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The Dimensions of Ordinal Well-Being Indexes: Using Orthogonal Weighting with the Kids Count Index

Ross Knippenberg (American Veterinary Medical Association)

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Ross Knippenberg †

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Abstract

Well-being indices such as the Gallup-Healthways Well-Being Index, the Kids Count Index, the Opportunity Index, the Healthy Planet Index, and the Human Development Index combine large amounts of data about geographic areas into a single number in order to compare economic and social well-being across time and space. The computation of these types of indices is all too often left without scrutiny. Are the proper variables being used? And what is the best way to determine the right variables? Even if the correct variables are used, what should be their relative importance? Many indices attempt to avoid these questions by weighting all variables equally, using the variables from a related index, or basing the calculations solely on availability of public data. This paper uses the example of the annual Annie E. Casey Foundation's Kid Count Index. The purpose of the Kids Count Index is to provide an objective and consistent third-party measurement of child welfare in every U.S. state. This paper suggests using a methodology based on Knippenberg (2014) that weights the variables into orthogonal dimensions. Variables are not the same thing as dimensions, so while most indices weight variables equally, this study recommends orthogonalization of the data so that dimensions are weighted equally. This minimizes the effect of correlation between variables. This procedure ensures that all underlying dimensions are equally represented in the final index calculation. This study recommends that well-being indices be constructed with special attention paid to the system of weighting between variables.

Keywords: Index Number Theory, Orthogonalization, Principal Components Analysis, Law of Cosines, Distance Metrics, Euclidean Distance, Kids Count Index, Well-Being Indices

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†Economics Division, American Veterinary Medical Association. Email: rknippenberg@avma.org

1 Introduction

In the widely popular book *Freakonomics*, authors Steven Levitt and Stephen Dubner ponder how to measure the effect of parenting on children. In one passage they write:

> "Still, the question of how much parents matter is a good one. It is also terribly complicated. In determining a parent's influence, which dimension of the child are we measuring: his personality? His school grades? His moral behavior? His creative abilities? His salary as an adult? And *what weight should we assign each of the many inputs* that affect a child's outcome: genes, family environment, socioeconomic level, schooling, discrimination, luck, illness, and so on?" [emphasis added] (Levitt and Dubner 2005, pg 141).

The authors posed this question under the assumption that it is unanswerable. In essence, it is the same question that the Gallup-Healthways Health Index and the Annie E. Casey Foundation's Kids Count Index attempt to answer. The idea of what variables or dimensions to include and what weights to assign those variables is a complicated issue. Most researchers are so unsure as to the correct weights that they either give arbitrary, equal weights to all the data involved, or they do not even attempt to discuss it. Many researchers are not even aware that when creating an index, they think that they are avoiding the issue by choosing to use an obscure statistical distance metric, but every known distance metric makes the implicit assumption that each variable included has an equal weight.

On the other hand, those researchers sho do decide to give different weights to variables are opening themselves to a great deal of criticism. A recent article in *The Economist*, "How to lie with indices" (2014a) lampooned creators of indices, particularly those of international country rankings. The point of the article was to reveal to the casual reader that very few people know how to create weights in index numbers, and thus also in ranking systems that are simply observations ordered by the value of their index number. The methodology, the author claims, is aimed to:

> "Get the presentation right. Leaving your methodology unpublished looks dodgy. Instead, bury a brief but baffling description in an obscure corner of your website, and reserve the home page for celebrity endorsements. Get headlines by hamming up small differences; minor year-on-year moves in the rankings may be statistical noise, but they make great copy." (*The Economist* 2014a)

The author goes on to offer a sardonic observation about indices for good causes being used too casually:

> "From human suffering to perceptions of corruption, from freedom to children's happiness, nowadays no social problem or public policy lacks one." (*The Economist* 2014a)

In this paper I will posit two answers to the question of index weighting, the first of which uses a unique algorithm based on the Law of Cosines, and the second of which uses the well-known statistical procedure of principal components. Both of these methods use statistical techniques to decide the dimensions of the data included as well as the weights assigned to each dimension. The principal components method is for comparison. The way that the well-being index is computed in this paper is very general and can be applied to any number of index numbers or to compute ordinal rankings based on index numbers.

The importance of including particular variables in wellness indicators is a hot topic. In particular, Decancq and Neumann (2014) compare five measures of well-being and point out that, while there are many similarities, the choice of an index and its ultimate use as a policy tool can have major implications based on the index used. Furthermore, the choice of an index has no "right" answer.

