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The Wellbeing of Nations: A Multidimensional Mixture Distribution Analysis of Poor Nation-Rich Nation Status

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Abstract

Recent concerns about the measurement of wellbeing have led to the progress of nations to be classified and studied in a multidimensional context, unfortunately this compounds the difficulties encountered in categorizing groups and assessing progress. Here a feasible methodology for defining classes in terms of the commonality of behaviours of the actors in a multidimensional setting is presented and techniques for assessing poverty, inequality, polarization and mobility within and between groups are proposed and implemented for 164 countries over the period 1990-2014 in that many variable setting without arbitrarily defining frontiers. The analysis detected a slowly evolving, relatively immobile world, over the period the poor group appears to have diminished in size (which may be interpreted as a reduction in the poverty rate and is reflective of some upward mobility). There appears to have been some reduction in inequality both within and between groups over the period though they appear to have become more polarized.

Keywords: Wellbeing, HDI, Mixture Models, Class membership, Mobility.

JEL classification: C14; I32; O1.

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1 Introduction

For most of the 20th Century real GDP or GNI per capita was used as a measure of societal wellbeing. As a measure of societal productive capacity, it was a proxy for capacity to generate "Consumption Wellbeing" and after suitable exchange rate adjustments it provided a useful instrument for international comparisons of poorness and wellness. Accompanying this approach was a long established practice of classifying agents (be they individuals, households or countries) into groups by employing "hard" boundaries in order to study aspects of group behavior (see for example Atkinson and Brandolini (2011), Banerjee and Duflo (2008), Citro and Michael (1995), Easterly (2001), Quah (1993, 1997) and Ravallion (2012)). In a multi-dimensional paradigm, Alkire and Foster (2011, 2011a) have proposed a many dimensioned poverty/deprivation measure which requires the specification of a boundary in each dimension. Determination of the cut-offs has frequently been a contentious matter which is not surprising given that their choice determines the nature of the classes and the outcomes of the classes being studied. Recent examples are the 2013 Gross National Income per capita categories published by the World Bank, and the cut-off points for the categories of human development index in the 2016 Human Development Report (United Nations Development Programme, 2016).

The end of the century saw increasing dissatisfaction with the measure of GDP per capita as proxy for well-being and as a basis for categorizing nations. Aside from some obvious measurement concerns (see for example Coyle, 2014) there was a real concern with the idea of equating "consumption utility" with "wellbeing". The 2008 Commission on the Measurement of Economic Performance and Social Progress (its findings are in Stiglitz, Sen and Fitoussi, 2010) set out to identify the limits of GDP as an indicator of economic and social progress, including the problems with its measurement. It also considered what additional information might be required for the production of more relevant indicators of social progress.

In the context of comparing nations, typical of the early attempts at expanding the dimensions of wellbeing is the Human Development Index (UNPD, 2016), first published in 1990, and commonly referred to as the HDI. Common concerns with these indices are questions regarding aspects of wellbeing that should be included in the analysis, how they should be aggregated, and how robust they may be to alternative assumptions on parameterizations (Ravallion, 2010). The HDI index which has a country as its basic agent is principally based on an equally weighted geometric mean aggregation of the three bounded dimensions of education (a combination of literacy and school enrolment rates), life expectancy (essentially a proxy for health status), and GNI per capita of a country. A problem with this approach, much like that of the GDP/GNI per capita, is that determining boundaries of the index in a particular fashion also determines the nature of the group in a way that is often prejudicial for analysis. Not only does it somewhat arbitrarily determine the nature of poorness and wellness but ultimately it affects the way transition and class mobility behavior is evaluated. In the one dimensioned paradigm this led Anderson, Pittau and Zelli (2014, 2016) to propose semi-parametric methods for determining nation status by reflecting commonalities in nations' behaviors, however now the classification problem is "many dimensioned".

The main goal of this study is to examine the progress of groups of nations in the modern era in the context of the joint distribution of the components of the HDI, without predetermining the number of groups or their boundaries. Additionally, tools are proposed for measuring the poverty, inequality and polarization of the groups in a many dimensioned context. To do this a semi-parametric technique for class categorization without resort to arbitrarily specified frontiers in a multidimensional context is proposed. The nature and progress (or otherwise) of these classes is analysed as is the extent of inequality, polarization and convergence of and mobility between them. Study of the poorest class characterizes the behaviour of the poor, and within class variability is used to study the progress of within class inequality. As in Hobijn and Franses (2001) the issue of convergence is examined by looking at the dynamics of the whole distribution of the indicators but, unlike them, the evolution of the joint distribution of the indicators is considered rather than the dynamics of the distribution of each indicator separately.

