier). That productivity residual is considered further in Part 2 of the paper where monetary problems are discussed[4].

Background

Using the simple one sector one commodity model, Read and Rymes (1968), Rymes (1971, 1983) and Cas and Rymes (1991) deduct the rate of growth of technical progress, τ_R, from the rate of growth of the stock of capital, K, in order to obtain the measure of the flow of waiting services, those primary inputs which earn net returns to capital, K_R:

$$\dot{K} = \dot{K}_R + \tau_R \qquad (1)$$

3. Indeed, "Consumption is treated as a direct delivery of final demand, and labour is regarded as the only primary input" Hulten (1979, 128), except for the initial capital imported from the past: "The import of capital from the past... is treated as a primary input since it is not produced within the economic system." Hulten (1979, 129).
4. Though we have greatly benefitted from reading Steven Keuning (1999), here we restrict our analysis to the services of banks as inputs in production.

where a dot over a symbol indicates a time percentage rate of growth. Equation (1) may be seen as a growth accounting equation applied to the (net) stock of capital taken as a measure of current period output and where inputs are givent by the flow of waiting, K_R, the latter being a measure of all primary inputs (both capital services measured in input units and labour services) required to produce the stock of capital, using the current year technology. Capital is thereby transformed into waiting units using the current year technology only. The difference between K and K_R, therefore, is the current year rate of technical progress, τ_R.

According to Rymes's fundamental equation (1) of the flow of waiting services, technical progress at the aggregate economy level is given by (see Rymes, 1983):

$$\tau_R = \dot{Q} - \alpha\dot{L} - (1-\alpha)(\dot{K} - \tau_R) \tag{2}$$

where α is the nominal labour income share in the value of aggregate output. Solving for technical progress, and letting τ stand for the conventional measure of technical progress, one has: (See also Lucas, 1988, 10)

$$\tau_R = \frac{\tau}{\alpha} \tag{3}$$

That equation holds in both the steady and non-steady state cases. The aggregate income-expenditure equation standing behind equation (2) may be written as:

$$pQ = wL + rK = wL + r_R K_R \tag{4}$$

where r and r_R are, respectively, the annual *gross* rental price of capital goods and waiting units. Differentiating (4) totally with respect to time leads to the following dual productivity equation:

$$\tau_R = \alpha\dot{w} + (1-\alpha)\dot{r}_R - \dot{p} \tag{5}$$

Measuring capital in waiting rather than in output units does not change its value on the market, nor its rental cost so that one has both:

$$p_R K_R = pK \quad \text{and} \quad r_R K_R = rK \tag{6}$$

where p (p_R) is the price of capital goods (waiting units) from which one obtains, through differentiation and using (1):

$$\dot{K} - \dot{K}_R = \dot{p}_R - \dot{p} = \dot{r}_R - \dot{r} = \tau_R \tag{7}$$

But since the price of waiting services grows at a rate that exceed the price of output by the rate of technical progress, one has:

$$\tau_R = \alpha\dot{w} + (1-\alpha)(\dot{r} + \tau_R) - \dot{p} \tag{8}$$

which solves, once more, for

$$\tau_R = \frac{\tau}{\alpha} \tag{9}$$

Note that in this interpretation of Rymes, both the quantity and the price of waiting services are in long-run equilibrium. That equilibrium is characterized by unchanging waiting/labour ratios and relative prices of the primary inputs along the steady state growth path[5]. The accumulation process becomes dominated by technical progress so that capital accumulation has no separate role in growth. In addition, multifactor productivity growth becomes identical to labour or waiting productivity growth. This particular result has been at the origin of much confusion about the Rymes' model and has sometimes lead to the erroneous interpretation that it reduces capital to indirect labour or that Harrod-Robinson neutral technical progress is labour augmenting only.

In non-steady state growth with full employment of both inputs, capital is considered as sufficiently variable for equilibrium to be maintained and the ratio of waiting to labour adjusts to their changing relative prices. Multifactor productivity then differs from labour productivity and waiting clearly appears as an independent and distinct source of growth.

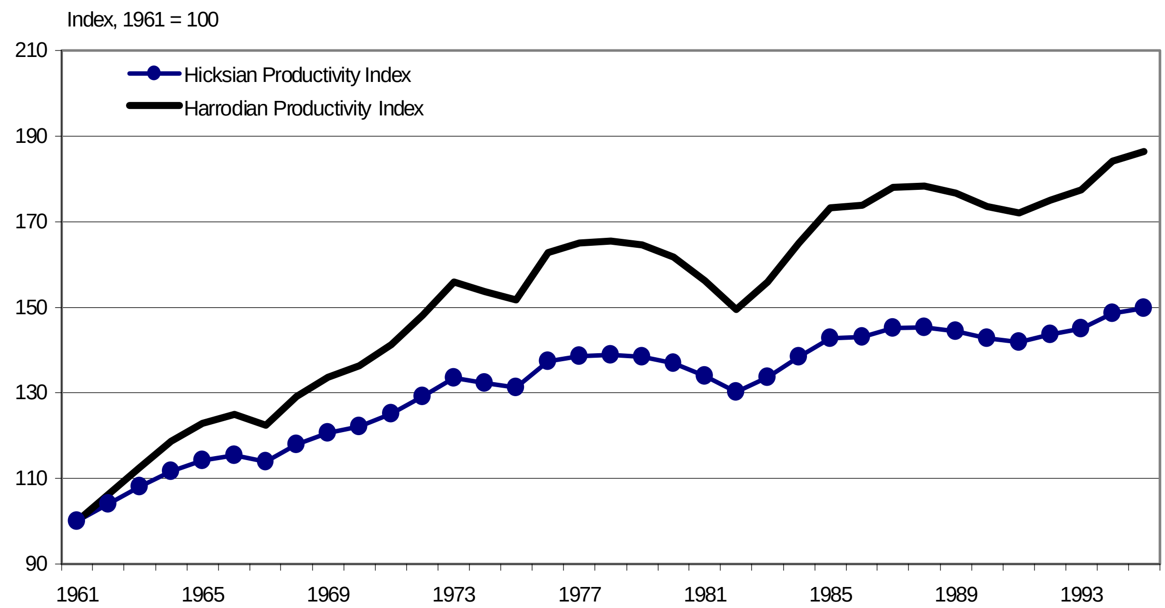
Durand's (1996) dynamic index of productivity growth was based on Rymes's concept of waiting briefly described above. The distinction with Rymes's index of productivity came from the recognition that the stock of waiting is accumulated through time as new investments add to the stock of capital. Subtracting the rate of productivity growth from the rate of growth of these investments, Durand obtained the rate of growth of the associated input requirements. These inputs were defined as the waiting inputs. As the investment goods are not immediately used up but carried forward into the future, the waiting inputs are similarly consumed in the following periods and depreciated in parallel to investment goods. Hence Durand accounted for the accumulated flow of inputs carried into the future (the stock of waiting), using the perpetual inventory method. The stock of waiting was thus seen as the stock of capital measured in the input units that were required to produce it *historically* rather than, as Rymes, by the amount of primary inputs that would be required in the current period to reproduce the whole stock of capital, using the *current* period technology only. These views can be reconciled fully, however, when it is recognized that waiting inputs depreciate both as capital goods depreciate when the

5. This is a result rather than an assumption and, outside the steady state growth path, the input ratios and their relative prices are variable.

latter experience wear and tear and become obsolete and as a result of technical progress in the production of capital goods which reduces the waiting requirements per unit of capital, thereby adding to the depreciation of previously accumulated waiting units[6].

