2019

IARIW-HSE

Special IARIW-HSE Conference "Experiences and Future Challenges in Measuring Income and Wealth in CIS Countries and Eastern Europe" Moscow, Russia, September 17-18, 2019

Asset Indexes and the Measurement of Inequality and Welfare: The Case of the South Caucasus and of Central Asia

Joseph Deutsch
Jacques Silber
Yongsheng Xu
Guanghua Wan

Paper Prepared for the IARIW-HSE Conference
Moscow, Russia, September 17-18, 2019
Session 7: Poverty and Wealth
Time: 13:15 – 15:15, September 18

Asset Indexes and the measurement of inequality and welfare: The Case of the South Caucasus and of Central Asia

Joseph Deutsch*

Jacques Silber**

Yongsheng Xu***

Guanghua Wan****,

* **Bar-Ilan University, Israel. Email: jdeutsch@biu.ac.il**

** **Bar-Ilan University, Israel; Senior Research Fellow, LISER, Esch-sur-Alzette, Luxembourg; Honorary Fellow, Centro Camilo Dagum, Tuscan Interuniversity Centre, Advanced Statistics for Equitable and Sustainable Development, Italy. Email: jsilber_2000@yahoo.com**

*** **Department of Economics, Andrew Young School of Policy Studies, Georgia State University, Atlanta, United States. Email: yxu3@gsu.edu**

******Fudan University, China. Email: guanghuawan@gmail.com**

August 2019

Not to be quoted without the authors' permission.

Abstract

This paper examines the evolution of inequality in the three South Caucasian and in four Central Asian countries, using an aggregate index based on consumer durables available to the households. Instead of using principal components analysis to aggregate asset indicators into an overall asset index, we propose an ordinal approach to using data on assets, when estimating the wealth of a household (or individual). Using two different approaches, item response theory and the concept of "order of acquisition of durable goods" we show that there tends to be an order of acquisition of assets. On the basis of such an order we then compute indices introduced recently to measure inequality and welfare when only ordinal variables are available.

Our empirical analysis is based on the Life in Transition Survey for the years 2006, 2010 and 2016, which cover Armenia, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan and Uzbekistan. As far as inequality is concerned, it appears that while during the first sub-period (2006-2010) inequality increased in some countries and decreased in some others, during the second sub-period (2010-2016) inequality increased in all countries. We also observed that there was important growth in both sub-periods and that this growth was sufficiently strong to counteract the impact of an increase in inequality in those sub-periods and countries where inequality increased. In other words not only was growth important during this period 2006-2016; so was also the increase in welfare.

J.E.L. Classification: D31 – I31

Key Words: asset indices - Item Response Theory - order of acquisition of durables goods – ordinal inequality - welfare

1. Introduction: Economic Growth and Inequality in the three South Caucasus countries and in Kazakhstan, Kyrgyzstan, Tajikistan and Uzbekistan

In 1991, following the fall of communism in the former Soviet Union, the countries in the South Caucasus and Central Asia, who had become independent, started their transition from central planning to market economies. The last decade of the twentieth century was characterized by a strong recession. As mentioned by Ganiev (2019), this contraction in economic activity was related to the breakdown of trade links among the former Soviet Republics, high inflation rates, the disappearance of many state-owned enterprises and for some of these countries to armed conflicts. It is only during the period 2000-2014 that these countries enjoyed a strong economic performance. While important inflows of foreign direct investment into natural resources, as well as an increase in commodity exports, were the main determinants of growth in Azerbaijan and Kazakhstan which are resource-rich countries, growth in Armenia, Kyrgyzstan and Tajikistan was related to important remittances from migrant workers. After 2014 the sharp drop in international commodity prices as well as a much slower growth in partner countries, such as the Russian Federation, significantly reduced growth in the seven countries of the South Caucasus and Central Asia which are the focus of the present study[1].

Despite the diversity of these seven countries, it is important to stress that they share a certain number of common features: they have all a relatively strong dependence on the economy of the Russian Federation and they all had to implement a transition from planned to market economies. Capannelli and Kanbur (2019a; 2019b) also stress that in all these countries the share in the economy of low-productivity agriculture and low-productivity of self-employment and informality were high, cross-border labor migration, in particular to the Russian federation, was important, spatial disparity in standards of living was significant. In addition there was a small market for non-resource production and employment was the main source of income. Moreover these countries have a relatively small population and include vast mountain or desert areas. Their economies are not very diversified, the private sector is still limited and social protection schemes are quite generous, an inheritance from the Soviet Union[2].

Despite these common characteristics there were important difference between these countries in per capita GDP, the latter being equal to $3180 in Tajikistan but $17398 in Azerbaijan (PPP$) in 2017. As stressed by Kanbur and Zhuang (2019), income inequality rose a lot, especially during the last decade of the twentieth

[1] We did not include Turkmenistan, one of the Central Asian countries, in our analysis because it did not participate in all the Life in Transition Surveys.

[2] For a detailed study of Central Asian countries, see, Pomfret (2019).

century: in 2000 the Gini index, based on consumption data, was thus equal to 0.4 in Georgia, 0.36 in Kazakhstan and Uzbekistan, 0.35 in Armenia, 0.34 in Azerbaijan, 0.31 in Kyrgyzstan and 0.26 in Tajikistan. While the previous data on the standard of living in these seven countries was based on per capita GDP while those on inequality were derived from consumption data, the present paper proposes an alternative method to measure standards of living, inequality and welfare. The idea is to adopt the asset approach to standards of living but while most studies taking such an approach use principal components analysis, we suggest using either item response theory or the concept of "order of acquisition of durable goods", a notion that was introduced by Paroush (1965; 1973). These two approaches emphasize the idea that there exists a most common order of acquisition of assets, so that we will be able to derive measures of inequality and welfare from the distribution of the individuals (households) along the most common path of asset acquisition. The paper is organized as follows. Section 2 describes shortly the asset approach to measuring standards of living. Section 3 summarizes the recent literature on ordinal inequality indices and on inequality-sensitive and additive achievement measures (welfare measures). Section 4 describes the data base and presents the empirical results of our investigation while Section 5 gives concluding comments.

2. Assets and the measurement of standards of living

It is well known that consumption is a better indicator of standard of living than income, among other reasons, first because consumption surveys have generally less missing values than income surveys, second because they are less subject to under- or over-reporting. Since consumption is a function of wealth (Modigliani and Brumberg, 1954; Friedman, 1957; Ando and Modigliani, 1963), and given that assets, in particular durable goods, are a reasonable proxy for wealth, Filmer and Pritchett (1999; 2001) suggested using information on assets when income or consumption data were not available or not reliable. To aggregate information on asset ownership into an overall measure of standard of living, Filmer and Pritchett (1999; 2001) used principal components analysis (PCA) and many subsequent studies taking an asset approach to the measurement of standards of living used also principal components analysis. This asset approach to evaluating standards of living has been applied to many issues (see, Filmer and Scott, 2011, for more details) such as inequality in health outcomes (Gwatkin et al., 2000, Bollen et al., 2002), child nutrition (Sahn and Stiffel, 2003), socio-economic inequalities in schooling (Ainsworth and Filmer, 2006), poverty change (Stifel and Christiaensen, 2007) or targeting public programs (Schady and Araujo, 2006, for Ecuador).