After a preliminary year-by-year study of the mixture distribution underlying the HDI which identifies a steady process of development, a hidden Markov model (HMM) is adopted in which the system being modeled is assumed to be a Markov chain with time-varying unobserved (hidden) states. These latent states are identified as different sub-populations of countries that share inherent circumstances of human development (or a similar set of functioning and capabilities, in Sen's words) that are fundamentally unobservable. Countries belonging to a specific state or category share in each period a common multivariate distribution of the (observable) outcome variables. Therefore the whole population distribution will be a mixture of these sub-distributions (components of the mixture). A HMM can be considered a generalization of a mixture model where the latent states, which originate the mixture components, are related through a Markov process rather than being time-independent. Within this framework, each country is not necessarily locked in a pre-defined category, but it may jump from one class to another one over time due to structural changes in its inherent characteristics. Category membership is partially determined by the commonality of observed behavior of category members: partial in the sense that only the probability of category membership in each category is determined for each country. Such an approach does not inhibit the size of classes or the nature of transitions between them and permits the study of class behaviors and characteristics including growth and transition properties.

We have analyzed a panel of 164 countries over the period 1990–2014. The work has been carried out in the context of population weighted distributions so that a nations population size is reflected in class size calculations.

The rest of the paper is organized as follows: Section 2 outlines our proposed method. Section 3 reports an illustrative study of the world multivariate distribution of the components of the HDI over the period 1990–2014. Section 4 concludes.

2 Empirical methods

An initial analysis of the data considered a year-by-year mixture distribution of the three outcomes under the assumption that they were jointly normally distributed and time independent. Under the hypothesis that the components belong to the multinormal family, the mixture density in a given period, can be written as:

$$f\left(\mathbf{y};\boldsymbol{\Psi}\right)=\sum_{j=1}^{k}w_j f_j\left(\mathbf{y};\boldsymbol{\mu}_j,\boldsymbol{\Sigma}_j\right), \tag{1}$$

where $f_j\left(\mathbf{y};\boldsymbol{\mu}_j,\boldsymbol{\Sigma}_j\right)$ denotes the multivariate normal density of the jth component with mean vector $\boldsymbol{\mu}_j$ and covariance matrix $\boldsymbol{\Sigma}_j$, and w_j represent the mixing proportions. The vector $\boldsymbol{\Psi} = (w_1, \cdots, w_{k-1}, \boldsymbol{\xi}')'$ contains all the unknown parameters of the mixture model; in this case $\boldsymbol{\xi}$ consists of the elements of the component means $\boldsymbol{\mu}_1, ..., \boldsymbol{\mu}_k$ and the distinct elements of the component-covariance matrices $\boldsymbol{\Sigma}_1, ..., \boldsymbol{\Sigma}_k$, that here are assumed diagonal.[1]

When it became clear that there were underlying inter-temporal relationships that could be exploited some inter-temporal restrictions were contemplated. Within this new approach measurements on each outcome and at each time period are considered independent only conditionally on an unobserved discrete latent variable. This leads to a hidden Markov model for panel data (see Bartolucci *et al.*, 2013, 2014; Farcomeni, 2015), which relies on similar assumptions but which assumes that the number of latent states is not constant over time. Relaxing these assumptions requires estimating the model in a Bayesian context.

Formally, let Y_{itr} denote the measurement for the r-th outcome at time t for country i. Assume there are k_t latent states, where the (unknown) latent state for country i at time t is denoted U_{it}, further assume $Y_{itr}|U_{it} = j, k_t = k \sim N(\mu_{jtkr}, \sigma^2_{jtkr})$, that is, when there are k groups and the i-th country belongs to the j-th one, the r-th outcome has mean μ_{jtkr} and variance σ^2_{jtkr}.

Given the sample size and the number of parameters to be estimated, any of two possible assumptions are made for the outcome-specific averages, namely, that they are time-constant (that is, $\mu_{jtkr} = \mu_{jkr}$) or that they have a linear trend of the kind

$$\mu_{jtkr} = \mu_{j1kr} + \beta_{jkr} t$$

Similarly, we might assume that variances are time-constant (that is, $\sigma_{jtkr} = \sigma_{jkr}$), or that we might make the GARCH-type assumption

$$\sigma^2_{jtkr} = \alpha^{2t}_{jkr} \sigma^2_{j1kr}.$$

This concludes the specification of the *manifest* distributions.

For the latent distribution we assume that U_{it} follows a time-homogeneous Markov chain with variable number of states, which is fully specified by initial distributions $\Pr(U_{i1} = j | k_1 = k) = \pi_{jk}$ and (possibly rectangular) transition matrices $\Pr(U_{it} = j | k_t = k, k_{t-1} = l, U_{i,t-1} = h) = \pi_{hjlk}$.

[1]This assumption has been removed for a sensitivity analysis but the results did not change significantly.

In summary, the latent variable follows a variable-support time-homogeneous Markov chain. Consequently, we simultaneously model the joint distribution of the three outcomes over time, taking into account dependence due to correlation and unobserved heterogeneity. The discrete latent distribution provides a natural way to cluster nations with respect to their measurements. We not only allow transitions between groups, but also year-specific number of clusters (components of the mixture). The (possibly rectangular) hidden transition matrices link the group compositions across years.