As an illustration of the importance of taking into account the endogenous nature of capital in measuring productivity growth, the following chart presents data for the Canadian business sector.

Harrodian and Hicksian Indices of Productivity Growth for the Canadian Business Sector, 1961=1995



A neoclassical perspective on waiting

Capital accumulation is not only due to the fact that capital goods can be produced over time with increasing efficiency. Accumulation per labour would occur even if capital producing industries would register no technical progress either because of substitution of capital to labour (along non-stationary growth paths) or because technical progress occurs in the production of consumption goods that makes profitable permanent increases in the capital/labour ratio (along either steady or non-steady growth paths). There is no absolute need, therefore, to account for capital accumulation through an

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6. It can be shown that setting the rate of depreciation on waiting, δ_R to $\delta+\tau_R$ where δ is the rate of depreciation on capital reconciles the two formulations of waiting as the accumulated waiting units based on older technology are transformed in that manner and in every subsequent period into equivalent waiting units based on the current period technology. Proofs can be obtained from the authors on request.

accounting framework that transforms capital goods, an output of the production system as well as one of its input, into some underlying measure of inputs associated with the production of capital goods. The latter may be accounted for in output units, such as the number of machines or capital goods in constant prices, rather than in input units, such as the saving or waiting units and labour inputs applied to their production.

From the restricted perspective of firms, capital goods are given at the beginning of each period and capital services that are obtained from these capital goods can be considered as exogenous to the same extent as the labour inputs they purchase, unless there is own-account production of capital goods such as goods in progress. From that perspective, waiting (or saving) is not an input into the production of commodities. Capital goods are outputs of the previous periods that were delivered to final demand, i.e. to the households or assets holders. They were induced to save more than they would have otherwise done without technical progress and their assets are rented to the business sector for use in the current period[7].

One may, however, integrate the household sector with the business sector and look at the global impact of technical progress by summing both of the previous impacts before hand: its impact on the production process and its impact on the accumulation of capital. Both of these impacts may be said to support growth in output. Output growth, therefore, might be decomposed as a function of the growth in the labour, waiting and technical progress, or as a function of the growth in the direct and indirect labour and technical progress in a pure labour theory perspective [8].

Hence, in that integrated view of the business and the household sectors, waiting rather than capital may be seen as the primary input that, together with labour, sustains output growth; the impact of technical progress may be seen as a pure shift in the production function rather than as both a shift of the production function and a move along that function through further capital accumulation as per Hulten.

The last point is of extreme importance as it brings a fundamental difference between neoclassical thinking together with the alternative formulation of that model presented here, on the one hand, and the labour theory of value, on the other hand. In the latter case, technical progress acts both directly and indirectly on output growth without households' saving behaviour playing any role in both steady and non-steady state growth[9]. In the neoclassical framework or its alternative Rymesian formulation, technical progress dominates the scene in steady state growth. However, the saving behaviour of households also plays a crucial role in non-steady state growth.

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7. One extremely important aspect of that process is that capital deepening, along the steady state growth path, is independent of the rate of saving or any other measure depicting saving behavior. Although households save, their saving behavior per se, has no influence on the growth path of output, capital, labour or any other state variable characterizing the economy.
8. The pure labour view nevertheless seems to neglect that the willingness of households to save rests on their receiving some capital income in exchange of the postponement of their present consumption for future consumption.
9. Only if their model remains defined in non-steady state growth, which we cast doubts on below.

Hulten's model

Hulten (1979, 1992), in a dynamic framework, obtains a measure of welfare growth by considering the stock of capital as an intermediate input. Welfare (defined as total wealth or the discounted value of the future consumption stream) is maximized under the standard neoclassical production function constraint. The welfare residual, therefore, encompassed the whole future time horizon rather than covering a single period as the other indices described above. As mentioned above, labour is considered as the sole primary input. In Hulten (1992), the dynamic welfare optimization problem is defined as:

$$\text{Maximize } W_0 = \sum_{t=0}^{\infty} \frac{p_t C_t}{\prod_0^t (1 + i_s)} \tag{10}$$

subject to

$$Q = F(K_{t-1}, \overline{L}_t, t) \tag{11}$$

where C is consumption, Q is real gross domestic product and where labour, L, is assumed to be exogenous (denoted by the bar over L) and p is the price of output. Capital is assumed to be endogenously produced by the economic system. The discount rates are assumed to be exogenous in this restricted context contrary to the case in which welfare is defined by a utility index.

Hulten compares the Harrodian residual of Rymes with the usual neoclassical residual (called the Hicksian residual) defined on gross domestic product and with his dynamic residual. Hulten (1992, p. S21), finds that the Hicksian rate of technical progress, τ, is related to the Harrodian rate of technical progress, τ_R, by exactly the same equation (3) (or (9)) given above that links Rymes' index to the neoclassical index.

The dynamic index of Hulten, τ_H, would also equals the Harrodian residual in steady state growth with a constant rate of technical progress as claimed by Hulten (1992, p. S21) and as could be seen in the one period horizon case:

$$\tau_H = \sum_{t=0}^{T} \omega_{Lt} \tau_R \tag{12}$$

where ω_{Lt} is the share of the wage bill of period *t (appropriately discounted)* in the present value of wealth. As total wealth is equated to the sum of the discounted wage bill and the value of the initial capital stock, the sum of the ω_{Lt} weights tends to one as T tends towards infinity. If τ_R is fixed, then, as claimed by Hulten, τ_H is equal to τ_R.

Hulten concludes that the change in national wealth, that he considers as a welfare indicator, is associated to the Harrodian rate of technical progress while similarly, the change in gross output is associated with the Hicksian (neoclassical) rate of technical progress:

> "This leads to the conclusion that the Hicksian and Harrodian concepts of technical change exhibits a complementarity that exactly parallels the complementarity of gross and net outputs as indicators of capacity and economic welfare." (Hulten (1992, S10).

However, that parallel may not be exactly as indicated by Hulten. In Hulten's model, net output only corresponds to the wage bill (plus the vanishing interest income on what remains of the initial capital stock) rather than net national income. In section III of his 1992 paper, however, Hulten discusses the measure of net output that corresponds to the maximum consumption path and concludes that this measure is given by net domestic product *Y*, that is, by the wage bill *plus* all interest income.