As the Economist author above notes, well-being indices have recently become popular. The Gallup HealthWays Well-Being Index is one such example. This index is an attempt to gather data from and rank all of the countries in the world on dimensions of well-being. Another popular well-being index, targeted specifically at children in the United States, is the Kids Count Index from the Annie E. Casey Foundation. The Kids Count Index uses 16 indicators to rank all of the U.S. states according to the well-being of children, with the goal that resources will be allocated according to need and that practices in successful states can be identified and replicated elsewhere.

The recent interest in summary indices, especially ones comparing geographic regions, has ignited a unique line of methodological literature looking at how these types of indices are created, what variables they contain, and how the result does or does not reflect reality.

Ordinal rankings pose a special challenge when creating index numbers because no distance function can be naturally defined. Sonne-Schmidt, Tarp and Østerdal (2015) suggest the solution is to use of a measurement of median outcomes. Madden (2009) finds that the exact choice of how ordinal data are incorporated into an index number is not particularly important. Though Knippenberg (2014) suggests that the importance of computation method depends entirely on how many variable dimensions are included in the index. The study by Bossert, Chakravarty and D'Ambrosio (2013) combines country-level statistical data with weights derived from a public survey on what factors survey respondents consider to be important when discussing poverty and material deprivation. This is a novel and commendable approach, but, again, the weights from the survey responses are likely to be highly correlated. Indeed, this is the idea behind weighting of intensity of survey responses in compositional data analysis. For example, see Vives-Mestres, Martin-Fernandez and Kenett (2016) on one way to control for both within-response (individual-level) correlation and between-response (variable-level) spurious correlation.

2 The Kids Count Index

Every year, the Annie E. Casey Foundation publishes an update to its Kids Count Index. The index transforms 16 variables in four categories into a single index number for each state. These categories and variables are listed in *Table 1.* Those index numbers are then compared against each other to create a ranking of U.S. states. Topping the list tend to be the New England states of

Vermont, Hew Hampshire, and Massachusetts. At the bottom of the list are typically Mississippi, New Mexico, and Nevada. The rankings for the 2013 Kids Count Index are listed along with the results at the end of this paper in *Table 3*.

Table 1: Kids Count Indicators

Economic Well-Being Indicators	1. Children in poverty 2. Children whose parents lack secure employment 3. Children living in households with a high housing cost burden 4. Teens not in school and not working
Education	5. Children not attending preschool 6. Fourth graders not proficient in reading 7. Eighth graders not proficient in math 8. High school students not graduating on time
Health	9. Low-birthweight babies 10. Children without health insurance 11. Child and teen deaths per 100,000 12. Teens who abuse drugs or alcohol
Family and Community	13. Children in single-parent families 14. Children in families where the household head lacks a high school diploma 15. Children living in high-poverty areas 16. Teen births per 1,000

The calculation of the state rankings takes the following steps: first, the mean value from each of these 16 variables is calculated. Then the distribution of mean values from each variable is assumed to follow a normal distribution. Each indicator's value from each state is normalized by converting it to a z-score (subtracting the simple, national mean and dividing by the standard deviation for each indicator). Each indicator is a negative measure, in that higher values indicate a worse situation and contribute more to a worse score. For example, a higher percentage of children living in poverty is a bad thing, so that the badness increases positively with the index. Because these variables all "point" in the same direction, no further normalizing is necessary: The z-scores are now simply added together, which creates a single index number for each state. This index number represents the sum of the number of standard deviations that the state falls away from the mean. These index numbers are then ordered so that the one with the largest negative value is the best-performing state, and the one with the largest positive value is the worst-performing state.

While I highly applaud the efforts of the researchers involved, I believe that the index can be made better and more reflective of reality, compared to the current methods being used. For example, the variables "The percent of children living in poverty," and "The percent of children living in high-poverty areas," are very similar. These two variables are not identical, because a child could be living in poverty in an area that is not impoverished, but for the most part, they go together the vast majority of the time. The same is true for many of the variables included in the study. The problem I see with the study is that the all of the variables are implicitly given equal weights. Though how can one justify equal weights to two variables that are nearly identical? One possible explanation is that the identical variables indicate dimensions that are more important than the non-identical ones: what could be more important to a child welfare index than measuring the percent of children in poverty? Here is an answer: children in single-parent households. This variable has been identified in numerous studies, such as Downey (1994), Carlson and Corcoran (2001) and Thomson, Hanson and McLanahan (1994), as one of the most important factors determining future poverty. The percentage is also included as a variable, but only as one variable, despite the fact that it may contribute more towards poor future and current well-being than simply living in poverty. Until I see a definitive study using turning-point analysis to give weights to these variables, I believe the best method is an orthogonal weighting process.