In order to fit this complex model, Bayesian techniques are employed. Trans-dimensional moves are obtained through a birth-and-death reversible approach, while full conditionals are available for all parameters except σ and α. For these parameters Adaptive Rejection Metropolis Sampling is used. To assess evidence for specific parameter configurations, the encompassing prior approach (Klugkist et al., 2005; Bartolucci et al., 2012) is used for dealing with discrete parameters, and Schwarz criterion for continuous ones.

With respect to overall variation, one way of considering the extent to which the world has become more unequal is to look at inequality or differences in the group distributions via a generalization of Gini's transvariation measure (Gini, 1916; 1959; Dagum, 1968; Anderson, Linton and Thomas, 2017). Suppose three different groups of countries have been identified, say Low, Medium and High human development group. Then, the transvariation measure is of the form:

$$
\begin{aligned}
3 \cdot Trans = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(& max(f_L(x,y,z), f_M(x,y,z), f_H(x,y,z)) \\
& - min(f_L(x,y,z), f_M(x,y,z), f_H(x,y,z))\Big) dxdydz
\end{aligned}
$$

where x,y and z are respectively relative ln per capita GNI, Life expectancy and Education and $f_L()$, $f_M()$ and $f_H()$ are the corresponding Low, Medium, and High human development distributions. The measure corresponds to an index between 0 and 1 of inequality of distribution which will be 0 when all distributions are identical and 1 when there is no overlap between distributions.[2] It treats all nations as equally

[2]For year-by-year comparison purposes, under some strong assumptions, the Transvariation statistic p can be considered asymptotically normal with a standard error $\sqrt{p(1-p)/3n}$ where n is the sample size.

important, in attaching the same weight to each distribution it can be interpreted as measuring the extent of distributional differences of the prospects for a representative low, medium and high developed poor, middle and rich nation increases in the measure signal diverging distributions, reductions correspond to increasing similarities or sigma convergence overall. It is also possible to construct a statistic which weights the comparison distributions by their relative importance in the mixture. This is of the form:

$$3 \cdot WTrans = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(max(s_L \cdot f_L(x,y,z), s_M \cdot f_M(x,y,z), s_H \cdot f_H(x,y,z)) - min(s_L \cdot f_L(x,y,z), s_M \cdot f_M(x,y,z), s_H \cdot f_H(x,y,z))\Big)dxdydz$$

Where s_k, $k = L, M, H$ is given by $w_k / \sum w_k$. Together, $Trans$ and $WTrans$ can be considered a multidimensional measure of world inequality.

In order to assess within class inequality and convergence in the context of the triple x, y and z (respectively Relative lnGNI per capita, Relative Life Expectancy and relative Education), note that for a given class in a given time period the distribution of the triple may be written as:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \nu \sim \frac{1}{\sqrt{2\pi|\Sigma|}}(\nu - \mu_\nu)'\Sigma^{-1}(\nu - \mu_\nu)\,, \text{where}\,\Sigma = \begin{pmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{pmatrix}.$$

It follows that:

$$\sqrt{|\Sigma|} = \sigma_x\sigma_y\sigma_z$$

is a measure of the overall relative variation in the class at that time and diminutions (increases) in it correspond to sigma convergence (divergence). Given that x, y and z are "base year" relative measures, this measure corresponds to a multivariate "coefficient of variation" where the base year mean is the standardizing factor.

3 The well-being of nations: categorization, convergence, mobility

3.1 Data and model choice

The analysis is carried out on a panel of 164 countries over a period spanning from 1990 to 2014. Data are taken from the Human Development Reports web-site[3] and have been analyzed every five years. Table 1 reports (weighted) means and standard deviations of the three variables involved in the HDI construction: per capita GNI, life expectancy at birth and years of education. There is one slight deviation from the HDI index, only one education variable (expected years of schooling) is used since including mean years of schooling would have involved too great a loss of data points. Per capita GNI are estimated in 2011 purchasing power parity.

Table 1: *Means and standard deviations of per capita GNI, life expectancy and years of schooling over time for the world population (164 countries)*

	Means			Standard Deviations		
Year	**GNI**	**Life_exp**	**Yrs_Educ**	**GNI**	**Life_exp**	**Yrs_Educ**
1990	8 661.10	65.19	9.58	11 918.8	8.45	3.07
1995	8 920.82	66.17	9.93	12 307.4	8.36	3.20
2000	9 949.40	67.43	10.42	13 713.3	8.48	3.16
2005	11 329.30	68.88	11.34	14 473.3	8.30	2.80
2010	12 915.70	70.35	12.34	14 143.8	7.68	2.54
2014	14 169.22	71.34	12.70	14 551.4	7.31	2.53

Given the sample size, the number of components has been assessed by using the Bayesian's Information Criteria (BIC) and for each year three components were selected (parsimony ruled whenever there was uncertainty in selecting three or four groups). The three components of the model are well-separated and can represent three different categories of human development (HD): low, medium and high HD. Initially progress in the three class model can be viewed in terms of its parametric structure year by year under the assumption that the outcomes were jointly independently normally distributed. To implement the model, per capita GNI has been log-transformed[4] and all the variables have been standardized with respect to the initial year 1990. So

[3]`hdr.undp.org/en/data.`

[4]Income is taken in logarithms "in order to reflect the diminishing returns to transforming income to human capabilities" (Anand and Sen, 1994, p.10); see also Brandolini (2008).

all the analyses performed are relative to the base year weighted average. Tables 2 and 3 report the year-by-year results. Note both year by year transvariation measures indicate significant overall convergence over the period as do the within group inequality measures.