Hence, Hulten (1992) uses two different measures of net income, net domestic product which corresponds to the maximization of the wealth function and, for all purposes, the wage bill, which corresponds to his dynamic index[10]. Therefore, the results that he obtains by defining value added as essentially comprising only the wage bill depends, and consistently so, on the very definition of the production process which comprises labour as the sole primary input. Had he chosen a different definition of net output and primary inputs consistently with net income comprising both labour and waiting income, he would have likely obtained a different result.

Hence, one may define a multi-period dynamic index τ_D, similar to τ_H, but defined in reference to net output and the primary inputs of waiting and labour that is related to our (single period) index by the relation:

$$\tau_D = \sum_{t=0}^{T} \omega_{yt} \tau_R^* \qquad (13)$$

where the weights ω_{yt} are now the shares of net income of year *t* in total wealth defined as the discounted value of *both* the flow of labour and waiting income. The star subscript on the productivity index indicates that it is defined on a net measure of output

10. One may wonder why the interest income paid on the capital stock acquired from the past is a primary input income while the interest paid on the capital accumulated after the initial period is an intermediate input income. This would be because the latter capital goods are endogenous while the initial capital stock is exogenous. Nevertheless, the initial capital stock is not manna from Heaven and had to be produced in the past. Hence the primary capital income would depend on the arbitrarily selected base year. If one climbs back indefinitely into the past, then $(1 - d)^{t-1}K_0$ tends towards a zero quantity that we could logically leave out of the model. Proceeding in such a way, one reconcile the equations of Hulten (1979) with those of Hulten (1992) and the weights in equation (12) would sum to unity as in equation (13).

and "pure" waiting services[11]. But then, Hulten's dynamic residual would correspond to our alternative residual, and the appropriate measure of net output would exclude only *economic* depreciation on capital. It is indeed, shown below that the productivity residual τ_R^* defined on net domestic product is the same as the productivity residual defined on gross domestic product. It therefore follows that this same last measure of net output would be suitable for both productivity and welfare analysis[12].

Bringing back an old debate, Hulten (1992) argued that the economy's output should be gross rather than net of capital depreciation for the purpose of measuring efficiency in production. That is, final demand output must be computed as including gross fixed capital formation contrary to the suggestion made earlier by Denison (1962)[13] that it include net investment and that aggregate output be measured by net national product.

Basically, Hulten's point is that even if utility depends on the inter temporal flow of consumption (corresponding to the net concept of output), this flow can only be sustained by producing capital goods and their replacement units. These units in a one-sector-one-good model are any units produced and there is no reason not to account for these units not used in consumption but used to replace the capital units consumed in the production process. In any case, the statistician observing production in the economy is observing the total amount of commodities produced whatever the eventual split of that production between consumption and investment.

Hulten (1992)'s argumentation, although intuitively highly appealing, is nevertheless not quite fully convincing. In the first place, what the statistician would observe would be the output gross of both the intermediate use of the commodities and of their use to replace worn out capital units. Hence, following that argument, the same gross output measure as used at the industry level should be used for the aggregate business sector as well. In the second place, nothing prevents the statistician from doing an imputation of depreciation costs to any production in the economy and, consequently, to measure that production net of depreciation as well as net of intermediate consumption. The fundamental issue, therefore, is whether depreciation is an intermediate or a primary input. That issue, however, is of limited significance in light of the result that our productivity residual is invariant to the choice of either one of the gross or net output concept.

Accounting for under utilisation of capacity in the short-run

Berndt and Fuss (1986) suggested that, if firms can remain in short-run equilibrium, then at the short-run cost minimization position on their isoquant map, one can define a shadow price for capital services at which that short-run equilibrium would also be a long-run equilibrium. This comes about by solving the input demand equations (from Shephard's (1953) lemma) for the quantity of labour and the price of capital, while

11. More precisely, where both output and waiting services are net of depreciation as described below.
12. Note that we defined net output as gross output minus the real value of *economic* depreciation, not *physical* depreciation as in Hulten so that net output and net income are always identical, whatever the pattern which is assumed for depreciation.
13. See also Denison (1989), p. 21.

maintaining the quantity of capital fixed. Since these equations are long-run equilibrium demand equations, the capital service price solution (its shadow price) must be consistent with long-run equilibrium. It turned out that under the assumption that expectations are realized, the shadow price of capital services is just equal to the residual price of capital services obtained by dividing the ex post capital income (as, for instance, per the economic accounts) by the stock of capital.

Berndt and Fuss concluded that measuring the capital cost share on the basis of that price or simply using the ex post capital cost share in the productivity equation would, thereby, account for the under utilisation of capital services, without any need to further adjust the rate of utilisation of capital itself.

But adjusting only for the capital cost share while still assuming that capital (or waiting) services are proportional to the capital (waiting) stock may not be sufficient when one compares how labour inputs are measured when firms hoard labour in the short-run, which is symptomatic of labour being also a quasi-fixed input. While hoarding labour, firms use less intensively their labour inputs by reducing the average number of hours worked per worker, *a*, so that hours worked, *H*, falls as a proportion to the number of workers, *N*:

$$H = aN \qquad \text{and} \qquad \dot{H} = \dot{a} + \dot{N} \tag{14}$$

The question is whether firms have the same flexibility in adjusting capital inputs use as they have in adjusting their use of labour inputs in the short-run. Under the same conditions as the Berndt-Fuss theorem, it can be said that it is the case. Indeed, looking at the dual of equation (14), one can relate the annual wage rate, w_N to the hourly wage rate w_H through the relation:

$$w_N = aw_H \qquad \dot{w}_N = \dot{a} + \dot{w}_H \tag{15}$$

When measuring productivity, one may use the number of workers (a stock variable), *N*, rather than hours worked, *H*, as a measure of labour input and state that, when firms hoard labour, they pay less for using the same labour services (*N* is fixed but w_N falls with *a*). Alternatively, one may use hours worked as a measure of labour input and state that firms pay the same price for labour (w_H) but use less labour inputs (as *H* falls with *a*).

In the same fashion, firms may be seen, when in a situation of excess capacity, as using the same amount of capital services (*K*) but paying less for these services (as *r* falls ex post according to the Berndt-Fuss model) or paying the same price for capital services as before (p_S to be defined) but using less capital services (*S*) such that[14]:

$$p_S S = rK \qquad \text{just like} \qquad w_H H = w_N N \tag{16}$$

14. For more detail derivations, see Durand, R. (2000), Using Slaves, Robots and Other Productive Machines, *Industry Canada*, Mimeo.