As mentioned previously, the most popular technique used to measure the standard of living via an asset approach is Principal Component Analysis (PCA), the first principal component being assumed to refer to the wealth of the households, given that it provides the highest level of discrimination between households. Correspondence Analysis (CA) is another approach that may be used to estimate the standard of living. This technique was introduced by Benzécri and Benzécri (1972) and it is probably preferable to use it, rather than principal components, when the variables are not continuous but categorical.

Another approach to analyze the ownership of assets is Item Response Theory, the idea being that the latent variable uncovered by this approach reflects the economic status of the household. IRT was originally introduced to analyze the results of psychometric tests but it has been used, for example, by Das et al. (2004) to measure the standard of living of households.

Finally another technique allowing to estimate the standard of living on the basis of asset data is based on the derivation of the most common sequence of acquisition of durable goods. This approach, borrowed from scale analysis in psychometrics, was introduced by Paroush (1965; 1973).

A summary of item response theory and of the Paroush approach is given in Appendix 1.

The present paper uses this Paroush approach as well as Item Response Theory to derive an aggregate measure of the standard of living at the household level. More details on these two approaches are given, for example, in Deutsch et al. (2015).

3. Ordinal variables and the measuring of inequality and welfare

Allison and Foster (2004) seem to have been the first to stress that inequality indices, commonly used when looking at income distributions, cannot be adopted when analyzing the degree of dispersion of ordinal variables, since small variations in the scale used may reverse the ordering of the frequency distributions that are being compared (see also, Zheng, 2011).

Following the work of Allison and Foster (2004), new inequality indices were proposed such as those introduced by Abul Naga and Yalcin (2008), Reardon (2009), Lazar and Silber (2013) and Lv, Wang and Xu (2015). The exact formulation of these indices is given in Appendix 2. These indices have been shown to have a certain number of desirable properties. Lv, Wang and Xu (2015), for example, who derived axiomatically two inequality indices when only ordinal variables are available, showed that their indices obeyed the axioms of focus, additivity, independence, perfect equality, invariance to simple switches, invariance to parallel shifts and polarization (see, Appendix 3, for more details).

In a recent paper Apouey et al. (2019) extended the work on inequality measurement in the presence of ordinal variables by deriving axiomatically inequality-sensitive and additive achievement measures based

on ordinal data. The index they introduced, which depends on the distribution of individual achievements, is a function of both the average level of achievement in society as well as of the degree of inequality of the distribution of achievements. Such a social achievement index was shown to satisfy the properties of Normalization, Independence, Weak Pareto Principle, Anonymity, an Equity Principle, and a Proportional Equality principle (see, Appendix 4 for the formulation of this achievement index as well as for an intuitive interpretation of these properties). Apouey et al. (2019) also showed that by relaxing the equity principle one obtained an index whose change over time reflects in fact the extent of pure growth in society.

4. The database and empirical results[3]

In this paper, we use the Life in Transition Surveys for the years 2006, 2010 and 2016. These surveys were conducted by the European Bank for Reconstruction and Development and they provide information on the seven countries that are the focus of the present paper: the three countries of South Caucasus, namely Armenia, Azerbaijan and Georgia, and four countries in Central Asia: Kazakhstan, Kyrgyzstan, Tajikistan and Uzbekistan.

Data on the following goods or services were available and used:

Dwelling (1), Access to Tap Water (2), Telephone (3), Access to Gas (4), Car (5), Second House (6), Bank Account (7), Computer (8), Access to Internet (9).

The orders of acquisition of these goods or services are given in Table 1, for all the seven countries and for the years 2006, 2010 and 2016, using both the Paroush approach and item response theory. In Uzbekistan, for example, in 2006, using the Paroush approach, the good ranked first is the dwelling, the one ranked second is the telephone, whereas the eighth rank corresponds to a bank account and the ninth to a computer.

[3] Some of the results for the years 2006 and 2010 appear in Deutsch et al. (2017)

Table 1: Orders of Acquisition in Armenia, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan and Uzbekistan, Based on Different Approaches

Approach	*Year*	*Country*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Reproducibility Index*	*Number of Observations*
Paroush approach	2006	Armenia	1	2	3	4	5	8	9	6	7	0.9599	1000
		Azerbaijan	1	3	2	4	5	8	9	6	7	0.9673	1000
		Georgia	1	2	3	4	5	6	9	7	8	0.9418	1000
		Kazakhstan	1	3	2	4	5	9	8	6	7	0.9271	1000
		Kyrgyzstan	1	2	3	4	5	6	9	7	8	0.9494	1000
		Tajikistan	1	2	3	5	4	6	9	7	8	0.9587	1000
		Uzbekistan	1	3	4	2	5	6	9	7	8	0.9593	1000
	2010	Armenia	2	1	3	4	5	9	8	6	7	0.9522	1000
		Azerbaijan	1	4	2	3	5	9	8	6	7	0.9658	1002
		Georgia	1	3	2	4	7	9	8	5	6	0.9507	1000
		Kazakhstan	1	3	2	4	6	9	8	5	7	0.9289	1000
		Kyrgyzstan	1	3	2	5	4	8	9	6	7	0.9533	1016
		Tajikistan	1	3	2	9	4	6	8	5	7	0.9616	1007
		Uzbekistan	1	4	3	2	5	8	9	6	7	0.9673	1500
	2016	Armenia	3	2	1	4	7	9	8	5	6	0.9306	1518
		Azerbaijan	1	4	2	3	7	9	8	6	5	0.9705	1504
		Georgia	1	3	2	4	7	9	8	5	6	0.9057	1506
		Kazakhstan	1	3	2	6	7	9	8	4	5	0.8882	1483
		Kyrgyzstan	1	3	2	7	5	9	8	6	4	0.9219	1499
		Tajikistan	1	3	2	9	4	8	7	5	6	0.9370	1486
		Uzbekistan	1	4	2	3	5	9	8	6	7	0.9128	1462

Approach	*Year*	*Country*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Number of Observations*
Item response theory	2006	Armenia	1	2	3	4	5	7	9	6	8	1000
		Azerbaijan	1	3	2	4	5	7	9	6	8	1000
		Georgia	1	2	3	4	5	6	8	7	9	1000
		Kazakhstan	1	3	2	4	5	9	7	6	8	1000
		Kyrgyzstan	1	2	3	5	4	6	8	7	9	1000
		Tajikistan	1	2	3	5	4	6	9	7	8	1000
		Uzbekistan	1	3	4	2	5	6	7	8	9	1000
	2010	Armenia	1	3	2	4	5	9	8	6	7	1000
		Azerbaijan	1	4	2	3	5	8	9	6	7	1002
		Georgia	1	3	2	4	6	8	9	5	7	1000
		Kazakhstan	1	3	2	4	6	9	8	5	7	1000
		Kyrgyzstan	1	3	2	5	4	7	9	6	8	1016
		Tajikistan	1	3	2	7	4	6	9	5	8	1007
		Uzbekistan	1	4	3	2	5	7	9	6	8	1500
	2016	Armenia	4	2	1	3	7	9	8	5	6	1518
		Azerbaijan	1	4	2	3	7	9	8	6	5	1504
		Georgia	2	3	1	4	8	9	7	5	6	1506
		Kazakhstan	2	3	1	6	8	9	7	4	5	1483
		Kyrgyzstan	2	3	1	7	5	9	8	6	4	1499
		Tajikistan	1	3	2	9	4	8	7	5	6	1486
		Uzbekistan	2	4	1	3	5	9	7	6	8	1462

Table 2a gives then the rank correlations between the countries for the year 2006, for each of the two approaches. Table 2b does the same for the year 2010 and Table 2c for the year 2016.