Table 2: *Estimated means, standard deviations and relative group sizes of the components in the year-by-year mixture model*

	Means			Std Deviations		
	Low	Medium	High	Low	Medium	High
	GNI per capita					
1990	-1.23	-0.02	1.15	0.287	0.253	0.308
1995	-1.18	0.02	1.31	0.431	0.187	0.172
2000	-1.11	0.09	1.36	0.417	0.172	0.186
2005	-1.00	0.24	1.41	0.402	0.157	0.174
2010	-0.91	0.39	1.44	0.328	0.149	0.137
2014	-0.81	0.46	1.46	0.332	0.138	0.133
	Life Expectation					
1990	-1.41	0.23	0.91	0.219	0.194	0.236
1995	-1.25	0.41	1.09	0.299	0.130	0.119
2000	-1.16	0.52	1.18	0.295	0.121	0.132
2005	-0.92	0.65	1.28	0.317	0.124	0.137
2010	-0.60	0.75	1.41	0.268	0.121	0.112
2014	-0.38	0.84	1.50	0.261	0.108	0.105
	Education					
1990	-1.40	0.25	0.87	0.167	0.147	0.180
1995	-1.04	0.39	1.22	0.536	0.232	0.214
2000	-0.78	0.59	1.47	0.549	0.226	0.245
2005	-0.43	0.82	1.61	0.459	0.179	0.198
2010	-0.12	0.97	1.74	0.383	0.173	0.160
2014	-0.03	1.03	1.81	0.385	0.160	0.154
	Relative group size					
	Low HD		Medium HD		High HD	
1990	0.26		0.45		0.29	
1995	0.30		0.45		0.25	
2000	0.31		0.41		0.27	
2005	0.32		0.40		0.29	
2010	0.31		0.41		0.28	
2014	0.32		0.40		0.28	

The following diagrams (Figures 1, 2, 3 and 4) illustrate the progress of the groups over time.

Relative education levels have also seen a big advance for the low HD with annualized growth rates of 5.7%, 3.2% and 3.9% respectively for Low, Medium and High

Table 3: *Transvariations and within group inequality measures of the year-by-year mixture model*

Year	Transvar	WTrans	Within group inequality Low HD	Medium HD	High HD
1990	0.9884	0.9938	0.0105	0.0072	0.0131
1995	0.8578	0.9206	0.0691	0.0056	0.0044
2000	0.9018	0.9491	0.0675	0.0047	0.0060
2005	0.8570	0.9268	0.0585	0.0035	0.0047
2010	0.6705	0.8218	0.0337	0.0031	0.0025
2014	0.6658	0.8090	0.0334	0.0024	0.0022

Figure 1: *Evolution of the estimated means of the components in the year-by-year model: per capita GNI*