3 Methodology

3.1 Index Weighting

The simplest way to compute in index number is to use shares as weights. A fine example of this is the consumer price index produced by the U.S. Bureau of Labor Statistics. Price index numbers of this type have one primary drawback: Which weights should the practitioner use? The weights from the previous period, as in a Laspreyes Index, or use the weights from the future period, as in the Paasche Index? Most would agree that, while the these two methods are intuitively appealing, a combination of the two is the most accurate. To that end, most modern price indices use a geometric mean of weights between the previous and current period.

When weights are not known, as in an index not of the expenditure type, an index should be weighted using a system that best reflects the objectives of the final measurement. If the final objective is well-defined, as in what causes an event A, then it's possible to work backwards to find the factors that cause A. However, for large studies where the causal relationships are only partially known, and the measure of the outcome variable is continuous with an arbitrary cut-off, then a researcher's best bet may be to use orthogonal, or unrelated factors. To that end, there are several possible types of index numbers, using various statistical methods.

One method, used for predictive modelling, is illustrated by the economic indicators comprising the Conference Board's Leading Economic Index (Levanon et. al. 2011). These indicators are selected based on their ability to predict a recession in a probit model, and the weights are set by each one's relative predictive performance, thereby giving the Leading Economic Index the best predictive power based on past relationships. For well-being indices, prediction is typically not the objective, since well-being and economic indicators are likely to be co-determined.

There are several papers in this literature. For an early review of socioeconomic indices, see Atkinson and Bourguignon (1982) who explore the problems of multidimensional measures of well-being. Bourguignon and Chakravarty (2003) take a novel approach in that they define a poverty line for each dimension of poverty, and any household that falls below that line in any dimension is considered to be in poverty which is a unique approach in that it altogether circumvents the problem of determining indicator weights. Another method, explored in Caruso, Sosa-Escudero and Svarc (2014), can be seen as converse from others. Their method uses a clustering approach to first identify a group of low socioeconomic-status families, and then they find the variables that most contribute to them being in that state of low socioeconomic status.

But some index numbers are created using principal components to determine factor loadings. Among the many studies promoting this method is Vyas and Kumaranayake (2006) which reviews the construction and validity of principal components analysis-based indices. Similarly, Gasparini et al (2011), Ferro Luzzi et al (2008), and Otoiua, Titan and Dumitrescu (2014) use principal components and factor analysis to determine the most important variables in determining poverty. The idea behind using principal components is that the weights are based on orthogonal factors. Orthogonal factors are those factors that are not correlated with one another. One main difference between the approach proposed below and those of predecessor studies is that dimension reduction is not an important objective. Rather, the Law of Cosines approach makes use of all of the available variables. Furthermore, data are needed at the household level to produce the previous studies, whereas this approach can make use of aggregate data, which is often much easier for a researcher to obtain.

The problem with using principal components is that the variables can include compositional (or ratio) data, in which correlation is spurious, as discussed in Pearson (1897), Aitchison (1986) and Pawlowsky-Glahn, Egozcue, Tolosana-Delgado (2007). Principal components analysis is not applicable when correlation is spurious (Aitchison, Barceló-Vidal and Pawlowsky-Glahn). Because of this, if an orthogonal weighting scheme is desired, then a different orthogonalization method is needed, which is proposed in Knippenberg (2014), and outlined in the following section. The Law of Cosines distance metric applies in all cases, even when the subject is not compositional data.

3.2 Orthogonal Weighting Using the Law of Cosines

In this section I propose methodology for adjusting the Kids Count Index, motivated by a desire to use orthogonal indicators in keeping with the knowledge of compositional data analysis from Aitchison (1986) and Knippenberg (2014).

Knippenberg (2014) derived an orthogonal weighting procedure for index numbers. This orthogonal weighting procedure is a two-step process. First the similarities between the variables are calculated to find the orthogonal dimensions, then the magnitudes are rotated onto orthogonal dimensions and a distance metric is applied to find the difference between the previous and current periods. This procedure has successfully been applied to international trade data where it was shown that patterns found in the composition of international trade simultaneously support multiple, opposing theories. An outline of the methodology is discussed below. This orthogonalization method gives equal weights to the latent, orthogonal dimensions.