Table 2a: Between Countries Rank Correlations in 2006, Derived from Table 1

		Armenia	Azerbaijan	Georgia	Kazakhstan	Kyrgyzstan	Tajikistan	Uzbekistan
Paroush Approach	Armenia	1.0000	0.9833	0.9500	0.9667	0.9500	0.9333	0.9000
	Azerbaijan	0.9833	1.0000	0.9333	0.9833	0.9333	0.9167	0.8833
	Georgia	0.9500	0.9333	1.0000	0.8833	1.0000	0.9833	0.9500
	Kazakhstan	0.9667	0.9833	0.8833	1.0000	0.8833	0.8667	0.8333
	Kyrgyzstan	0.9500	0.9333	1.0000	0.8833	1.0000	0.9833	0.9500
	Tajikistan	0.9333	0.9167	0.9833	0.8667	0.9833	1.0000	0.9000
	Uzbekistan	0.9000	0.8833	0.9500	0.8333	0.9500	0.9000	1.0000
Item response theory	Armenia	1.0000	0.9833	0.9667	0.9167	0.9500	0.9667	0.8667
	Azerbaijan	0.9833	1.0000	0.9500	0.9333	0.9333	0.9500	0.8500
	Georgia	0.9667	0.9500	1.0000	0.8833	0.9833	0.9667	0.9333
	Kazakhstan	0.9167	0.9333	0.8833	1.0000	0.8667	0.8500	0.8167
	Kyrgyzstan	0.9500	0.9333	0.9833	0.8667	1.0000	0.9833	0.8833
	Tajikistan	0.9667	0.9500	0.9667	0.8500	0.9833	1.0000	0.8500
	Uzbekistan	0.8667	0.8500	0.9333	0.8167	0.8833	0.8500	1.0000

Note: Between countries rank correlations are calculated using results presented in Table 1 for the Paroush approach and item response theory.

Table 2b: Between Countries Rank Correlations in 2010, Derived from Table 1

		Armenia	Azerbaijan	Georgia	Kazakhstan	Kyrgyzstan	Tajikistan	Uzbekistan
Paroush Approach	Armenia	1.0000	0.9000	0.9000	0.9333	0.9167	0.6500	0.8667
	Azerbaijan	0.9000	1.0000	0.9333	0.9667	0.9333	0.6000	0.9667
	Georgia	0.9000	0.9333	1.0000	0.9833	0.8833	0.6333	0.8833
	Kazakhstan	0.9333	0.9667	0.9833	1.0000	0.9333	0.6833	0.9167
	Kyrgyzstan	0.9167	0.9333	0.8833	0.9333	1.0000	0.8167	0.9000
	Tajikistan	0.6500	0.6000	0.6333	0.6833	0.8167	1.0000	0.5167
	Uzbekistan	0.8667	0.9667	0.8833	0.9167	0.9000	0.5167	1.0000
Item response theory	Armenia	1.0000	0.9667	0.9667	0.9833	0.9333	0.8167	0.9000
	Azerbaijan	0.9667	1.0000	0.9667	0.9500	0.9333	0.8000	0.9667
	Georgia	0.9667	0.9667	1.0000	0.9833	0.9333	0.8500	0.9167
	Kazakhstan	0.9833	0.9500	0.9833	1.0000	0.9000	0.8000	0.8833
	Kyrgyzstan	0.9333	0.9333	0.9333	0.9000	1.0000	0.9500	0.9000
	Tajikistan	0.8167	0.8000	0.8500	0.8000	0.9500	1.0000	0.7500
	Uzbekistan	0.9000	0.9667	0.9167	0.8833	0.9000	0.7500	1.0000

Note: Between countries rank correlations are calculated using results presented in Table 1 for the Paroush approach and item response theory.

Table 2c: Between Countries Rank Correlations in 2016, Derived from Table 1

		Armenia	Azerbaijan	Georgia	Kazakhstan	Kyrgyzstan	Tajikistan	Uzbekistan
Paroush Approach	Armenia	1.0000	0.9000	0.9500	0.9000	0.8000	0.6500	0.8667
	Azerbaijan	0.9000	1.0000	0.9667	0.8833	0.8167	0.5833	0.9333
	Georgia	0.9500	0.9667	1.0000	0.9500	0.8500	0.7000	0.9333
	Kazakhstan	0.9000	0.8833	0.9500	1.0000	0.9167	0.8167	0.8167
	Kyrgyzstan	0.8000	0.8167	0.8500	0.9167	1.0000	0.9000	0.7833
	Tajikistan	0.6500	0.5833	0.7000	0.8167	0.9000	1.0000	0.6500
	Uzbekistan	0.8667	0.9333	0.9333	0.8167	0.7833	0.6500	1.0000
Item response theory	Armenia	1.0000	0.8667	0.9333	0.8500	0.7500	0.5167	0.8500
	Azerbaijan	0.8667	1.0000	0.9333	0.8500	0.8000	0.5833	0.8667
	Georgia	0.9333	0.9333	1.0000	0.9500	0.8000	0.6333	0.8667
	Kazakhstan	0.8500	0.8500	0.9500	1.0000	0.8667	0.7500	0.7333
	Kyrgyzstan	0.7500	0.8000	0.8000	0.8667	1.0000	0.8833	0.7167
	Tajikistan	0.5167	0.5833	0.6333	0.7500	0.8833	1.0000	0.6167
	Uzbekistan	0.8500	0.8667	0.8667	0.7333	0.7167	0.6167	1.0000

Note: Between countries rank correlations are calculated using results presented in Table 1 for the Paroush approach and item response theory.

It appears that, in most cases, these correlations are quite high (generally well above 0.7 or 0.8).

The rank correlations between the different approaches for each country and year are presented in Table 3. Here also the correlations between the two approaches, for a given year, are generally high, whatever the country analyzed.

In Table 4, we give, separately for each approach and country, the correlations between the rankings observed in 2006 and 2010, 2006 and 2016, and 2010 and 2016. Here also the correlations are high.