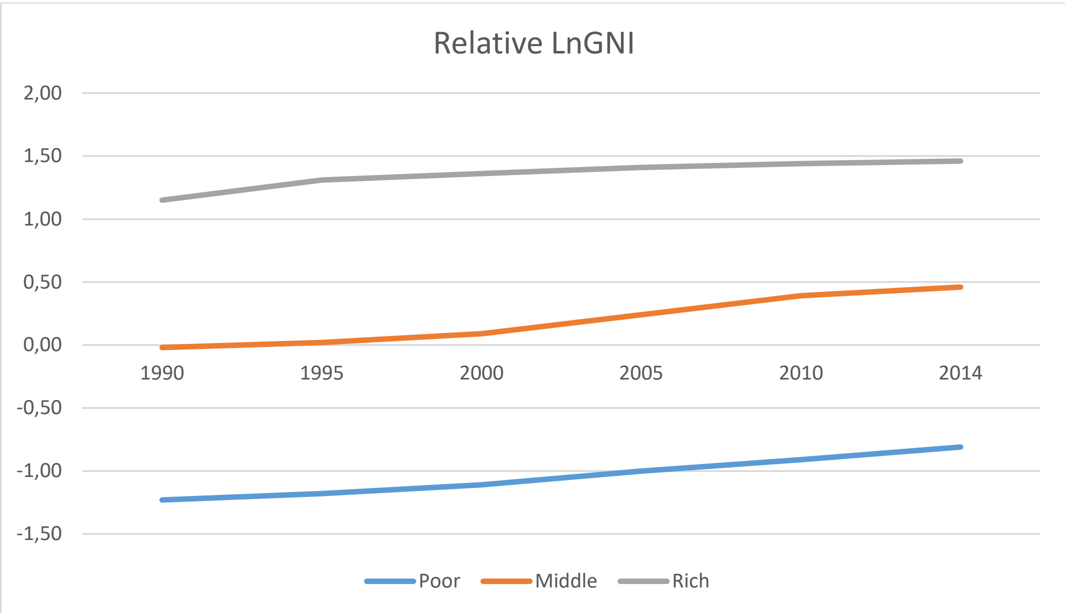


Figure 2: *Evolution of the estimated means of the components in the year-by-year model: life expectancy*

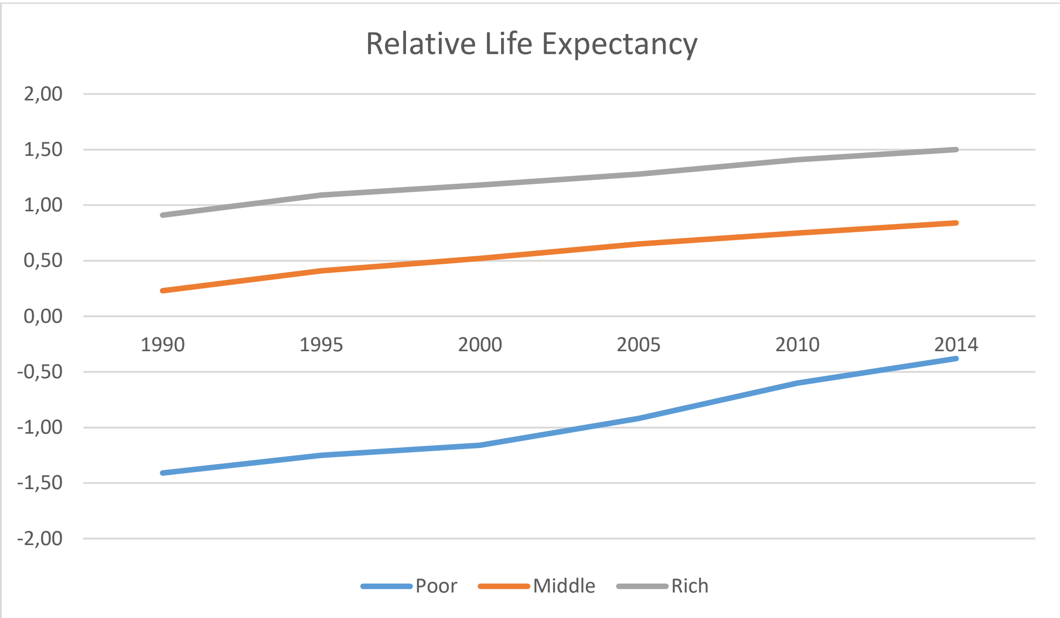


Figure 3: *Evolution of the estimated means of the components in the year-by-year model: education*