Since the rate of return falls below the equilibrium rate of return in a downturn (as short-run minimum cost are above long-run minimum costs), the latter can be used to both adjust the capital income share and the rate of utilisation σ, consistently with the Berndt-Fuss model, by setting[15]:

$$\sigma = \gamma\frac{\rho}{\rho*} \quad \text{an} \qquad S = \gamma\frac{\rho}{\rho*}K \qquad p_S = p\rho* = r* \tag{17}$$

where γ is a normalization constant transforming the stock variable K into the flow variable S and where ρ is the ex post gross rate of return :

$$\rho = (\iota + \delta - \dot{p}) \tag{18}$$

where ι is the nominal rate of return, δ is the rate of economic depreciation and p the price of output which, in this simplified world, also stands for the price of capital goods. The gross rate of return $\rho*$ is the observed market rate of return or the rate that corresponds to the full capacity utilisation of the capital stock. The rental price of capital services, r, is therefore given by $p\rho$. Under both steady state and non-steady state growth with full employment and full capacity utilisation[16], $\rho = \rho*$ so that capital services vary in fixed proportion to the stock of capital as in the traditional model.

The price of capital services, p_S, corresponds uniquely to a full capacity utilisation of capital while the standard rental price of capital services, r, may correspond to a situation where under utilization of capacity is present.

Transposed into waiting units, the price of waiting services is to be denoted by p_{SR}. One has then the following primal and dual productivity equations:

$$\tau_R = \dot{Q} - \alpha\dot{L} - (1-\alpha)\dot{S}_R = \dot{Q} - \alpha\dot{L} - (1-\alpha)[\dot{K}_R + \dot{\rho} - \dot{\rho}^*] \tag{19}$$

and

$$\tau_R = \alpha\dot{w} - (1-\alpha)\dot{r}_R^* - \dot{p} = \alpha\dot{w} - (1-\alpha)[\dot{p}_R + \dot{\rho}^*] - \dot{p} \tag{20}$$

15. Which is equivalent to measuring the rate of utilisation of labour, a, in (16) by the ratio of the annual wage rate to the hourly wage rate instead of H/N.
16. Along a transient path between two steady state growth path when capital can still be considered as a variable input that adjusts fully to changing relative primary input prices.

Note that the standard model results by setting the ex post gross rate of return to its market value in (19) while the dual equation is identical to Rymes dual equation if there is full employment equilibrium along a transient path between two steady state growth path. Using the primal and the dual fundamental equations respectively in equations (19) and (20) gives:

$$\tau_R = \dot{Q} - \alpha\dot{L} - (1-\alpha)[\dot{K} + \tau_R + \dot{\rho} - \dot{\rho}^*] \quad \text{or after solvin} \quad \tau_R = \frac{\tau}{\alpha} \tag{21}$$

and

$$\tau_R = \alpha\dot{w} + (1-\alpha)[\dot{p} + \tau_R + \dot{\rho}^*] - \dot{p} \quad \text{or after solvin} \quad \tau_R = \frac{\tau}{\alpha} \tag{22}$$

These equations are similar to those of the basic model except that the neoclassical measure of productivity growth, τ, is itself altered in both (21) and (22) when there is excess capacity. But equation (22) may be re-written as

$$\tau_R = \alpha(\dot{w} - \dot{p}) + (1-\alpha)(\dot{p} + \tau_R - \dot{p} + \dot{\rho}^*) = \alpha(\dot{w} - \dot{p}) + (1-\alpha)(\tau_R + \dot{\rho}^*) \tag{23}$$

which solves for the real wage as:

$$\dot{w} - \dot{p} = \tau_R - \left(\frac{1-\alpha}{\alpha}\right)\dot{\rho}^* \tag{24}$$

while the real capital service price is given by:

$$\dot{r}_R^* - \dot{p} = \dot{p} + \tau_R + \rho^* - p = \tau_R + \dot{\rho}^* \tag{25}$$

Under steady state growth, both of these equations solve for the same real input price growth rate which is equal to the rate of growth of productivity. When the gross rate of return is decreasing, that is when the capital/output ratio is increasing or when the waiting/ labour ratio is increasing, then the real price of waiting services is growing at a slower pace than technical progress and vice versa for the real wage rate. The converse situation applies when the gross rate of return is increasing so that we have the following cases:

(1) Steady state growth path:

$$\dot{\rho}^* = 0 \qquad \dot{K} = \dot{Q} \qquad \dot{K}_R = \dot{L} \qquad K - \dot{K}_R = \tau_R \qquad \dot{w} - \dot{p} = \tau_R = \dot{r}_R - \dot{p} \tag{26}$$

(2) Substitution of capital (waiting) for labour:

$$\dot{\rho}^* < 0 \qquad \dot{K} > \dot{Q} \qquad \dot{K}_R > \dot{L} \qquad K - \dot{K}_R = \tau_R \qquad \dot{r}_R - \dot{p} < \tau_R < \dot{w} - \dot{p} \tag{27}$$

(3) substitution of labour for capital (waiting):

$$\dot{\rho}^* > 0 \qquad \dot{K} < \dot{Q} \qquad \dot{K}_R < \dot{L} \qquad K - \dot{K}_R = \tau_R \qquad \dot{r}_R - \dot{p} > \tau_R > \dot{w} - \dot{p} \tag{28}$$

If in case (1) multifactor and labour productivity grows at the same rate, as waiting services grows at the same rate as labour services, in case (2), multifactor productivity grows at a lower rate than labour productivity while the converse occurs in case (3).

When there is excess capacity, the ex post rate of return differs from its market value so that the primal equation becomes:

$$\tau_R = \dot{Q} - \alpha\dot{L} - (1-\alpha)[\dot{K}_R + \dot{\rho} - \dot{\rho}^*] \tag{29}$$

while the dual equation remains unaltered. In that case, when the ex post rate of return decreases below the market rate of return, that is when ρ falls and at a higher rate than $\rho*$, then:

$$\dot{\rho} - \dot{\rho}^* < 0 \tag{30}$$

and productivity grows at a higher rate than in the standard model. Conversely, when the ex post of return grows and at a faster rate than the market rate, then productivity, as estimated by (29) grows at a smaller rate than in the standard model. But in the downturn, standard productivity estimates tend to fall so that the alternative measure presented here tends to fall by less. Conversely, in the upturn of a business cycle, the conventional measure of productivity tends to increase faster than normal so that the alternative measure is below the standard measure so that, in general, one has:

$$\tau_R^{SL} < \tau_R < \tau_R^{SH} \tag{31}$$

where we have denoted by the superscript S the standard measure with L in the low growth situation and H in the high growth situation. Hence, the alternative measures derived here tends to fluctuate less over the business cycle than the standard measure of productivity. This applies to the neoclassical measure as well.

It is important to correct the productivity residual from the incidence of changes in capacity utilisation over the short-run so that this residual provides, neglecting a host of other potential problems, as close to a pure estimate of technical progress as possible. The short-run behaviour of the corrected residual may then be scrutinized so as to determine if it is not subject to real influences under some endogenous growth model as studied in Part 2 of this paper.