Table 3: Rank Correlations Between the Two Approaches in 2006, 2010 and 2016

Year	**Country**	**Rank correlation between the Paroush approach and the item response theory**
2006	Armenia	0.9833
	Azerbaijan	0.9833
	Georgia	0.9833
	Kazakhstan	0.9833
	Kyrgyzstan	0.9667
	Tajikistan	1.0000
	Uzbekistan	0.9500
2010	Armenia	0.9500
	Azerbaijan	0.9833
	Georgia	0.9667
	Kazakhstan	1.0000
	Kyrgyzstan	0.9833
	Tajikistan	0.9500
	Uzbekistan	0.9833
2016	Armenia	0.9833
	Azerbaijan	1.0000
	Georgia	0.9667
	Kazakhstan	0.9667
	Kyrgyzstan	0.9833
	Tajikistan	1.0000
	Uzbekistan	0.9667

Table 4: Rank Correlations Between the Orders of Acquisition Observed in 2006, 2010 and 2016

Country	Order	Rank correlation between the orders observed in 2006 and 2010	Rank correlation between the orders observed in 2006 and 2016	Rank correlation between the orders observed in 2010 and 2016
Armenia	**Paroush**	0.9667	0.8667	0.9000
Armenia	**IRT**	0.9333	0.7667	0.8500
Azerbaijan	**Paroush**	0.9667	0.9000	0.9333
Azerbaijan	**IRT**	0.9667	0.8333	0.9167
Georgia	**Paroush**	0.8000	0.8000	1.0000
Georgia	**IRT**	0.8667	0.6833	0.9000
Kazakhstan	**Paroush**	0.9833	0.8667	0.9167
Kazakhstan	**IRT**	0.9667	0.7667	0.8667
Kyrgyzstan	**Paroush**	0.9167	0.6833	0.8667
Kyrgyzstan	**IRT**	0.9500	0.6167	0.7667
Tajikistan	**Paroush**	0.8000	0.7167	0.9500
Tajikistan	**IRT**	0.9167	0.7167	0.8667
Uzbekistan	**Paroush**	0.9333	0.8500	0.9667
Uzbekistan	**IRT**	0.9000	0.7833	0.8833

In Table 5, we give, for the approach of Paroush[4], the cumulative percentages of individuals having zero, one, two, …, nine goods or services. This Table shows clearly that in the Caucasus wealth increased over time, since the percentage of individuals without any goods or services decreased over time, becoming nil for some countries already in 2010. We also observe that the cumulative percentage of the individuals having eight out of the nine goods decreased generally over time, indicating that the percentage of individuals having all the nine goods increased over time.

For the four countries of Central Asia, the story is different, since in Kazakhstan, Kyrgyzstan and Tajikistan there was no one without any of the goods and services in 2006, while the percentage of these individuals was positive in 2006 and 2010. In Uzbekistan, on the contrary, the percentage of individuals without any good or service was positive in 2006 but nil in 2010 and 2016. We also observe that the cumulative percentage of individuals with eight goods decreased over time in Kazakhstan and Kyrgyzstan, did not change in Tajikistan since it was 100% in 2006, 2010 and 2016. In Uzbekistan the percentage was 100% in 2006 and 2010 but slightly smaller in 2016.

[4] The results using item response theory are available upon request form the authors.

Table 5 Cumulative Distribution of the Numbers of Commodities in the Most Common Path of Acquisition (Paroush Approach)[5]

Approach	*Year*	*Country*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Paroush	2006	Armenia	0.597	3.433	12.985	31.493	76.269	94.478	96.269	97.910	99.254	100.000
		Azerbaijan	0.415	15.768	41.079	52.420	90.456	97.787	98.617	99.170	100.000	100.000
		Georgia	0.698	30.541	46.946	62.827	87.086	94.939	98.778	99.127	99.651	100.000
		Kazakhstan	0.000	32.189	46.137	63.519	87.124	94.206	97.210	98.712	99.571	100.000
		Kyrgyzstan	0.000	44.340	66.981	74.686	92.610	97.799	99.371	99.528	100.000	100.000
		Tajikistan	0.000	73.353	87.262	93.119	97.072	99.707	99.854	100.000	100.000	100.000
		Uzbekistan	0.146	21.460	47.737	70.803	90.365	97.810	99.416	99.416	99.708	100.000
	2010	Armenia	0.160	0.320	2.400	21.760	65.920	82.080	86.880	95.840	98.560	100.000
		Azerbaijan	0.000	0.562	17.275	27.669	78.230	92.416	94.242	99.579	99.860	100.000
		Georgia	0.000	13.863	35.358	56.386	77.103	78.505	90.966	97.664	99.377	100.000
		Kazakhstan	0.214	2.998	12.848	29.122	58.887	73.662	81.585	96.788	98.287	100.000
		Kyrgyzstan	0.000	2.080	33.280	73.600	92.160	94.880	96.800	99.840	100.000	100.000
		Tajikistan	0.291	11.773	57.994	81.977	94.767	98.401	99.709	99.855	100.000	100.000
		Uzbekistan	0.000	0.366	8.700	30.037	75.183	94.231	98.077	99.634	100.000	100.000
	2016	Armenia	0.000	0.264	3.689	9.486	27.404	30.830	59.684	88.538	98.024	100.000
		Azerbaijan	0.000	0.176	2.993	5.546	40.669	55.194	74.736	98.944	100.000	100.000
		Georgia	0.000	1.029	10.635	23.499	39.966	42.367	57.461	76.158	95.197	100.000
		Kazakhstan	0.225	1.348	6.966	11.461	12.809	22.921	42.022	70.337	97.303	100.000
		Kyrgyzstan	0.295	1.329	18.612	40.768	54.505	69.719	84.638	95.716	99.705	100.000
		Tajikistan	0.123	1.601	50.739	80.296	88.300	91.995	96.921	99.384	100.000	100.000
		Uzbekistan	0.000	1.190	28.061	40.136	66.667	77.551	84.524	94.728	99.660	100.000

[5] Results based on item response theory may be obtained upon request from the authors.

Table 5 allows us also to compute various ordinal inequality indices. The results are presented for each country and separately for 2006, 2010 and 2016 in Table 6. In 2006 it appears, whatever index we look at, that inequality was highest Kazakhstan and lowest in Tajikistan. In 2016 inequality was highest in Georgia and lowest again in Tajikistan. If we look at the changes over time, we see that in Armenia inequality increased between 2006 and 2010 as well as between 2010 and 2016. In Azerbaijan inequality decreased between 2006 and 2010 but increased between 20010 and 2016. In Georgia inequality increased during both sub-periods. In Kazakhstan inequality did not vary much from one period to the other. In Kyrgyzstan inequality decreased between 2006 and 2010 but increased between 2010 and 2016, reaching then a much higher level than the one observed in 2006. In Tajikistan inequality increased during both periods but remained much lower than in the other countries surveyed. Finally in Uzbekistan inequality decreased between 2006 and 2010 but increased between 2006 and 2010, but its level in 2016 was much higher than what it was in 2006.

Note that these trends may be observed, whatever ordinal inequality index one selects in Table 6.

Table 6: Ordinal Inequality Indices

Approach	***Year***	***Country***	***Reardon Index***	***Abul Naga – Yalcin Index***	***Lv, Wang and Xu Index with parameter α equal to 0.1***	***Lv, Wang and Xu Index with parameter α equal to 0.5***	***Lv, Wang and Xu Index with parameter α equal to 0.9***
Paroush	2006	Armenia	0.2955	0.2985	0.0000	0.0042	0.3078
		Azerbaijan	0.3370	0.3751	0.0000	0.0043	0.3254
		Georgia	0.3940	0.4128	0.0000	0.0063	0.3528
		Kazakhstan	0.4042	0.4177	0.0000	0.0071	0.3529
		Kyrgyzstan	0.3369	0.3630	0.0000	0.0047	0.3129
		Tajikistan	0.1793	0.2214	0.0000	0.0022	0.1863
		Uzbekistan	0.3330	0.3596	0.0000	0.0045	0.3380
	2010	Armenia	0.3281	0.3230	0.0000	0.0048	0.3177
		Azerbaijan	0.2884	0.2915	0.0000	0.0035	0.2924
		Georgia	0.4668	0.4427	0.0000	0.0094	0.3819
		Kazakhstan	0.4373	0.4133	0.0000	0.0076	0.3732
		Kyrgyzstan	0.2623	0.2846	0.0000	0.0033	0.2946
		Tajikistan	0.2524	0.2875	0.0000	0.0028	0.2901
		Uzbekistan	0.2474	0.2711	0.0000	0.0026	0.2917
	2016	Armenia	0.3990	0.3898	0.0000	0.0061	0.3521