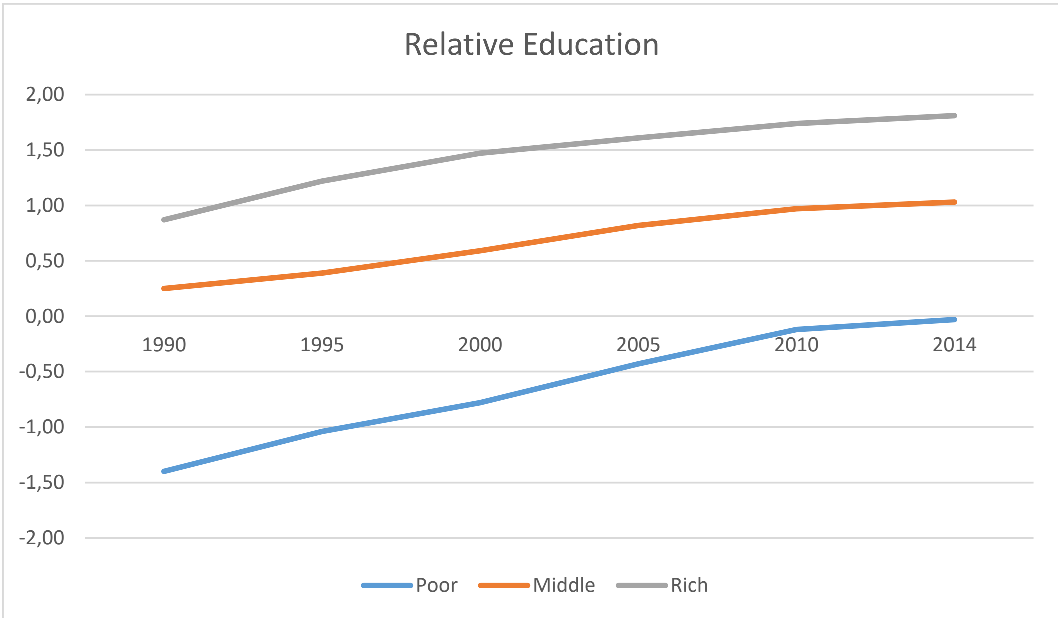
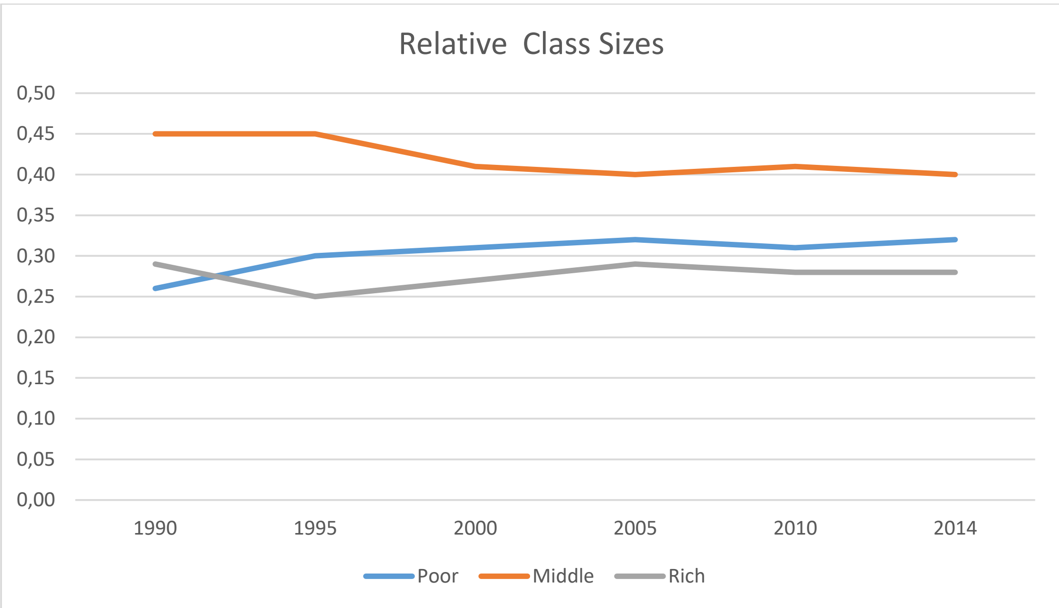


Figure 4: *Evolution of the estimated weights of the components in the year-by-year model.*



classes respectively.

In terms of relative class sizes the Low HD class has grown over the period with a shrinking Medium HD Class and a relatively stable High HD class. Their respective annual GNI growth rates are 1.75%, 2% and 1.29%. With respect to life expectancy the Low class has experienced some catch-up with the Medium and High classes (whose Relative Life Expectancy gap persists) with an annualized growth rate of 4.3% compared to 2.5% for the two upper classes.

The extent to which the classes are polarizing or converging can be studied using a multi-dimensional bi-polarization measure (Anderson, Linton and Leo, 2012) based upon kernel estimates, between two unimodal group distributions i,j, with relative population sizes w_i, w_j , given by:

$$\mathrm{POL}_{i,j} = \frac{0.5}{w_i + w_j} \left(w_i \cdot f_i(x_{mi}, y_{mi}, z_{mi}) + w_j \cdot f_j(x_{mj}, y_{mj}, z_{mj}) \right) \\ |(x_{mi}, y_{mi}, z_{mi}) - (x_{mj}, y_{mj}, z_{mj})|$$

Where $|(x_{mi}, y_{mi}, z_{mi}) - (x_{mj}, y_{mj}, z_{mj})|$ is the Euclidian distance between the modal points (x_{mi}, y_{mi}, z_{mi}) and (x_{mj}, y_{mj}, z_{mj}). In the present context with independent multivariate normal distributions in a mixture distribution this may be written as:

$$\mathrm{POL}_{i,j} = \frac{0.5}{\sqrt[3]{2\pi}\,(w_i + w_j)} \left(w_i \cdot \frac{1}{\sigma_{xi}\sigma_{yi}\sigma_{zi}} + w_j \cdot \frac{1}{\sigma_{xj}\sigma_{yj}\sigma_{zj}} \right) \\ |(x_{mi}, y_{mi}, z_{mi}) - (x_{mj}, y_{mj}, z_{mj})|$$

This measure, together with approximate standard errors, are reported in Table 4. Note the trending polarization between all groups especially post 2005.

The smoothly trending processes illustrated in the foregoing suggests a model in which the progress of the classes is systematically linked with past class structure informing the present. To reflect this, the model now entertained is the hidden Markov model with time-varying number of latent states described in Section 2.

The final results are based on the assumption that the component means are fixed while the variances are not equal between components and each component variance may vary over time according to a specific GARCH-type regression.