Index numbers are a combination of indicator variables, each of which is paired with a weight. The weights must sum up to an arbitrary constant number, called the "closure" of the sample space. This is most typically scaled to equal 1 or 100. The closure ensures that any assignment of weights must occur on the n-simplex. For each variable x_i, $i = 1, ...n$, let the associated weight be denoted by w_i where

$$\sum_{i=1}^{n} w_i = 1 \tag{1}$$

Given this notation, an index number can be expressed for each observation y_j, $j = 1, ..., k$ as

$$y_i = \sum_{i=1}^{n} x_i, w_i \tag{2}$$

In well-being indices like the Kids Count Index or the Opportunity Index, the closure of the sample space is not explicitly defined. However, it is discernable from the calculation that every factor has an equal weight. The problem is that some variables included in the index may be highly correlated, indicating that they are likely quantifying different measurements of the same factor dimension. This could lead to some factor dimensions to have a higher or lower weight than desired. Is this acceptable in a normative sense? That is up to the researcher to determine, but by no means should it be ignored.

The process has two major steps. The first is step is to measure the similarity between the well-being indicators. This could be accomplished in a number of different ways, and is not necessarily a contribution of this paper. However, the second step is to use the similarity data to create a measure of distance on orthogonal dimensions, which is the real methodological contribution of the paper.

The first step in the orthogonalization procedure is to measure similarity between the indicators used. While two indicators like, "the percent of children living in poverty," and "the percent of children living in high-poverty areas," are not identical, most would agree that there is a high degree of overlap between them because a child who lives in poverty is likely also living in a high-poverty area. So the first task is to measure what is known as revealed comparative advantage in international trade; that is, to measure which states suffer from a higher level of each indicator, and which do not, compared to overall national average. Here the national average is the simple mean of each indicator, and where each state is equally weighted:

$$RCA_{c,i} = \frac{x_{c,i}/X_c}{x_{w,i}/X_w} \geq 1. \tag{3}$$

Where each $i = 1, ..., 16$ denotes each of the 16 indicators, $s = 1, ..., 50$ denotes each U.S. state, and $w = 1$ denotes the national average. As such, $RCA_{c,i}$ equals 1 if the inequality is satisfied, meaning that state c has a welfare indicator at least as large as that of the national average, and 0 otherwise.

Next, using the above-calculated binary outcomes of the revealed comparative advantage calculations, compute the similarity between indicators i and j, denoted $\phi_{i,j}$:

$$\phi_{i,j} = \min\{Prob(RCA_{c,i}|RCA_{c,j}), Prob(RCA_{c,j}|RCA_{c,i})\} \tag{4}$$

In words, the above expression is asking for every instance in which indicator j is above average, how often is indicator i above average, and vice versa? The similarity between i and j is then the minimum of those conditional probabilities. Because indicators are not necessarily normally distributed, some are more or less likely to be above average than are others, and the minimum function is used to account for this relative prevalence.

Next, put all of these pair-wise values into a similarity matrix and denote it by Φ. This matrix represents a measure of the pair-wise similarity among all of the included welfare indicators. However, this could just as easily be a correlation measure or some other similarity measure. Again, the above methodology is not the focus or innovation of this study.

The entries in the similarity matrix Φ are displayed in *Table 2*. The last row is a summation that indicates the overall level of similarity between that indicator and fifteen others included in the calculations. The smallest value is 7.00 for “Teens who abuse drugs or alcohol”, showing that this indicator is least like the others, while the highest value is 10.73 for “Children living in high poverty areas”, showing that this indicator is most like the others.