		Azerbaijan	0.3427	0.3789	0.0000	0.0040	0.3291
		Georgia	0.5515	0.5304	0.0000	0.0128	0.4096
		Kazakhstan	0.4216	0.4002	0.0000	0.0094	0.3617
		Kyrgyzstan	0.4631	0.4594	0.0000	0.0079	0.3856
		Tajikistan	0.2836	0.3202	0.0000	0.0042	0.2849
		Uzbekistan	0.4597	0.4361	0.0000	0.0084	0.3752
Item response theory	2006	Armenia	0.2840	0.2920	0.0000	0.0040	0.3029
		Azerbaijan	0.3324	0.3725	0.0000	0.0042	0.3237
		Georgia	0.3859	0.4086	0.0000	0.0057	0.3501
		Kazakhstan	0.3900	0.4101	0.0000	0.0065	0.3477
		Kyrgyzstan	0.2792	0.3182	0.0000	0.0037	0.2795
		Tajikistan	0.1793	0.2214	0.0000	0.0022	0.1863
		Uzbekistan	0.3299	0.3580	0.0000	0.0042	0.3371
	2010	Armenia	0.3230	0.3204	0.0000	0.0046	0.3149
		Azerbaijan	0.2919	0.2935	0.0000	0.0036	0.2938
		Georgia	0.4088	0.3938	0.0000	0.0083	0.3593
		Kazakhstan	0.4373	0.4133	0.0000	0.0076	0.3732
		Kyrgyzstan	0.2247	0.2634	0.0000	0.0023	0.2805
		Tajikistan	0.2474	0.2844	0.0000	0.0026	0.2884
	2016	Armenia	0.3797	0.3755	0.0000	0.0057	0.3434
		Azerbaijan	0.3427	0.3789	0.0000	0.0040	0.3291
		Georgia	0.5571	0.5398	0.0000	0.0128	0.4102
		Kazakhstan	0.4561	0.4436	0.0001	0.0105	0.3718
		Kyrgyzstan	0.4640	0.4598	0.0000	0.0079	0.3859
		Tajikistan	0.2836	0.3202	0.0000	0.0042	0.2849
		Uzbekistan	0.4249	0.4112	0.0000	0.0075	0.3612

In Table 7 we present results concerning the welfare index introduced by Apouey et al. (2019). As mentioned previously, when the parameter α tend towards 1, this index ignores inequality in the standards of living so that its change over time reflects pure growth. It then appears that in all the countries, but Uzbekistan, there was important growth in both sub-periods, 2006-2010 and 2010-2016. In Uzbekistan there was growth only during the first sub-period. If we now take a different value of the parameter α, like 0.9 or 0.5, we look at a welfare change that takes into account both growth and inequality change. It then appears, that welfare increased in both sub-periods for all the countries except again Uzbekistan. In the latter country welfare increased only during the first sub-period. These conclusions hold whether we assume that α is equal to 0.9 or 0.5. Note that these conclusions are based on ownership frequencies derived from the Paroush approach[6].

Table 7: Apouey, Silber and Xu Welfare Index I_{SX} in 2006, 2010 and 2016 and 2013 (Paroush approach)

Year	Country	I_{SX} with $\alpha \to 1$	I_{SX} with $\alpha = 0.9$	I_{SX} with $\alpha = 0.5$	I_{SX} with $\alpha = 0.1$
2006	Armenia	0.43	0.54	0.90	0.990
	Azerbaijan	0.34	0.44	0.82	0.978
	Georgia	0.31	0.40	0.77	0.961
	Kazakhstan	0.31	0.40	0.77	0.966
	Kyrgyzstan	0.25	0.33	0.70	0.953
	Tajikistan	0.17	0.23	0.59	0.925
	Uzbekistan	0.30	0.40	0.78	0.974
2010	Armenia	0.50	0.60	0.94	0.998
	Azerbaijan	0.43	0.54	0.91	0.998
	Georgia	0.39	0.48	0.84	0.984
	Kazakhstan	0.50	0.59	0.92	0.994
	Kyrgyzstan	0.34	0.44	0.85	0.994
	Tajikistan	0.28	0.38	0.79	0.981
	Uzbekistan	0.44	0.55	0.92	0.999
2016	Armenia	0.65	0.74	0.97	0.999
	Azerbaijan	0.58	0.68	0.96	0.999
	Georgia	0.62	0.70	0.94	0.998
	Kazakhstan	0.71	0.78	0.96	0.996
	Kyrgyzstan	0.48	0.58	0.91	0.994
	Tajikistan	0.32	0.42	0.83	0.992
	Uzbekistan	0.45	0.55	0.89	0.996

[6] Results based on item response theory are available upon request from the authors.

In Table 8 we give the per capita G.D.P. (computed at PPP) and the Gini index in 2017, as published by the World Bank. We then combine these two measures to derive a welfare index which corresponds to what Atkinson called "equally distributed equivalent level of income", the only difference being that we use the per capita G.D.P. rather than disposable income, and the Gini index rather than the Atkinson index of inequality. Data on the Gini index and hence on the welfare index are not available in 2017 for Azerbaijan and Uzbekistan. Among the five remaining countries, it appears Kazakhstan has the highest per capita G.D.P. and highest level of welfare and Tajikistan the lowest.

Table 8: Per capita GDP, Gini index and welfare* measure in 2017

Country	Per capita G.D.P. (PPP) ($)	Gini Index	Welfare Measure
Armenia	**9648**	**0.336 (2017)**	**6406**
Azerbaijan	**17398**		
Georgia	**10699**	**0.379 (2017)**	**6644**
Kazakhstan	**26410**	**0.275 (2017)**	**19147**
Kyrgyzstan	**3726**	**0.273 (2017)**	**2970**
Tajikistan	**3180**	**0.340 (2015)**	**2099**
Uzbekistan	**6865**		

* "equally distributed equivalent level of per capita GDP", using the Gini index.

We then compare the results of Table 8 with those presented in Tables 6 and 7 where we computed ordinal inequality indices and inequality sensitive measures of achievement (welfare) on the basis of the notion of order of acquisition of assets. Our comparison focuses however only on the ranking of the different countries. It first appears that the ranking of the seven countries obtained on the basis of the per capita G.D.P. and that of the Apouey et al. (2019) index when the parameter $\alpha \to 1$ (the case where this welfare measure ignores inequality) is almost identical, the only difference being that Kyrgyzstan and Uzbekistan permuted their rank (5 and 6).

Differences in ranking are more important when comparing the Gini index and the Reardon ordinal inequality index. In both cases Georgia has the highest level of inequality and Armenia and Kazakhstan permuted their rank (3 and 4). The main difference is that according to the Gini index Kyrgyzstan has the lowest level of inequality (among these five countries) while Tajikistan has rank 2, the opposite being true when working with the Reardon ordinal inequality index.