In order to choose among the four possible models (constant/variable means x constant/variable variances) initially the most complex model was fitted (with trends in

Table 4: *Polarization measures between components from the year-by-year model (in brackets the approximated standard errors)*

	Low vs. Medium	Low vs. High	Medium vs. High
1990	20.43	21.81	10.80
	(2.88)	(2.32)	(2.74)
1995	17.80	29.04	20.83
	(2.76)	(2.65)	(3.65)
2000	20.05	22.09	20.71
	(2.85)	(2.32)	(3.56)
2005	25.02	26.80	25.07
	(3.25)	(2.63)	(4.05)
2010	26.89	47.86	32.97
	(3.53)	(3.62)	(4.80)
2014	32.14	51.39	39.77
	(3.95)	(3.83)	(5.32)

the mean and variances) but was found to be rather unstable largely because the number of countries is fixed and relatively small as compared to the number of parameters involved in such model.

Reducing the number of parameters by assuming fixed means and variances leads to stable but biased solutions, as this strong form of homogeneity is rejected by the data. Models with trends only in the dimension of means *or* in the dimension of variances are more stable for the data at hand. We have proceeded by separately fitting them.

Based on the reduction in the log likelihood or deviance, the model with trends in the cluster-specific variances has been preferred to the model with trends in means. The GARCH-type model with constant means shows almost all α_{jkr} parameters to be non-zero with convincing evidence.

These results hold for any choice of plausible values of fixed number of latent states. Having chosen the model with constant relative means and varying standard deviation, attention was focused on assessing the year-specific number of latent states/components and the estimated parameters.

The first hypothesis to be tested is that the number of components k of the multivariate distribution remains fixed (the alternative being that the number varies over time). The null hypothesis of k fixed is strongly not rejected (the estimated probability of rejecting the null is 0.002). Conditionally fixed k over time its actual value has to be assessed and there was overwhelming evidence in favor of $k = 3$ with respect to $k = 1; 2; 4; 5$, as assessed by practically any measure. For instance, testing $k = 3$ against $k = 2$ (which is the second most likely), the probability of rejecting the null smaller than 0.001. Incidentally, this endogenously determined clustering contrasts the four categories proposed by the 2014 Human Development Report for country grouping the HDI.

3.2 Characteristics of the components

The estimated parameters of the hidden Markov model in which relative means are kept fixed and variances are allowed to vary are reported in Table 5.[5]

Table 5: *Estimated parameters of the HMM model. Components are labeled: low HD, medium HD and high HD.*

	log(GNI)	**Life_exp**	**Yrs_Educ**
		means	
low HD	-0.808	-0.424	-0.384
medium HD	0.004	0.570	0.419
high HD	1.366	1.308	1.604
		standard deviations - 1990	
low HD	0.843	1.148	0.985
medium HD	0.631	0.417	0.516
high HD	0.521	0.495	0.739
		parameter α	
low HD	0.912*	0.861*	0.885*
medium HD	0.915*	1.063	1.138*
high HD	0.918	0.832*	0.911*

Note: Asterisk $*$ means significantly different from 1 at least at 5% level.
For each year other than 1990, the standard deviation can be calculated as: $\sigma_{jtr} = \alpha^t_{jr}\sigma_{j(1990)r}$, where j is the generic component, r the generic variable and t is time. $t = 1$ stands for 1995, $t = 2$ stands for 2000,..., $t = 5$ for 2014.

The components are well separated and reflect the different stages of human development of the three groups. An interesting feature is the reduction of variability for all

[5] We also estimated a hidden Markov model with varying means and constant variances. The means of the three components show an increasing trend in all the dimensions, corroborating the previous results obtained with the year-by-year model. Interestingly enough, the coefficients of the trend estimated for the three groups are insignificantly different from each other for GNI and for Education. Instead, for Life expectancy, the slope of the trend of the poor group is significantly steeper than those of the middle and rich groups, indicating a catching-up process of the poor group in this dimension.

the variables in both the Low and the High HD group, indicating a substantial process of polarization. Instead the Medium HD group distribution shows a significant squeeze in per capita income and an increase in education.

Table 6 reports the relative group sizes estimated with the hidden Markov model.

Over the period the relative size of the groups has changed considerably with the poor group membership diminishing somewhat (interpretable as a reduction in the poverty rate) with a corresponding increase in the middle and rich group relative size. Notice this is substantially different from the year-by-year model where the middle class declined in size and the poor class grew.

Table 6: *Relative group size of the components of the HMM model*

	Relative group size		
Year	Low HD	Medium HD	High HD
1990	0.41	0.43	0.16
1995	0.41	0.42	0.16
2000	0.40	0.43	0.17
2005	0.38	0.45	0.17
2010	0.35	0.47	0.18
2014	0.34	0.47	0.19

Table 7 reports the results of the transvariation calculations and the within group sigma calculations. Both un-weighted and weighted transvariation measures record an increase in variation (sigma-divergence) up to 2005 with a diminution (sigma convergence) thereafter. This is consistent with the Low HD versus High HD polarization observed post 2005 in the unrestricted estimates. With regard to within group variation sigma convergence is observed for Low HD and High HD groups while the Middle HD group appears to be diverging somewhat. The standard error of the estimates yields an approximate value of 0.014 and a standard error for differences of 0.02 suggesting that the only significant change is the 2005–2014 reduction.

