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RANKING INCOME DISTRIBUTIONS WHEN NEEDS DIFFER

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We derive criteria for ranking income distributions where households differ in equity-relevant non-income characteristics ('needs'), using methods which do not require cardinal specifications of equivalence scales. We consider comparisons for situations where the distributions of needs differ (e.g. cross-nationally or intertemporally), building on the results of Atkinson and Bourguignon (1987) and Atkinson (1992). The modifications required when the individual rather than the household is the income-receiving unit are also discussed. We illustrate the methods with an analysis of changes in social welfare and poverty in the U.K. between 1981 and 1986.

INTRODUCTION

To compare distributions of income for populations consisting of households which differ in non-income equity-relevant characteristics ("needs," for short), analysts typically use an equivalence scale. Deflating each household's raw income by its equivalence scale rate converts the income distribution for each heterogeneous population into a homogeneous distribution of "equivalent" income. Thence analysts are able to apply the great range of income distribution comparison methods developed in the theoretical literature, virtually all of which assume homogeneous populations. This practical advantage of equivalence scales is, however, offset by a significant disadvantage: specification of an equivalence scale requires strong assumptions about the relationship between income and needs, and there may not be wide agreement about what the appropriate assumptions should be. (The continuing co-existence of many different equivalence scales is evidence for this view.) Moreover there is growing evidence that the results of distributional comparisons are sensitive to the choice of equivalence scale.[1] This paper considers methods for making income distribution comparisons which employ weaker (and hence less controversial) assumptions about the income-needs relationship than equivalence scales do.

Specification of an equivalence scale embodies three assumptions. The first and most basic is the choice of household characteristics with which to summarise differences in needs (e.g. household size and household composition): specification of these implies a partition of the population into subgroups differentiated by needs (e.g. single adults, childless couples, couples with children).[2] The second assumption summarises judgements about the *ranking* in terms of needs of each household type relative to all the others. The third summarises *how much* more

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[1]For a more detailed discussion of the assumptions embodied in equivalence scales and the evidence about sensitivity of comparisons to equivalence scale choice, see Coulter *et al.* (1992*a*, 1992*b*).

[2]We make no distinction in the the theory between a "houschold" and a "family".

needy a household from one group is assumed to be relative to every other household, and is the aspect about which there is probably least agreement. For example, the equivalence scale recommended by the OECD (1982) in its *List of Social Indicators* incorporates the assumption that a couple with two children aged two and four years is 1.58 times more "needy" than a childless couple at each money income level. By contrast the ratio is 1.36 according to the McClements (1977) scale used extensively in the U.K. by government and academics. These are not isolated cases: as Buhmann *et al.* (1988, Table 1) detail, there is a wide range of equivalence scale relativities in common use.

In this paper we develop methods which weaken the third type of assumption, building on the recent work of Atkinson and Bourguignon (1987) and Atkinson (1992).

Our results on social welfare extend those of Atkinson and Bourguignon (1987) to situations where the marginal distribution of needs differs. Although Atkinson and Bourguignon's results can be used to assess, say, the reform of income taxation in one particular country (since pre- and post-reform income distributions contain the same households), in general they cannot be used to assess secular trends in the income distribution for one country, or for comparisons across countries.

Atkinson's (1992) results concerning poverty can be applied when the proportions of different household types differ. So too may the modified sequential dominance conditions we derive. However, we argue below that our formulation of the function summarising aggregate poverty is more appropriate than Atkinson's.

The advantage of the formulations we use is that they allow a clear distinction to be made between welfare-improving (resp. poverty-reducing) income distribution change and a welfare-improving (resp. poverty-reducing) population composition change. Moreover the formulations easily accommodate the modifications to analysis which are required when the income-receiving unit is assumed to be the person rather than the household. Atkinson and Bourguignon (1987) and Atkinson (1992) give little attention to the issue of the income-receiving unit.

In some other respects our paper parallels those by Atkinson and Bourguignon (1987) and Atkinson (1992). We derive, as they do, "sequential dominance" conditions which ensure that one income distribution is unambiguously preferred to another distribution, whatever the structure of needs relativities. Our results too provide ordinal measures rather than cardinal indices.[3]

Throughout the paper, we suppose that there is agreement as to which household characteristics are relevant to assessments of income distributions and that, on the basis of these judgements, the population is partitioned into n mutually exclusive socially homogeneous subgroups.

[3]The sequential dominance conditions, if satisfied, allow one to say one distribution is preferred to another, but not by how much (which is the price to be paid for weakening the assumptions about social judgements). The results are analogous to those of Atkinson (1970) who proved that rankings of income distributions according to all standard inequality measures are equivalent to Lorenz dominance, except that differences between households along a second dimension, needs, are now incorporated in addition to differences in income. (Atkinson, 1970, assumed a homogenous population.) Coulter *et al.* (1992*a*, section 7) survey the difficulties of characterising changes in inequality (rather than social welfare or poverty) when there are differences in needs.

Let $F_i(x)$ be the distribution function for the income x of households in the ith subgroup (where $i=1, \ldots, n$), and θ_{iF} be subgroup i's population share (the number of households in subgroup i, divided by the total number of households). If A is any income level exceeding the maximum one in any subgroup, then we have $F_i(A)=1$, for all i, and $\sum_i \theta_{iF}F_i(A)=\sum_i \theta_{iF}=1$. We shall compare this income distribution with another distribution characterised similarly, using subgroup distribution functions $G_i(x)$ and population shares θ_{iG}, where $i=1, \ldots, n$.

Sections I and II contain our results for comparisons of social welfare and poverty respectively (with proofs provided in an Appendix). In both sections we assume that the household is the income-receiving unit. The modifications to the analysis which are required when the income-receiving unit is, instead, the individual are discussed in Section III. In Section IV, we illustrate the methods with an analysis of changes in social welfare and poverty in the U.K. between 1981 and 1986. Section V provides concluding comments.