Finally when comparing the Gini based "equally distributed level of per capita G.D.P." and the Apouey et al. (2019) welfare index, we may observe that the ranking is almost identical when the parameter α is equal to 0.9, the only difference being that Armenia and Georgia permuted their ranks (2 and 3). The differences

in ranking are somehow more important when comparing the Gini index and the Apouey et al. (2019) welfare index when the parameter α is equal to 0.5.

Table 9: Comparing the ranking of countries

Country	Per capita GDP in 2017	Apouey et al. index with $\alpha \to 1$ (2016)	Gini index (in 2017)	Reardon index in 2016 (Paroush approach)	"Equally distributed level of per capita GDP" in 2017	Apouey et al. index with $\alpha=0.9$ (2016	Apouey et al. index with $\alpha=0.5$ (2016
Armenia	**4**	**2**	**3**	**4**	**3**	**2**	**1**
Azerbaijan	**2**	**4**	**n.a.**		**n.a.**		
Georgia	**3**	**3**	**1**	**1**	**2**	**3**	**3**
Kazakhstan	**1**	**1**	**4**	**3**	**1**	**1**	**2**
Kyrgyzstan	**6**	**5**	**5**	**2**	**4**	**4**	**4**
Tajikistan	**7**	**7**	**2**	**5**	**5**	**5**	**5**
Uzbekistan	**5**	**6**	**n.a.**		**n.a.**		

5. Concluding comments

This paper takes an ordinal approach to using data on assets when estimating the wealth of a household (or individual). This approach is based upon the conjecture that there tends to be an order of acquisition of durable goods. More specifically, we assume that households behave as if they were implicitly assigning an order of importance to the various assets that they may acquire.

Our empirical analysis, based on data collected by the Caucasus Barometer and the Life in Transition Survey and covering states in South Caucasus and Central Asia, shows that there exist such an order and that it does not really depend on the statistical approach adopted and was quite similar in 2006, 2010 and 2016.

Using ordinal inequality indices we observed that during the first sub-period (2006-2010) inequality increased in Armenia, Georgia, Kazakhstan, and Tajikistan but decreased in Azerbaijan, Kyrgyzstan and Uzbekistan. But in the second sub-period (2010-2016) inequality increased in all seven countries, the increase in Kazakhstan being relatively small.

Using the Apouey et al. (2019) welfare index when the parameter α was close to 1, we were able to conclude that there was significant growth in all seven countries, in both sub-periods. When taking inequality into account, we found that there was in all sub-periods an increase in welfare, even when inequality rose, the only exception being Uzbekistan where welfare did not really increase between 2006 and 2010. This clearly

indicates that growth was sufficiently important to counteract the negative effect on welfare of an increase in inequality in those countries and sub-periods where inequality increased.

Finally we attempted to compare our results with what a more traditional approach, using the Gini index and data on per capita G.D.P would give. It then appears that the ranking of the five countries for which we had also relevant data on the Gini index, was quite similar, at least, as far as the standard of living and welfare are concerned.

References

Abul Naga, R. H., and T. Yalcin. 2008. Inequality Measurement for Ordered Response Health Data. *Journal of Health Economics* 27(6): 1614–1625.

Ainsworth, M., and D. Filmer. 2006. Children's Schooling: AIDS, Orphanhood, Poverty, and Gender. *World Development* 34: 1099–1128.

Allison, R. A., and J. Foster. 2004. Measuring Health Inequalities Using Qualitative Data. *Journal of Health Economics* 23: 505–524.

Ando, A. and F. Modigliani (1963) "The "Life Cycle" Hypothesis of Saving: Aggregate implications and Tests," *American Economic Review* 53(1), Part 1:. 55–84.

Apouey, B., J. Silber and Y. Xu (2019) "On inequality-sensitive and additive achievement measures based on ordinal data," *Review of Income and Wealth*. Available at https://doi.org/10.1111/roiw.12427

Ayala, L., and C. Navarro. 2007. The Dynamics of Housing Deprivation. *Journal of Housing Economics* 16(1): 72–97.

Ayala, L., and C. Navarro. 2008. Multidimensional Housing Deprivation Indices: With Application to Spain. *Applied Economics* 40(5): 597–611.

Benzécri, J.-P., and F. Benzécri. 1980. *Pratique de L'Analyse des Données, I, Analyse des Correspondances*, *Exposé Elémentaire*, Paris: Dunod Bordas.

Bollen, K. A., J. L. Glanville, and G. Stecklov. 2002. Economic Status Proxies in Studies of Fertility in Developing Countries: Does the Measure Matter?. *Population Studies* 56: 81–96.

Capannelli, G. and R. Kanbur, eds (2019a) *Good Jobs for Inclusive Growth in Central Asia and the South Caucasus. Regional Report*, Asian Development Bank, Manila, Philippines.

Capannelli, G. and R. Kanbur (2019b) "Introduction and Overview," Chapter 1 in Capannelli and Kanbur, op. cit.

Cappellari, L., and S. P. Jenkins. 2007. Summarizing Multiple Deprivation Indicators. In S. P. Jenkins and J. Micklewright (Eds.), *Inequality and Poverty: Re-examined* (pp. 166–184). Oxford: Oxford University Press.

Das, J., S. Dercon, J. Habyarimana, and P. Krishnan. 2004. Public and Private Funding of Basic Education in Zambia: Implications of Budgetary Allocations for Service Delivery. (Africa Region Human Development Working Paper Series). Washington, DC: The World Bank. Retrieved from http://siteresources.worldbank.org/AFRICAEXT/Resources/AFRHD62DasZambia022504FINAL.pdf

Deutsch, J. and J. Silber. 2008. The Order of Acquisition of Durable Goods and the Multidimensional Measurement of Poverty. In N. Kakwani and J. Silber (Eds.), *Quantitative approaches to multidimensional poverty measurement*. Basingstoke: Palgrave-Macmillan.

Deutsch, J., A. C. Guio, M. Pomati, and J. Silber .2015. Material Deprivation in Europe: Which Expenditures are Curtailed First? *Social Indicators Research*, 120: 723–740.

Deutsch, J., J. Silber and G. Wan (2017) "Improved Indicators for Income Inequality Using Asset Indexes and Dichotomous Variables: The Case of Central and Western Asia," Background Thematic Paper of the report of Capannelli and Kanbur (2019a).

Dickes, P. 1983. Modèle de Rasch pour items dichotomiques: Théorie, Technique et application à la mesure de la pauvreté. Nancy: Université de Nancy II.

Dickes, P. 1989. Pauvreté et Conditions d'Existence. Théories, Modèles et Mesures. Document PSELL no. 8. Walferdange: CEPS/INSTEAD.

Filmer, D., and L. Pritchett. 1999. The Effect of Household Wealth on Educational Attainment: Evidence from 35 Countries. *Population and Development Review* 25: 85–120.

Filmer, D., and L. Pritchett 2001. Estimating Wealth Effects Without Expenditure Data—or Tears: With an Application to Educational Enrollments in States of India. *Demography* 38: 115–132.

Filmer, D., and K. Scott. 2012. Assessing Asset Indices. *Demography* 49: 359–392.

Friedman, M. (1957) *A Theory of the Consumption Function*, Princeton University Press.