Table 2: Similarity Matrix Φ

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0.75	0.46	0.54	0.58	0.68	0.74	0.71	0.69	0.38	0.46	0.42	0.46	0.73	0.67	0.67
2	0.75	1	0.56	0.47	0.5	0.69	0.66	0.78	0.69	0.38	0.44	0.34	0.56	0.83	0.72	0.59
3	0.46	0.56	1	0.48	0.37	0.52	0.56	0.68	0.62	0.41	0.24	0.38	0.55	0.63	0.56	0.46
4	0.54	0.47	0.48	1	0.6	0.76	0.74	0.75	0.66	0.4	0.48	0.44	0.48	0.67	0.64	0.64
5	0.58	0.5	0.37	0.6	1	0.6	0.44	0.43	0.34	0.37	0.53	0.42	0.42	0.43	0.4	0.54
6	0.68	0.69	0.52	0.76	0.6	1	0.74	0.71	0.55	0.4	0.48	0.44	0.48	0.6	0.64	0.64
7	0.74	0.66	0.56	0.74	0.44	0.74	1	0.79	0.72	0.41	0.52	0.41	0.52	0.73	0.67	0.63
8	0.71	0.78	0.68	0.75	0.43	0.71	0.79	1	0.72	0.43	0.5	0.39	0.54	0.77	0.68	0.61
9	0.69	0.69	0.62	0.66	0.34	0.55	0.72	0.72	1	0.41	0.48	0.38	0.55	0.77	0.69	0.62
10	0.38	0.38	0.41	0.4	0.37	0.4	0.41	0.43	0.41	1	0.48	0.43	0.43	0.57	0.4	0.63
11	0.46	0.44	0.24	0.48	0.53	0.48	0.52	0.5	0.48	0.48	1	0.33	0.35	0.43	0.52	0.63
12	0.42	0.34	0.38	0.44	0.42	0.44	0.41	0.39	0.38	0.43	0.33	1	0.38	0.47	0.36	0.42
13	0.46	0.56	0.55	0.48	0.42	0.48	0.52	0.54	0.55	0.43	0.35	0.38	1	0.8	0.69	0.62
14	0.73	0.83	0.63	0.67	0.43	0.6	0.73	0.77	0.77	0.57	0.43	0.47	0.8	1	0.7	0.6
15	0.67	0.72	0.56	0.64	0.4	0.64	0.67	0.68	0.69	0.4	0.52	0.36	0.69	0.7	1	0.64
16	0.67	0.59	0.46	0.64	0.54	0.64	0.63	0.61	0.62	0.63	0.63	0.42	0.62	0.6	0.64	1
sum	9.93	9.96	8.48	9.75	7.98	9.93	10.27	10.49	9.9	7.51	7.86	7	8.83	10.73	9.97	9.93

If two well-being indicators are orthogonal to one another, then the similarity matrix would have an entry of zero. The conditions necessary for equal weighting of these indicators in an index is that either all indicators are orthogonal - in which the similarity matrix can be represented simply by the identity matrix $\Phi = I$ - or that all off-diagonal similarity measures are equal to a constant c. Another interesting fact is that the highest entry in the similarity matrix - the two well-being indicators which are most highly correlated - are “Children whose parent lack secure employment” and “Children in families where the household head lacks a high school diploma.” This is not surprising, as other research has shown that the unemployment rate is highest for U.S. men who lack a high school diploma. The lowest entry in the table is for the similarity between “Children living in households with a high housing cost burden” and, not surprisingly, “Child and teen deaths per 100,000."

Now following the example set by the original Kids Count Index, which converted each indicator to a z-score by subtracting the national mean and dividing by the national standard deviation. The original calculation also did this, and then simply added up the z-scores from all 16 indicators. Because each indicator measures a negative aspect of child welfare, the state with the highest sum was the worst, and the state with the lowest sum (the least of the negative indicators) was the best. I leave the indicators as z-scores going into the next step.

The key point here is that, in simply adding up the 16 indicators, there is an implicit assumption that each indicator is orthogonal to every other one. Because the similarity matrix is not the identity matrix, this is not the case. In terms of distance metrics, this simple adding of z-scores is equivalent to a manhattan distance metric that measures the distance from the origin (the national average for each of the indicators) to a state's location as defined by the 16-dimensional space of child well-being. My argument is that this is not as accurate as it could be, and instead of implictly using a manhattan distance metric, a better method would be one that accounts for the entries in the similarity matrix. The mathematical calculations to do so are the subject of the following subsection.

3.3 Law of Cosines Distance Metric

Once I have calculated the similarity between all welfare indicators, how can I use this information to compare each state? The answer is to map the welfare indicators of each state into a welfare space, as defined by the information in the similarity matrix, Φ, and then to find the relative positions of each state in that space.

Plotting a state c and the national average on this line, I can find the distance between them as a straight line on the triangle's hypotenuse, using the Pythagorean Theorem:

$$\rho_{c,d}^2 = (x_{c,1} - X_1)^2 + (x_{c,2} - X_2)^2 \tag{5}$$

The above is an example of a distance metric, with the implicit assumption that indicators x_1 and x_2 are completely unrelated and uncorrelated. However, this is clearly not the case and this information needs to be incorporated into the distance measure. For that reason, think of each indicator $x_{c,i}$ of a state's welfare vector x_c is its own vector that spans all of x_c, but is with zeros as entries j for which $j \neq i$.