Table 7: *Transvariations and within group inequality measures of the HMM model*

Year	Transvar	WTrans	Within group inequality		
			Low HD	Medium HD	High HD
1990	0.8898	0.8802	0.9532	0.1358	0.1906
1995	0.8947	0.8726	0.6624	0.1503	0.1326
2000	0.9015	0.8878	0.4604	0.1663	0.0923
2005	0.9087	0.8964	0.3199	0.1841	0.0642
2010	0.9081	0.8998	0.2223	0.2038	0.0447
2014	0.8748	0.8758	0.1545	0.2256	0.0311

3.3 Mobility and polarization

The 5-year and implicit 25-year transition matrices (obtained as the 5-year transition matrix to the power of 5) are given in Table 8.

Table 8: *The estimated 5-year (hidden) transition matrix*

Final year	Initial Year		
	Low HD class	Medium HD class	High HD class
5 year			
Low HD class	0.971	0.003	0.005
Medium HD class	0.025	0.927	0.005
High HD class	0.004	0.058	0.991
25 year			
Low HD class	0.863	0.016	0.021
Medium HD class	0.106	0.733	0.021
High HD class	0.031	0.251	0.958

Following Anderson (2016), for the 5-year transitions this yields a Mobility Index of 0.037 which corresponds to a slowly evolving long run process with a considerable lack of mobility between the classes. What mobility there is tends to be upward, though the upward advancement index of 0.5169 is insignificantly greater than 0.5, similarly the polarization index did not indicate significant polarization (0.529). In the longer run the 25-year Mobility Index of 0.147 which corresponds to a slowly evolving long run process with a lack of mobility between the classes. A polarization index of 0.597 suggests some significant polarization in the system (> 0.5 with a standard error 0.038). What little mobility there is, is upward, with an upward advancement index of 0.5748 indicating significant upward mobility (> 0.5 standard error 0.038) which is consistent with a diminishing poor group. A polarization index of 0.5970 suggests some significant polarization in the system (> 0.5 with a standard error 0.038). All of this corresponds to a slowly evolving class structure.

Looking at country specific results in detail few changes in classes are observed in the vast majority of cases, which accords with the rigidity of the transition matrix. In tune with the suggestion of some upward mobility, decreasing probability of Low HD class membership and increasing probability of Medium and High HD class membership the changes that were detected were upward. Notable movers were former members of the USSR, Croatia, Hungary, Lithuania, Poland, Romania and Slovakia all moved into the High class from the Medium class and the Czech Republic and Slovenia consolidated their positions in the High HD class. Other "movers" from the Medium to the High class were Argentina, Bahamas, Barbados, Chile, Costa Rica, Oman and Saudi Arabia. Movers from the Low class to the Medium class were Bolivia, Botswana and Guatemala. China can be seen to be consolidating its Medium class position and India, though remaining in the Low group can be seen to be moving toward the Medium HD group.

4 Conclusions

Recent concerns about the measurement of wellbeing have led to the progress of nations to be classified and studied in a multidimensional context. Perhaps the most popular multi-dimensioned measure is the Human Development index. Unfortunately, increasing dimensionality, whilst better reflecting wellbeing, compounds the difficulties encountered in categorizing groups largely with regard to the arbitrary choice of boundaries (Ravallion 2010). In a one dimensional setting Anderson, Pittau and Zelli (2014, 2016) circumvented this problem by defining classes in terms of the commonality of behaviours of the actors. The downside of this approach is that agents can no longer be definitively placed in a class, all that be discerned is the probability that an agent is in a particular class. However, this was shown not to hinder analysis and it did circumvent the problems associated with arbitrarily determined boundaries by classifying groups according to the commonalities of their behaviours.

Here a feasible methodology for performing a similar analysis in a multidimensional setting has been presented and the progress of 164 nations has been examined over the period 1990-2014. In that context measures of poverty, inequality, polarization and mobility have also been proposed and implemented. Contrary the usual four group classification (UNPD, 2016) three groups, Low Human Development (HD), Medium HD and High HD, with a commonality of behaviours were established. The analysis detected a slowly evolving, relatively immobile world, over the period the Low group diminished in size (which may be interpreted as a reduction in the poverty rate) reflective of some upward mobility. While there was some evidence of reduced inequalities both within and between groups, the transition structure did characterize polarizing behaviour with the groups in essence growing apart (this was also detected in the elementary year-by-year analysis). For the most part, countries stayed within their groupings though some advancement was seen for former Soviet Socialist Republics and for some South American nations.

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