Fusco, A., and P. Dickes. 2008. The Rasch model and multidimensional poverty measurement. In N. Kakwani and J. Silber (Eds.), *Quantitative Approaches to Multidimensional Poverty Measurement* (pp. 49–62). New York: Palgrave Macmillan.

Gailly, B., and P. Hausman. 1984. Des Désavantages Relatifs à une Mesure Objective de la Pauvreté. In G. Sarpellon (Ed.), *Understanding Poverty* (pp. 192–216). Milan: Franco Angeli.

Ganiev, B. (2019) "Economic Performance and Prospects," Chapter 2 in Capannelli and Kanbur, op. cit.

Guio, A.-C., D. Gordon, and E. Marlier. 2012. Measuring Material Deprivation in the EU: Indicators for the Whole Population and Child-specific Indicators. Eurostat Methodologies and working papers, Luxembourg: Office for Official Publications of the European Communities (OPOCE).

Guttman, L. 1950. *The Basis for Scalogram Analysis*. In Stouffer et al. Measurement and Prediction, *The American Soldier*, Vol IV. New York: Wiley.

Gwatkin, D. R., S. Rutstein, K. Johnson, R. Pande, and A. Wagstaff. 2000. Socio-economic Differences in Health, Nutrition, and Population Within Developing Countries. Washington, DC: HNP/Poverty Thematic Group, The World Bank. Retrieved from http://web.worldbank.org/WBSITE/EXTERNAL/TOPICS/EXTHEALTHNUTRITIONANDPOPULATION/EXTPAH/0,,contentMDK:20219043~ menuPK:400482~pagePK:148956~piPK:216618~theSitePK:400476,00.html

Kanbur, R. and J. Zhuang (2019) "Growth and Inclusiveness," Chapter 3 in Capannelli and Kanbur, op. cit.

Lazar, A., and J. Silber. 2013. On the Cardinal Measurement of Health Inequality When Only Ordinal Information is Available on Individual Health Status. *Health Economics* 22: 106–113.

Life in Transition Survey. 2016. European Bank for Reconstruction and Development, London, UK.

Lv, G., Y. Wang, and Y. Xu. 2015. On a New Class of Measures for Health Inequality Based on Ordinal Data. *Journal of Economic Inequality* 13(3): 465–477.

Modigliani, F. and R. Brumberg (1954) "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," In Kurihara, K., Ed., *Post Keynesian Economics*, New Brunswick: Rutgers University Press, pp. 388-436.

Paroush, J. 1965. The Order of Acquisition of Consumer Durables. *Econometrica* 33(1): 225–235.

Paroush, J. 1973. Efficient Purchasing Behavior and Order Relations in Consumption. *Kyklos* 26(1): 91–112.

Pérez-Mayo, J. 2004. Consistent Poverty Dynamics in Spain. IRISS Working Paper Series N0. 2004–09, Differdange, Luxembourg.

Pérez-Mayo, J. 2005. Identifying Deprivation in Spain: A New Approach. *Applied Economics* 37: 843–955.

Pomfret, R. (2019) *The Central Asian Economies in the Twenty-First Century: Paving a New Silk Road*. Princeton University Press, Princeton, NJ.

Reardon, S. F. 2009. Measures of Ordinal Segregation. *Research on Economic Inequality*, vol. 17. Emerald: Bingley, UK; 129–155.

Sahn, D. E., and Stifel, D. 2003. Exploring alternative measures of welfare in the absence of expenditure data. *Review of Income and Wealth* 49: 463–489.

Schady, N., and M. C. Araujo. 2008. Cash transfers, conditions, and school enrollment in Ecuador. *Economía* 8(2): 43–70.

Stifel, D., and Christiaensen, L. 2007. Tracking poverty over time in the absence of comparable consumption data. *World Bank Economic Review* 21: 317–341.

Szeles, M., and A. Fusco, A. 2013. Item response theory and the measurement of deprivation: Evidence from Luxembourg data. *Quality & Quantity* 47(3): 15–45.

Zheng, B. 2011. A New Approach to Measure Socioeconomic Inequality in Health. *Journal of Economic Inequality* 9(4): 555–577.

Appendix 1: On Item Response Theory
and the Concept of Order of Acquisition of Durable Goods

Item Response Theory (IRT)

IRT was originally introduced to analyze the results of psychometric tests, the idea being that the probability of a correct response to an item is a mathematical function of person and item parameters. The person's parameter is considered as a single latent trait, like the degree of intelligence of the individual. The different parameters of IRT are assumed to measure the difficulty of the question, its discriminatory power (slope) indicating how steeply the rate of success of individuals varies with their ability. There is also a third parameter which we ignore to simplify the presentation.
IRT produces S-shaped curves for each question and the difficulty of a question is represented by the position reached by the curve on the x-axis when there is a probability of 0.5 on the y-axis (this axis varies evidently between 0 and 1). These item response curves allows on to rank the different items according to their position on the latent scale.
Item response theory was later applied to the measurement of deprivation, the idea being that poverty is a latent variable difficult to measure. For illustrations of the use of IRT in poverty analysis, see, for example, Dickes (1983; 1989), Gailly and Hausman (1984), Pérez-Mayo (2004; 2005), Cappellari and Jenkins (2006), Ayala and Navarro (2007; 2008), Fusco and Dickes (2008), Guio, Gordon and Marlier (2012), Szeles and Fusco (2013).

The Order of Acquisition of Durable Goods

This approach is probably less known and we will present it in somehow greater detail.
Assume that consumers can buy three durable goods, D1, D2 and D3. Table 1-1 shows all the possible ownership combinations of these three goods. It is easy to derive that there are 2^3=8 possible outcomes. In Table 1-1, the digit 1 (0) indicates that the consumer owns (does not own) the good and each possible outcome is called a consumers' profile.

Table 1-1: Possible Consumer Profiles when there are Three Goods

Ownership Profile	The household owns Good D1	The household owns Good D2	The household owns Good D3
1	0	0	0
2	1	0	0
3	1	1	0
4	1	1	1
5	0	1	1
6	0	1	0
7	0	0	1
8	1	0	1

Suppose now that the order of acquisition is D1, D2 and D3. It should then be clear that the only possible outcomes are profiles 1 to 4 and that no consumer will have profiles 5 to 8. In such a case one could say that there is a perfect scale, in the sense that there will be a one-to-one correspondence between the profile of the consumer and his/her ranking in the wealth scale.

It is however likely that some consumers will deviate from the path of acquisition. Imagine, for example, a consumer with profile 6 in Table 1-1. If the order of acquisition is D1, D2, D3, and if we use as measure of distance the sum of the absolute values of the differences between the numbers appearing in profile 6 and those corresponding to profiles 1 to 4 (the only possible profiles), then the closest profiles in the path of acquisition of this consumer will be profiles 1 or 3 with a deviation S equal to 1 (see, Table 1-1).

More generally, call S_i the smallest deviation for an individual (or household) with profile *i* and N_i the number of such individuals (households). Guttman (1950) then defined what he called a "reproducibility index" R, as

$$R = 1 - \frac{\sum_i N_i S_i}{k \sum_i N_i} \qquad (1\text{-}1)$$

In (1-1) k is the number of goods; S_i is the deviation and N_i the number of individuals with deviation S_i. Guttman (1950) proved that this index R varies between 0.5 and 1. When there is a perfect scale $S_i = 0$ for all consumers and R=1.