Now comes the innovation. According to Gentle (2007), a similarity matrix gives extra information about the orientation of a set of vectors: "The cosine of the angle between two vectors is related to the correlation between the vectors, so a matrix of the cosine of the angle between the columns of a given matrix would also be a similarity matrix. "(pg 298). Assuming that a reverse of Gentle's logic holds, then given a similarity matrix, one can use the similarity measures as the proportion of an angle between any two entries *within* a vector [1].

[1]My interpretation of this passage is a key point. Note that Gentle (2007, pg 37) defines $Corr(x, y) =$

The vectors representing differences in export shares can be represented by line segments which are not parallel to the axes. The angle between these line segments is given by a simple transformation of the economic indicators' similarities: $(1 - \phi_{i,j})90°$. Two goods which are always revealed as exceeding the national average together, $\phi_{i,j} = 1$, can be thought of as highly correlated and indistinguishable from one another, and are extremely likely to be measuring the same underlying factors. In the extreme case, the angle between the two indicators is zero degrees when they measure exactly the same variable, since $(1 - \phi_{i,j})90° = 0$. On the other extreme if $\phi_{i,j} = 0$, then indicators i and j are completely different and are very likely to be measuring completely unrelated variables. In this case the angle between the two indicators would be: $(1 - \phi_{i,j})90° = 90°$. In the last case we refer to the two indicators as being orthogonal.

The distance from a U.S. state to the national average in the two-dimensional case is given by the Law of Cosines:

$$\rho_{c,d}^2 = (x_{c,1} - X_1)^2 + (x_{c,2} - X_2)^2 + 2(x_{c,1} - X_1)(x_{c,2} - X_2)\cos((1 - \phi_{1,2})90\frac{\pi}{180}) \tag{6}$$

The length of the triangle's third side on the hypotenuse is given by the square root of *Equation 6*. For the purposes of this study, the distance formula needs to account for all 16 indicators included in the Kids Count Index. Using the result from Ding (2008), who derives the Law of Cosines in n-dimensions, I have:

$$\rho_{c,d}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(x_{c,i} - X_i)(x_{c,j} - X_j)\cos((1 - \phi_{i,j})90\frac{\pi}{180}) \tag{7}$$

Alternately, in matrix form, let Φ be defined as above, and let $\vec{x}_i$ denote a vector of the difference in the indicators between state i and the national average. Then the matrix representation is simply:

$$y_i = \vec{x}_i \cos(\Phi)\vec{x}_i^{\intercal}, \tag{8}$$

Note that y_i is a scalar. Here I am dividing the Law of Cosines measure by the square root of 2, which is the largest possible distance between any two points on a unit simplex. I subtract 1 from the measure so that it approximates similarity, rather than distance, which is more inline with previous literature and is intuitively appealing. This measure makes sense because I need to find the distance *along* the (n-1)-dimensional hyperplane representing all possible combinations of export share vectors. Given that I have information on the lengths of export share vectors and the angle between any two of them, this distance measure is appropriate.

This derivation of the Law of Cosines distance metric should not be confused with other derivations of the Law of Cosines. The main difference here is that I am assuming the shape of the space

$cos(angle(x_c, y_c)$, and because $-1 \leq Corr(x, y) \leq 1$, the direct corollary of this should hold for similarity, where $0 \leq \phi_{c,d} \leq 1$ is more restrictive, and two objects cannot have a negative similarity. If I interpret each element of an export vector to be itself a vector projected in as many dimensions as there are goods (whereas a vector entry for plain data would be orthogonal to every dimension except its own), then this interpretation reveals a rather simple equation.

is *not* Euclidean. Distance in the linear algebra sense depends on the coordinate system being Euclidean, and results from algorithms in papers such as Pan, Kotani, and Ohmi (2004) depend on knowing the Euclidean coordinate system. Instead I am assuming that any data vector given is not in canonical form, or, equivalently, that the the space is not an orthogonal one. The whole point is that the similarity matrix gives researchers additional information not contained in any one data vector. This information is about the *shape of the space* in which the measurements lie, which is not necessarily Euclidean space. A basic concept in measure theory says that just because something can measure be measured, doesn't mean that what is measured correctly represents anything.

This is one of the metrics to take into account similarity between variables, or non-canonical form data matrices. This similarity is important, for two reasons. The first is that a researcher should never assume that pair-wise similarity between economic indicators is zero or homogeneous, as seen in the previous discussion of the product space. The second is that leaving out the notion of similarity is akin to leaving out the notion of covariance in other statistical analyses. Indeed Gentle (2007) goes so far as to say that similarity and correlation have the same interpretation: as the cosine of the angle between two vectors (pg. 298). A researcher would never run a regression without accounting for covariates. Analogously, I hope that the reader now finds any metric not accounting for similarity to be just as nonsensical.