Measuring productivity on net output

case (1): net current price output is equated to the wage bill

Using our alternative definition of capital services, *S*, we may define an associated measure of net real output, *q*, given by:

$$q = Q - S \tag{32}$$

In a one-sector-one-good model, *S* is measured in the same output units as *Q*, and it has the same price *p* from which follows that the price of *q* is also *p*. That justifies the subtraction in (32). Deriving (32) and subtracting the rate of growth of labour gives the total labour productivity index, say τ_L:

$$\tau_L = \dot{q} - \dot{L} = \frac{\dot{Q} - (1-\alpha)\dot{S}}{\alpha} - \dot{L} = \frac{\tau}{\alpha} \tag{33}$$

which is consistent with the result obtained from the general integration rule referred to above. All productivity equations taking capital as endogenous that compared in this paper are equal to partial labour productivity defined on net output.

Equation (33) shows[17] more explicitly that Rymes' *standard* Harrodian measure of productivity is identical to total labour productivity on net output when net output is defined as gross domestic output minus capital services. That measure of net output leaves only the wage bill as part of the national income as in Hulten's model.

The dual equation corresponding to (33) which uses *p* for both the price of output *Q* and and *w f*or the price of labour is:

$$\tau_L = \dot{w} - \dot{p} = \frac{\alpha\dot{w} + (1-\alpha)\dot{r}^* - \dot{p}}{\alpha} = (\dot{w} - \dot{p}) + \frac{(1-\alpha)\dot{\rho}^*}{\alpha} \tag{34}$$

Equation (34) implies that the market gross rate of return must be constant for the last equality to hold, i.e. that the economy be evolving along a steady state growth path. That result had to be expected as the only input price is the price of labour (which should be the same for both direct and indirect labour). This would confirm the equivalence of Rymes' standard model with Hulten's model only under steady state growth[18].

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17. Capital services in that case need to be defined using our alternative formulation since it would make no sense to subtract the whole stock of capital from gross value added. In the neoclassical case, there simply does not exist, indeed, any measure of the *level* of capital services. The proportionality assumption of the flow of services to the stock of capital is insufficient to derive the level measure. Note further that, according to (32), the value of net production is equal to the wage bill.
18. This result may be obtained in a simpler manner by deflating the capital stock by the wage rate in order to obtain a measure of the capital stock in labour (indirect input) units.

Hence, Hulten's rate of productivity growth differs from Rymes' rate of productivity in non-steady state growth, unless he would account for capital income and introduce waiting as an additional input. Alternatively, one may interpret the condition that the gross rate of return must be constant in Hulten's case as the fact that his model is only defined along the steady state growth path. Hulten's model would be a particular case of the more general model of Rymes that rests on two primary inputs.

case (2): net current price output is defined as net domestic product

One advantage of the alternative capital service formulation developed in the preceding section is that it allows a decomposition of capital services into two major components: economic depreciation, $\delta_R K_R$, and pure waiting, ιK_R. In fact, only the latter could really be assimilated to waiting as it corresponds to the income received by asset holders for delaying their consumption. The depreciation component is not paid to households and, consequently, is not part of national income. Rather, depreciation should be considered as part of the intermediate inputs. Fixed capital consumption is no different than inventory depletion of raw materials and the latter is considered as an intermediate input in national input-output accounting.

What matters is the real value inputted into the production process or the loss of real value of the stock of capital, whatever the origin of that loss, be it from physical wear and tear or obsolescence. Consequently, the net output concept that we favour is the one given by gross value added minus *economic* depreciation.

Treating depreciation as an intermediate input leads naturally to its accounting in output units, i.e., as δK, rather than as $\delta_R K_R$. Total capital services could then be written as:

$$S'' = \gamma\frac{(\iota - \dot{p})}{\rho*}K_R + \gamma\frac{\delta}{\rho*}K = \tilde{S}_R + D \qquad (35)$$

where $\tilde{S}_R$ is the waiting component of capital services and D, its economic depreciation component.

Note that the price of pure waiting services is the same as before since one must have:

$$\tilde{p}_{SR}\ \tilde{S}_R = p(\iota - \dot{p})K = p_R(\iota - \dot{p})K_R \qquad (36)$$

which, given the definition of $\tilde{S}_R$ in (35) gives:

$$\tilde{p}_{SR} = \frac{p_R(\iota - \dot{p})K_R}{\gamma\frac{(\iota - \dot{p})}{\rho^*}K_R} = \frac{p_R\rho^*}{\gamma} = p_{SR} \quad (37)$$

We now proceed by treating depreciation as an intermediate input and restricting the definition of waiting to correspond only to the interest component in the definition of waiting services given in equation (35) above, that is to the primary input part. Our alternative index of productivity, τ_R^* - referred to above in equation (13) - becomes defined on output net of depreciat i o $\tilde{q}$:

$$\tilde{\tau}_R = \dot{\tilde{q}} - \tilde{\alpha}\dot{L} - (1-\tilde{\alpha})\dot{\tilde{S}}_R = \tilde{\alpha}\dot{w} + (1-\tilde{\alpha})\dot{p}_{SR} - \dot{p} \quad (38)$$

where $\tilde{\alpha}$ is the labour income share in the net value of output $p\tilde{q}$. We have the following identity relating $\tilde{q}$ to Q:

$$p\tilde{q} = pQ - p\delta K \quad (39)$$

Hence, once more, the price of net output in (39) is the same as the price of gross output and:

$$\tilde{q} = Q - \delta K \quad (40)$$

Integration over depreciation in the case of the integrated index does not follow Durand's general integration rule as depreciation on the output side of the equation is in output units, δK, while it is in input units, δK_R, on the input side. Intuitively, it follows that the productivity index remains invariant when netting a primary input from outputs and inputs[19]. The standard neoclassical productivity measure, however, as it defines depreciation in output units on both sides of the productivity equation, varies when switching from a gross output to a net output basis according to the general integration rule.

To conclude, Rymes' measure of technical progress, modified so as to take into account under capacity utilization of capital, is invariant to the choice of output in either steady state or non-steady state growth with and without full employment of both primary inputs. From that perspective, the choice of an output measure is somewhat irrelevant. It is only relevant in the standard neoclassical formulation of technical progress where capital is considered as exogenous and is at the source of the debate over the use of the gross versus the net output. Under full integration of the business sector with the household sector, therefore, the productivity residual cannot be distinguished from the welfare residual.