The computation of this reproducibility index, is made, assuming a given order of acquisition. Paroush (1965, 1973) suggested to compute the coefficients of reproducibility for all possible orders of acquisition. The population most common order of acquisition will then be considered as that order of acquisition which has the highest coefficient[7] R.

It should be clear that the estimation of the most common order of acquisition requires a very high number of computations. Assume, as was the case in our empirical investigation, that there are nine goods. Then, for each individual household i in the sample, the determination of the minimum distance S_i from his/her profile to one of the possible profiles in the path of acquisition is based on ten comparisons.

For example, the number of respondents for the survey conducted in Armenia in 2016 was 1518. As a consequence, 15180 (=1518× 10) comparisons are needed in order to determine the reproducibility index R for a given order of acquisition. But this procedure has to be repeated 362880 (= 9!) times. This is the total number of possible orders of acquisition, resulting from nine durable goods. Therefore the total number of iterations needed to find the order of acquisition with the highest index of reproducibility R is 15,180 × 362,880 = 5508518400.
Note that this kind of computation had to be made for each of the seven countries and each of the three years.

[7] Paroush added that this reproducibility coefficient should however be greater than 0.9.

Appendix 2: Measuring inequality in the presence of ordinal variables

Following the work of Allison and Foster (2004) a new inequality index was introduce by Abul Naga and Yalcin (2008), which can be used with ordinal variables. Let p_k be the proportion of individuals with wealth status W_k. Assume that the K wealth status categories are ordered by increasing wealth status and call $P_k = (p_1 + \cdots + p_k)$, the sum of the proportions p_k. Abul Naga and Yalcin (2008) then proposed to use the following index I_{AY} to measuring ordinal inequality:

$$I_{AY} = 1 - \left[\frac{2\sum_{k=1}^{K-1}|P_k - 1| - 1}{(K-1)}\right]. \quad (2\text{-}1)$$

Note that, when using this index, changing the numerical scale does not change the value of the index. In addition, inequality is minimal when everyone is located at the median (i.e., has the same wealth status). Conversely, inequality is maximal (equal to 1) if half the population is located at the lowest wealth status and the other half at the highest wealth status.

Other ordinal inequality indices have been proposed (Reardon 2009; Lazar and Silber 2013), the first being:

$$I_{REARDON} = \frac{1}{(K-1)}\sum_{k=1}^{K-1} 4P_k(1 - P_k) \quad (2\text{-}2)$$

The approach of Lv et al. (2015) amounts to measuring overall wealth inequality (in the case of ordinal variables) by first measuring the inequality between any two different wealth outcomes, then aggregating these inequalities via a simple weighted sum, in which the further apart the two wealth outcomes, the higher the weight attached to the inequality between these two wealth outcomes. They proposed the two following indices:

$$I_{LWX1} = \sum_{k=1}^{K}\sum_{h\neq k}\left(\frac{2}{(K-1)}\right)|h - k| f_h f_k \quad (2\text{-}3)$$

where K is the number of possible wealth outcomes and f_h and f_k are the proportion of individuals with wealth outcomes h and k, respectively, and

$$I_{LWX2} = \sum_{k=1}^{K}\sum_{h\neq k}\alpha^{K-1-|h-k|} f_h f_k \quad (2\text{-}4)$$

with $0 < \alpha < 1$.

Desirable properties of ordinal inequality indices

Lv et al. (2015) showed that the two previous indices obey a certain number of axioms. In the case of wealth inequality, when wealth is an ordinal variable, these axioms may be stated as follows:

- **Focus:** This property implies that any additional information about individuals, such as their gender or age, should not play any role in constructing an index of wealth inequality.
- **Additivity:** An index of wealth inequality should be the sum of all "individual" wealth inequalities. The measure sums up all possible inequalities of any two different wealth outcomes.
- **Independence**: This property requires that any change in the degree of wealth inequality between two wealth outcomes, h_i and h_j, as a consequence, say, of an increase in the frequency of wealth outcome, h_j, is independent of the wealth frequency of wealth outcome, h_j.
- **Perfect equality**: If everyone has the same wealth outcome, then wealth inequality is equal to zero.
- **Invariance to simple switches**: When all individual wealth outcomes are clustered on two wealth outcomes, a simple switch of the frequencies of these two wealth outcomes leaves the index of wealth inequality unchanged.
- **Invariance to parallel shifts**: When all individual wealth outcomes are clustered on two wealth outcomes, a parallel shift of the entire frequency distribution leaves the index of wealth inequality unchanged.
- **Polarization**: A "median preserving" change in the spread of a frequency distribution increases its inequality. A simple illustration is a move from the distribution $\{0, 0.2, 0.8, 0\}$ to the distribution $\{0.2, 0, 0, 0.8\}$.

Appendix 3: Inequality-sensitive and additive achievement measures based on ordinal data

In a recent study, Apouey et al. (2019) derived axiomatically new classes of measures of the level of achievement in a population when the achievement variable is ordinal. The main result of their analysis is that a social achievement index h satisfies Normalization, Independence, Weak Pareto Principle, Anonymity, Equity Principle, and Proportional Equality if and only if

$$h(s) = \frac{1}{T}\sum_{k=1}^{K} p_k(s)\frac{1-\alpha^{K-k}}{1-\alpha^{K-1}} \quad (3\text{-}1)$$

with $0 < \alpha < 1$ and where *s* refers to the achievements, ranked by decreasing levels, *K* to the number of achievement categories, $p_k(s)$ to the number of individuals with achievement level *k* and *T* to the total number of individuals. Normalization implies that the social achievement index varies between 0 and 1. The axiom of independence assumes that if the achievement of one individual changes without affecting the achievement of any other individual, the resulting change in the social achievement index is independent of the initial achievements of those other individuals. Given that the achievement are ranked by decreasing level, the Weak Pareto Principle implies that if every individual has the same achievement level k, the social achievement will be higher than if every individual has the same achievement level k', with $k' < k$. Anonymity stipulates that, in the measurement of social achievement from individual achievements, only the level of achievements matters, the other characteristics of the individuals having no impact on the overall level of social achievement. The Equity principle requires that, other things being the same, changes in the achievements of two individuals from two further-apart levels to two "closer" levels will increase the level of social achievement. Finally, the interpretation of the axiom of Proportional Equality is as follows. Assume an original achievement vector, where everyone has the same achievement level k. If now the achievement level of everyone changes one level up, to $k + 1$, or if the achievement level of everyone changes one level down, to $k - 1$, the ratio of the changes in the social achievement index of two such changes is independent of the initial achievement level k: $\frac{h(k-1,\ldots,k-1)-h(k,\ldots,k)}{h(k,\ldots,k)-h(k+1,\ldots,k+1)}$ is independent of k.

When the parameter α tends towards 1, the axioms of equity (and evidently proportional equality) will not hold and the social achievement index will be expressed as

$$h(s) = \frac{1}{T}\sum_{k=1}^{K} p_k(s)\frac{K-k}{K-1} \qquad (3\text{-}2)$$

It is easy to show that in such a case we obtain

$$h(s) = \frac{1}{(K-1)}\sum_{k=1}^{K-1} F_k(s) \qquad (3\text{-}3)$$

where $F_k(s) = \sum_{j=1}^{k} p_j(s)$, that is, $F_k(s)$ refers to the cumulative relative frequency of the various achievement categories.