The final step is to turn the y_i entries into an ordinal set by ranking them from the best to the worst. Just like with the original indexes' calculation, the one with the lowest index number value (the least of the negative indicators) is the best state, and the one with the highest index number value is the worst.

4 Discussion

An article in *The Economist* demonstrated the importance of ordinal ranking indices. It told of the billions of dollars poured into education based on the OECD's Programme for International Student Assessment, and highlighted the policy implications from indices such as the World Bank's Ease of Doing Business Index and the U.S. State Department's Trafficking in Persons Report, all of which have drummed up sensationalist rhetoric based on where a country falls on the list. However, this article was also quite critical, and stated that such indices are based on shaky data, questionable assumptions, and are mainly used as policy tools. The Kids Count Index is similar. Though it ranks U.S. states instead of countries, the index is used as a policy tool to both praise the best-performing states and shame the worst-performing ones. The difference with the Kids Count Index is that it is based on good data: U.S. state data is generally quite reliable, and the Annie E. Casey Foundation has made impressive efforts by setting up offices in states to collect the best data possible. I would only add that the final computation can be improved.

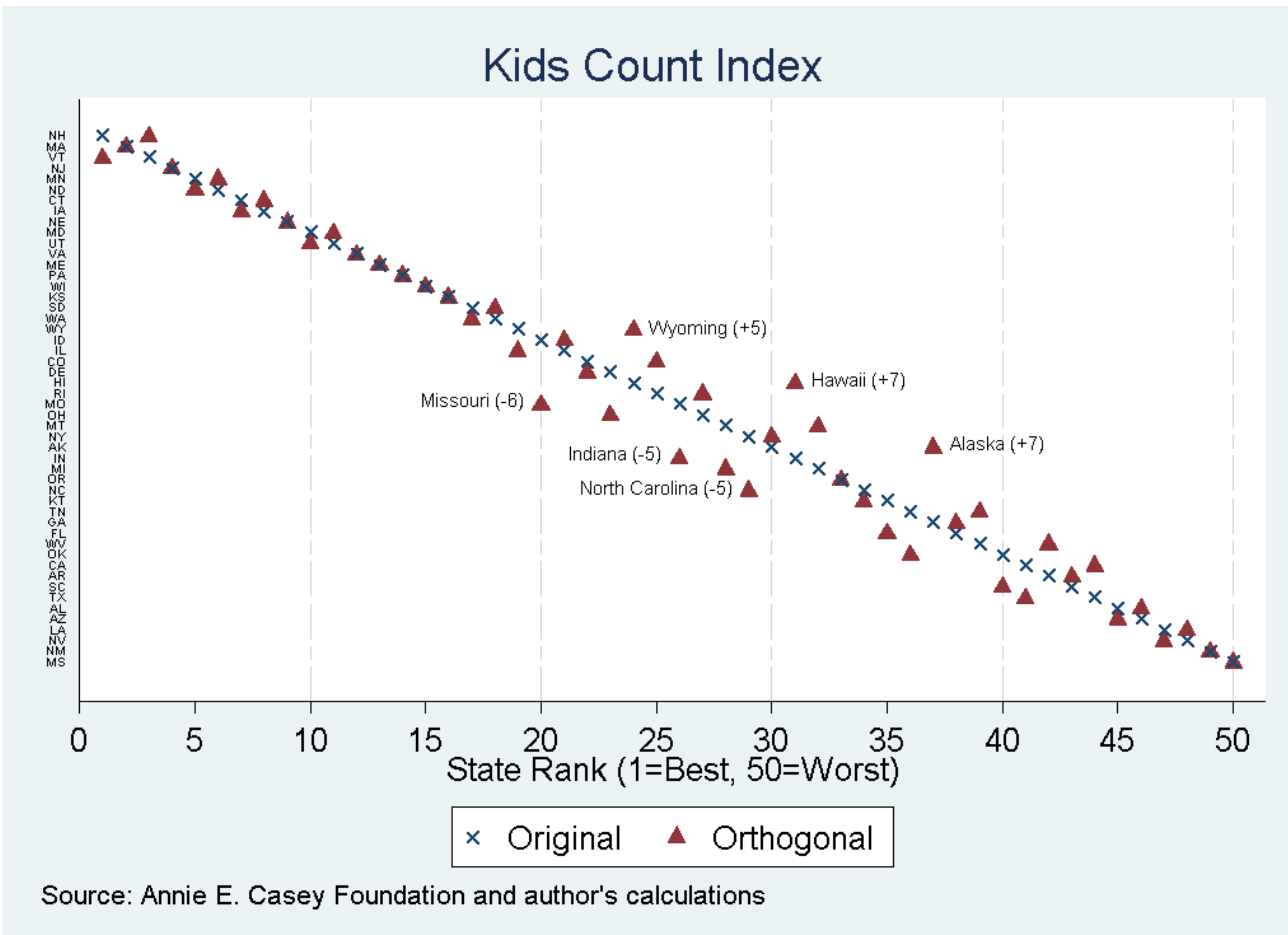
The results of the analysis show some changes in the relative rankings of states. *Table 3* presents the 2013 state rankings, the rankings using orthogonal weights, and the change in rank between the two methods, while *Figure 1* graphs this same information. The reader can see that Alaska