19. Proof of that result can be obtained from the authors.

Part 2

Introduction

What happens if there is a change in waiting, i.e., an increase or decrease in waiting, and it affects Harrodian outcomes and measures of technical change when Keynes's problem is considered. The Keynesian problem is that increased waiting can not only take the form of additions to capital, the classical case where our previous analysis holds, but it can also take the form of desired additions to the flow of liquidity services. In Keynesian economics, if the increased desires for liquidity are not met by an increased supply from the banking and Central Banking systems, the increased waiting doesn't appear as additions to the capital stock but as wasted savings and the unemployment of labour and waiting.[20]

The fundamental problem is that the rate of interest as set by the Monetary Authorities plays a crucial *real* role in Keynesian economics. The lower is 'the' rate of interest set by the Monetary Authorities, the higher will be the rates of capital accumulation, technical progress and rates of return to capital. Rates of growth, rates of capital accumulation and rates of return to waiting, even with the enormous problems associated with depreciation ignored, cannot be determined independently of the monetary arrangements. We live in monetary economies where the techniques of monetary produced by banks and the Central Banks, always have real consequences. This is where the monetary aspects of Keynesian and Cambridge economics come into play in the Harrodian measurement of total factor productivity.

Once the Keynesian problem is admitted then it is not just that measures of total factor productivity will reflect the rate of capital accumulation, but will also reflect the effects of Keynesian unemployment and policy steps designed to ameliorate the

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20. In classical or neoclassical economics, an increased desire for liquidity is said to be an excess demand for money. If money prices have the requisite flexibility, 'real' balances will grow to accommodate the increased demand for them, the Keynesian theoretical problem set aside and any unemployment which might result is said to be the result of short run stickiness in prices so that in the long run the Keynesian problem of the unemployment of labour and waiting is laid to rest and productivity advance and its measurement is unaffected by monetary policy. See Friedman (1976) While it is often stated that one would not rely upon 'real balance' effects in the real world, their existence nevertheless is said to prove that Keynes's contention that there may exist a less than full employment equilibrium is without theoretical foundation. As Friedman (321) says,

> "The Pigovian and the more far-reaching [Friedman's] answer to Keynes's proposition have been extremely important on a theoretical level in assuring that there is no basic flaw in our theoretical analysis. But I hasten to add that in my opinion neither corresponds to effects that are empirically important in the kind of economic fluctuations that actual economies experience."

Without a fiat money base, this argument against Keynes's theory collapses. We are now in monetary world, if one may set aside circulating currency, with no fiat money base and no determination of price levels by 'real' balance effects. Keynes's problems remain.

Keynesian problem. Measures of total factor productivity will not only then reflect the effects, if any, of government policies designed to advance the rate of improvement in information technology but will also reflect the success, or lack thereof, of macro-economic policies designed to ensure full employment and the bringing about of the most efficient rate of introduction of technical progress. In short, the state of technology and its advance is not given but is also partly a function of policy. Technology and its advance are not part of the fundamentals like natural resources but are a function of the application of macro economic policy.

This is a huge topic. In this paper we reduce it down to how Harrodian measures of total factor productivity will reflect Keynesian monetary policy.[21] In both Keynes's **TREATISE ON MONEY** and **THE GENERAL THEORY**, since waiting or savings could take two related forms, the acquisition of capital, as in classical economic analysis, but also the acquisition of liquidity, should the monetary authorities fail to conduct economic policy such that the desired amount of liquidity, consistent with non-inflationary full employment are not forthcoming then the economic system would either experience rising prices or rising unemployment of working and waiting.

It is extremely important to realize that monetary policy is now conducted without central bank reserves. We shall assume in this paper that central bank money has vanished. There is no 'real balance effect' to set aside Keynes's contention that unemployment equilibria can exist. (See Rymes, 1998) and Rogers and Rymes (1997, 2000)

Money and Keynesian Emendations to Harrodian Measures of Technical Progress

Consider the following accounts in the SNA 1993 format for a 'stylized' competitive bank in the contemporary Canadian context:

$$\begin{aligned} i_{OD}O_D + i_{OT}O_T + \delta_{OD}O_D + \delta_{OT}O_T - i_{DD}D_D - i_{DT}D_T + \delta_{DD}D_D + \delta_{DT}D_T + i_{CR}C_R \\ = WL + \pi - p_i P_i K_i + P_j M_j + i_{DR}D_R + \delta(C_R + D_R) \end{aligned} \quad (1)$$

where the notation is standard, save for bank profits π and economic depreciation $-p_i P_i K_i$ (the subscript for banking is implied), but $i_{OD}O_D$stands for the interest rate(s) and nominal amounts of demand loans (called overdrafts), $i_{OT}O_T$ for time overdrafts and similarly for demand and time deposits and the δ's stand for service charges or carrying charges, expressed as rates, the bank may levy for services rendered with overdrafts and deposits. The entries $i_{CR}C_R$ and $i_{DR}D_R$ refer to interest receipts of the bank should it experience positive settlement balances in clearings with the Central Bank and interest payments should it experience negative settlement balances. The entry $\delta_{CB}(C_r + D_r)$ represents carrying charges the Bank might levy on the bank's settlement balances for services, which can be priced, rendered by the Bank.

21. We extend the work of Rymes in "Effect of monetary policy on productivity in Canada.", Carleton Economic Papers, forthcoming in Statistics Canada ***Probing the New Economic Realities***, publication of the proceedings of a conference in Ottawa, 23-24 March 1999.

Ignore the entries pertaining to the Bank for a moment. Reference or 'pure' rates are conceived in the SNA 1993 such that the gross output of a bank becomes:

$$(i_{OD} - i_{RD})O_D + (i_{OT} - i_{RT})O_T + \delta_{OD}O_D + \delta_{OT}O_T \\ + (i_{RD} - i_{DD})D_D + (i_{RT} - i_{DT})D_T + \delta_{DP}D_D + \delta_O D_T \quad (2)$$

so that if the same reference rate i_{RD} is used for demand overdrafts and demand deposits and i_{RT} for time overdrafts and deposits and it is assumed that banks pursue balanced equilibria portfolio policies such that

$$O_D = D_D \text{ and } O_T = D_T \quad (3)$$

then

$$-i_{RD}O_D + i_{RD}D_D = 0 \quad (4)$$

as does

$$-i_{RT}O_T + i_{RT}D_T = 0 \quad (5)$$

and the accounts for the banks are unchanged by the introduction of the reference rate. However, if one argues, for example, that individuals are 'paying' for the services of banks by foregoing the reference rate on (say) demand deposits and actually obtain a low (sometimes zero and perhaps negative) rate of interest on such deposits, the measure $[(i_{RD} - i_{DD}) + \delta_{DD}]D_D$ is the 'true' price times quantity with respect to demand deposits where it is assumed that while δ_{DD} is the actual market rental or carrying cost of demand deposits, $i_{RD} - i_{DD}$ is the implicit carrying cost involved in the willingness of depositors, to hold deposits and earn only i_{DD} on their lending to the bank, compared to i_{RD}. Similarly, the measure [`$(i_{OD} - i_{RD}) + \delta_{OD}]O_D$ is the 'true' price times quantity with respect to demand overdrafts where again it is assumed that while δ_{OD} is the actual market carrying cost of demand overdrafts, $i_{OD} - i_{RD}$ is the implicit carrying cost involved in the willingness of over drafters to hold overdrafts when the borrowing rate of interest exceeds the reference rate. The reference rates seem to embody the idea that if borrowers and lenders could get together costlessly[22]. the reference rate would be the cost less borrowing-lending rate. Similar arguments would be made for time overdrafts and deposits with *higher* reference or 'pure' rates.

Central Banks and Reference Rates

If the reference rate is crucial [23] for the measurement of bank output and if the reference rate is affected by Bank policy, then it is important to understand how the Bank affects banks' output and productivity.

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22. For comments on how meaningful such an assumption of such costlessness is, see Clower (1995)

We begin by examining one of the inputs into banks noted above: the service charges the Bank may levy for the transactions and portfolio services it provides to banks (or through their House if that intermediary were taken into account) together with any interest paid on negative settlement balances plus any interest received on positive settlement balances. In other words, I assume that the nominal Bank's gross output is

$$i_{Dr}D_r - i_{Cr}C_r + \delta_{CB}(D_r + C_r) \qquad (6)$$

which is, of course, subject to the same problem which holds for private banks. It should therefore be transformed in the standard UN way, that is,

$$[i_{Dr} - i + \delta_{CB}]D_r + [i - i_{Cr} + \delta_{CB}]\ C_r \qquad (7)$$

where again i is a pure reference rate. Banks who clear through the Bank and run negative and positive settlement balances obtain liquidity services from the Bank. Explicit service charges levied by the Bank are Bagehot or implicit prices paid by banks for liquidity or lender of last resort services provided to them by the Bank. In the case of banks running negative settlement balances, the Bagehot price is $i_{Dr} - i$ while for those running positive settlement balances, it is $i - i_{CR}$.

It is the property of the Bagehot prices set by the Bank that banks' portfolio policies (i.e. their overdrafts and deposits) would be such that the bank's expected negative (positive) settlement balances would be zero, if and only if the Bank is neutral with respect to the setting of settlement balance positions.

The opportunity cost of a bank being in an expected negative settlement balance is the Bagehot price $i_{Dr} - i$ if i were the overnight rate the bank could earn on demand overdrafts. The Bagehot price $i_{Dr} - i$ if i were the overnight rate the bank could earn on demand overdrafts. The opportunity cost of a bank being in an expected positive settlement balance is the Bagehot price $i - i_{CR}$. If i, the reference rate, is half-way between the rate charged on negative settlement balances and that paid on positive settlement balances, then, at the margin it is equally costly or profitable for the bank to be in a zero, positive or negative, expected settlement balance position.

When the price and output of banking and the economy as a whole is that desired by the Bank, no non-neutral action will be undertaken by it. The overall banking system will be in a zero settlement bank position and 'the' reference rate - the rate halfway between i_{Dr} and i_{Cr} - will be such that the bank's prices of $[(i_D - i) + \delta_0]$ and $[(i - i_D) + \delta_D]$ and outputs O and D will be that desired by the Bank. The nominal amounts of O and D will be such that the Bank's view as to a desired price level (or rate of change in prices) is also being met.

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23. "Clearly, the linch pin of an integrated approach to macroeconomic measurement of financial services built around the rental price of money concept is the opportunity cost rate/benchmark rate/reference rate..." Dennis Fixler and Kim Zieschang, "The productivity of the banking sector integrating financial and production approaches to measuring financial service output", a paper presented at the Centre for the Study of Living Standards Conference on Service Centre Productivity and the Productivity Paradox, Ottawa, 11-12 June 1997, 22. See the earlier papers by Fixler and Zieschang, (1991, 1992)"

We see immediately not just in the Canadian context but more general for all monetary economies with advanced central banking policies that it is the Bank which sets the reference rate. If one takes, as one of the reference rates, the overnight rate then that rate is set by the Bank. That reference rate will be affected as well by fundamentals but it is vitally important to realize that the Bank plays a central role in its determination. Since all other rates are related to the overnight rate by such arguments as the term structure of rates or in general the Keynesian liquidity theory of the determination of all rates including rates of return on non human and human capital, 'the' reference rates, for there is clearly a need for more than one in measuring bank output, indeed all real interest rates are not given as fundamentals but are also a function of monetary policy.

Growth Accounts for Banks

We can rewrite the bank accounts in a general way, ignoring the difference between demand and time overdrafts and deposits, as:

$$\begin{aligned}(i_{O} - i + \delta_{O})\frac{O}{P} + (i - i_{D} + \delta_{D})\frac{D}{P} &= \frac{WL}{P} + (R - p)\frac{P_{i}K_{i}}{P} + \frac{P_{M}M}{P} \\ &+ (i_{Dr} - i + \delta_{CB})\frac{D_{r}}{P} + (i - i_{Cr} + \delta_{CB})\frac{C_{r}}{P}\end{aligned} \tag{8}$$

where P is the general level of the prices of consumption goods. A representative bank in equilibrium will have

$$\frac{O}{P} = \frac{D}{P} \tag{9}$$

and

$$\frac{D_{r}}{P} = \frac{C_{r}}{P} \tag{10}$$

Then, the standard total factor productivity growth measure for banks would be (where ^ indicates growth rates and greek symbols indicate shares)

$$\begin{aligned}&\alpha(\hat{O} - \hat{P}) + \beta(\hat{D} - \hat{P}) - [\gamma\hat{L} + \delta\hat{K} + \varepsilon\hat{M}] = t_{\beta} \\ &= [\gamma\hat{W} + \delta\widehat{\overline{(R - p_{i})P_{i}}} + \varepsilon\hat{P}_{M}] - [\alpha(\widehat{\overline{i_{O} - i + \delta_{O}}}) + \beta(\widehat{\overline{i - i_{D} + \delta_{P}}})]\end{aligned} \tag{11}$$

where R is the rate of return to capital. The weighted rates of growth in real terms of overdrafts and deposits, the stocks standing for the services of the banks demanded by and supplied to holders of overdrafts and deposits, will be equal to standard measures of the rates of growth of inputs plus the standard residual. Equally, the weighted rates of growth of the rentals of such inputs will equal the weight rates of growth of their own rates of interest on overdrafts and deposits plus the standard residual.

The growth accounts for banking expressed for the Harrodian measures would take into account productivity advance in all industries producing inputs used by banks. In that sense, banking adds no new complication to Harrodian measures of total factor productivity at the industry or aggregate level. Yet much illumination is provided. Banks have been investing heavily in computers and given the hedonic approach to the construction of price and quantity index numbers, the recorded growth of non-human capital in banking has been remarkable and at the same time standard measures of total factor productivity in banking have been low. Once the Cambridge Correction is made so that, understanding that Harrodian technical progress in banking cannot be ascertained independently of such advances in the computer producing industries and their suppliers as well, the Harrodian productivity measures in such sectors as banking appear more reasonable. This is just another illustration of just how treacherous the traditional growth accounting procedures are.