Table 3: Kids Count Rankings

State	Original Rank	Orthogonal Rank	Change
Alabama	45	46	1
Alaska	30	37	7
Arizona	46	45	-1
Arkansas	42	43	1
California	41	44	3
Colorado	22	25	3
Connecticut	7	8	1
Delaware	23	22	-1
Florida	38	35	-3
Georgia	37	38	1
Hawaii	24	31	7
Idaho	20	21	1
Illinois	21	19	-2
Indiana	31	26	-5
Iowa	8	7	-1
Kansas	16	16	0
Kentucky	35	34	-1
Louisiana	47	48	1
Maine	13	13	0
Maryland	10	11	1
Massachusetts	2	2	0
Michigan	32	28	-4
Minnesota	5	6	1
Mississippi	50	50	0
Missouri	26	20	-6
Montana	28	32	4
Nebraska	9	9	0
Nevada	48	47	-1
New Hampshire	1	3	2
New Jersey	4	4	0
New Mexico	49	49	0
New York	29	30	1
North Carolina	34	29	-5
North Dakota	6	5	-1
Ohio	27	23	-4
Oklahoma	40	36	-4
Oregon	33	33	0
Pennsylvania	14	14	0
Rhode Island	25	27	2
South Carolina	43	40	-3
South Dakota	17	18	1
Tennessee	36	39	3
Texas	44	41	-3
Utah	11	10	-1
Vermont	3	1	-2
Virginia	12	12	0
Washington	18	17	-1
West Virginia	39	42	3
Wisconsin	15	15	0
Wyoming	19	24	5

and Hawaii worsened by seven ranks, while Missouri improved by six places. Many states improved in ranking while others declined. Many states didn't change at all. The average absolute change was two places. So the changes made are not particularly groundbreaking, as the researchers who originally developed the index surely knew what they were doing and made good decisions in what to include in the original index. The changes are marginal, but, I believe, represent a stronger theoretical foundation. So while the changes in general are not large, the changes for a few particular states are indeed so. If I were a federal policymaker and about to make a decision on the amount of funding that were to go to a particular state, this change could sway my decision. And because allotment of government funds is a zero-sum game, ranking does matter. For example, I know that policymakers in the state of Colorado follow the Kids Count Index very closely, because many funding decisions are made on the basis of a state's relative ranking, and particularly on how that ranking changes over time.

Figure 1: Kids Count Index, Original and Orthogonal



The fact that the rankings change so little is comforting. It means that the creators of the index have done a good job in their choice of variables: The computation method is robust in that it stands up to scrutiny, and the variables chosen appear to minimize parallel measurements. It also adds evidence to the fact that the orthogonalization technique described herein appears to be not only theoretically valid, but valid in applications as well.

The above methodology is being proposed as only second best in this case. It is second best because it depends only on correlation, or co-relatedness of the variables involved. The best possible way to create an index in this setting is by establishing cause and effect. We know some of the things that decrease child well-being, and some of the things that are symptoms. The best approach to creating a well-being index is to create an all-inclusive economic model with empirically validated cause-and-effect relationships. Otherwise the calculation is an exercise that cannot distinguish causes from symptoms.

5 Conclusion

This paper demonstrates an orthogonal weighting technique using the Kids Count Index, a ranking of child well-being across U.S. states. This method reduces the heterogeneous similarity between the 16 indicators used in the Kids Count Index. The results show relatively little change in ranking, signifying that the components of the original index are good indicators of child well-being.

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