Return now to the all important discussion of the role of Central Banks. If the Bank of Canada wants to constrain the banks and the economy, it raises the overnight rate by raising the bands where $i_{Dr} = i + .25i$ and $i_{CR} = i - .25i$ become $i_{Dr} = i^* + .25i^*$ and $i_{CR}^* = i^* - .25i^*$ so that if the Bank wants i to move to i^*, it sets $i_{DR}^*$and i_{CR}^*such that i^*, the rate halfway between, will be higher than i. As previously argued, the banks immediately know that the expected cost of being in a negative settlement balance has increased and of being in a positive position has decreased and will adjust their own portfolio policies attempting to contract overdrafts and expand deposits. Their actions result in an immediate rise in the overnight rate and in a general rise in overdraft and deposit rates causing the non bank public to attempt to shift away from deficit to surplus inter temporal transaction streams, such real effects affecting prices, the ultimate objective of the Bank. The system entails, of course, that if the overnight rate does not move in accordance with the Bank's wishes, it can enforce by putting the banking system as a whole into a net negative settlement balance position by a draw down out of which, no matter how much pre-settlement clearings are attempted in the House, the banks cannot escape until the Bank of Canada relents by the reverse redeposit mechanism.

From the argument that banks will try to substitute deposits for overdrafts, in the growth accounting framework, one would expect the rate of growth of real overdrafts to fall relative to that of deposits. Since overdrafts determine deposits, however, both would fall and the overall rate of growth of banking output would decline relative to the rest of the economy.

Temporary equilibrium total factor productivity in banking would decline. There is no observed change in the flow of central banking liquidity services because the net settlement balances position of the banking system remains unchanged at zero. There need also be no change in the flow of factor services such as labour and capital in the banking system. Yet what has happened is that the rise in the price of the liquidity services of the Bank has caused the output of the banks to contract. And to the extent that banks gross outputs enters either as intermediate inputs into all other industries and as well into final demand, the observed total factor productivity in the Harrodian framework for all other industries and the aggregate economy would also decline.

The fundamental point is that in a monetary economy, the flow of total factor productivity, which is the changing efficiency of the primary inputs labour and the flow of waiting attached to the ownership of capital (Rymes, 1996), cannot be ascertained without taking into account the changing cost and quantity of the flow of liquidity services of the banks and the Central Bank!

Similarly, if the Bank lowers the bands, it seeks to shift the banks to pursue portfolio policies involving a relative expansion in overdrafts as compared to deposits and the predicted movement in overdraft and deposits rates, vis-à-vis the lower reference rate, involving the overall economy seeking a deficit inter temporal path. As the growth rate of real overdrafts and deposits rises so does the temporary equilibrium rate of growth of total factor productivity for the banks.

If Bank policy is effective without use of settlement balance policies, then as real overdrafts and deposits change, so, adjusted for the partial elasticity of bank output with respect to settlement balances, will settlement balances without further affecting total factor productivity of the banks. However, if the settlement balance enforcement mechanism is employed, then in the relative contraction case, the relative decrease in real overdrafts and deposits will be associated, temporarily with a rise in real constant dollar net negative settlement balances, a factor which would entail a further reduction in the temporary equilibrium total factor productivity of the banks. In the relative expansion case, the relative increase in real overdrafts and deposits will be associated, again briefly, with a rise in real constant dollar net positive settlement balances, a factor which would positively modify the temporary equilibrium total factor productivity of the banks. Why? The use of negative or positive settlement balances, via redeposit and drawdown mechanisms or repos to enforce Bank policy with respect to the reference rate adds an element of inefficiency compared with the case where by the market embraces the Bank's view as to what the overnight rate should be.

The banking imputation problem which plagues National Accounts (and their users) has been 'resolved'. Value added in banking makes sense. Value added in all other industries will be reduced. Other industries will be availing themselves of the services of banks by holding overdrafts and deposits. The remaining gross output of the bank (not appearing in inter bank transactions) would appear as parts of components of final demand. Total gross domestic product would be higher, given the measurement of bank output by the application of the SNA and potential Canadian use of reference rates.[24]

From equilibrium to equilibrium, where the Bank's 'spreads' or 'bands' policies [25] would result in a changed overnight rate, the observed changes in the banks's settlement balances would be zero. Yet in the case of a contraction in monetary policy, while the real change in the banks's inputs would be zero, because of their higher prices, the real flows of services associated with banks overdrafts and deposits would fall. Since these

24. For a detailed discussion of current Canadian procedures and adaptations planned based on the SNA 1993, see Lal 1994.

25. The Bank can also widen or narrow the bands, as well as changing their level. In Canada, to make paper transactions more expensive, the Bank sets the bands much wider for paper clearings than for Large Value Transfer clearings with the banks.

components of the gross outputs of the banks would appear as intermediate and even final inputs into industries and households, the overall economy would be shown as exhibiting a fall in the growth rates of Harrodian productivity. At the basic level, what is happening is that the supply of liquidity services by the Bank has been reduced, the supply of liquidity service by the banks has also decreased so that for the overall economy one says that the level of output is lower because the flow of liquidity services from the Central bank has fallen, even if the flow of liquidity services in real terms supplied by the Bank is difficult to measure. Clearly the Harrodian measurers must be supplemented by Keynesian ones, which attempt to capture the phenomenon that the flow of waiting services entails as well the flow of liquidity services in a monetary economy.

Conclusion

The first part of this paper has compared alternative measures of productivity growth which all consider capital goods as produced inputs. These measures have been shown to be identical under steady state growth. In non-steady state growth, only the Harrodian measure of technical progress that admits the existence of two primary inputs is defined. Measures that admits the existence of only labour are undefined in non-steady state growth as they cannot account for the substitution process that is going on between direct and indirect labour as the gross rate of return on capital is changing. One-input-based measures of productivity implicitly assumes that the gross rate of return on capital is constant.

We have shown that the debated choice between net or gross domestic product was irrelevant as our productivity residual defined on the inputs of working and waiting is invariant to the choice of either of these output measures. There would not be any need to use a different measure of output for analysing productivity growth and welfare.

The results that we have achieved in this paper shed further light on the somewhat paradoxical observation that real income per capita and labour productivity grow more in line together than does real income per capita and traditional measures of multifactor productivity. Clearly, this is because the traditional neoclassical productivity growth index fails to take into account the productivity gains made in the production of capital goods. Once corrected for that bias, indeed, multifactor productivity Harrodian measures tend to grow approximately at the same rate as the widely used measure of partial labour productivity over the long run, offering some support to the conjecture that technical change tends to be Harrod neutral.

In the second part of the paper, we argue that the Harrod measures must be embedded in more general Keynesian measures, where effects of monetary policy on banking output, Harrodian total factor productivity measures for banks and for the economy as a whole should be attempted. Such attempts are designed to capture the Lavington (1934) and Keynesian ideas that the services of waiting are generally affected by the services of liquidity in a monetary economy.

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