I. Social Welfare Comparisons for Heterogeneous Households

We assume, following Atkinson and Bourguignon (1987), that social welfare for the population W_F is the average of the subgroup social welfares:[4]

$$W_F=\sum_{i=1}^{n} \theta_{iF} \int_0^A u_i(x)\, dF_i(x)=\sum_{i=1}^{n} \theta_{iF}W_{iF}. \tag{1}$$

Here $u_i(x)$ is the increasing and concave function summarising the social valuation of income for a subgroup i household:

$$u_i'(x)\geq 0 \quad \text{and} \quad u_i''(x)\leq 0 \quad \text{for all } x \quad \text{and all } i=1, \ldots, n. \tag{2}$$

Now let us suppose that the subgroups can be ranked in descending order:

$$u_1'(x)\geq u_2'(x)\geq \ldots \geq u_n'(x)\geq 0 \tag{3a}$$

and

$$u_1''(x)\leq u_2''(x)\leq \ldots \leq u_n''(x)\leq 0 \tag{3b}$$

for all x. By (3a) subgroup 1 is the "neediest"—it has highest social marginal valuation of income at each income level—and group n is the least "needy." For groups in-between, "differences in social marginal valuations between groups become smaller as we move to higher income levels. It may not be unreasonable to suppose that we become less concerned about differences in needs at higher incomes" (Atkinson and Bourguignon 1987, p. 360).[5] Hence assummption (3b).

[4]Our results are not in fact conditional on the interpretation of the θ_is as subgroup population shares; these could represent any other set of subgroup weights. With our chosen interpretation, each individual income unit receives an equal weight in the social aggregation process. By contrast with this "utilitarian" approach, a quasi-"Rawlsian" one might set $\theta_i=1$ for the neediest group, and $\theta_i=0$ for the other groups.

[5]It should be noted that the $u_i(x)$, and the poverty contribution functions introduced below, incorporate views about both economies of scale and "pure" differences in needs. The former depend primarily on household size (broadly speaking, the number of adults and children), whereas the latter concern grounds for treating households differently even if they have the same composition (e.g. because of old age, disablement etc).

To extend the results about social welfare of Atkinson and Bourguignon to the case of different distributions of needs, we need one additional assumption:

$$u_i(A) = u_{i+1}(A) \quad \text{for all } i, \tag{4}$$

where A is the maximum conceivable income level. Together with the assumptions in (2) and (3), this implies that $u_i(x) \leq u_{i+1}(x)$ at each x, i.e. that those in more needy groups derive lower utility from a given level of income than those in less needy groups. This represents a significant weakening of the assumptions made in the equivalence scale literature, where it assumed not only that a more needy group has lower welfare than a less needy group if they have the same money incomes, but also *how much* lower at every income level.

The assumptions imply our main result concerning social welfare comparisons.

Theorem 1. $W_F \geq W_G$ for all $u_i(x)$, $1 \leq i \leq n$, satisfying (3) and (4) if

$$\sum_{i \leq j} \int_0^x [\theta_{iG} G_i(y) - \theta_{iF} F_i(y)] dy \geq 0 \quad \text{for all } x \quad \text{and all } j = 1, \ldots, n.$$

The dominance conditions are based on comparisons of cumulated areas between distribution functions scaled by subgroup population shares. The procedure is a sequential one, undertaken first for the neediest subgroup alone, and then successively adding the next most needy subgroup and repeating the cumulated area comparisons, until finally the incomes from all the subgroups are used. Unambiguous social welfare rankings are achieved if the cumulated differences in areas are non-negative at each stage.

Necessary conditions for this criterion are obtained at $x = A^*$, the highest income present with non-zero frequency in either distribution:

$$A^* \sum_{i \leq j} (\theta_{iG} - \theta_{iF}) + \sum_{i \leq j} (\theta_{iF} \mu_{iF} - \theta_{iG} \mu_{iG}) \geq 0 \quad \text{for all } j = 1, \ldots, n \tag{5}$$

where μ_{iF} (μ_{iG}) is the mean of distribution F (G) for subgroup i. These conditions on the differences in subgroup shares and (scaled) mean incomes are demanding.

Notice that the dominance criterion in Theorem 1 may be rewritten as

$$\sum_{i \leq j} \int_0^x \theta_{iG} [G_i(y) - F_i(y)]\, dy + \sum_{i \leq j} (\theta_{iG} - \theta_{iF}) \int_0^x F_i(y)\, dy \geq 0. \tag{6}$$

When the distribution of needs is fixed across applications ($\theta_{iG} = \theta_{iF}$, all i), then the second term on the left hand side disappears and the sequential dominance conditions reduce to those derived by Atkinson and Bourguignon (1987). In this special case, the dominance conditions are equivalent to a set of sequential Generalised Lorenz dominance conditions, as they show.

In general however it is clear that the impact on social welfare of income distribution changes [the first term in (6)] may be reinforced—or offset—by population composition changes (the second term). To put things another way, allowing for differences in population composition need not make the dominance criterion more restrictive. We return to this point in the empirical illustration (Section IV).

II. Poverty Comparisons for Heterogeneous Households

We retain the assumption that the population can be partitioned into n mutually-exclusive socially homogeneous subgroups, and define aggregate poverty as the average of the poverties in the subgroups:[6]

$$(7) \qquad P_F = \sum_{i=1}^{n} \theta_{iF} \int_0^A p_i(x)\, dF_i(x) = \sum_{i=1}^{n} \theta_{iF} P_{iF}.$$

Here $p_i(x)$ is the poverty contribution of income x for a household in subgroup i. This function is assumed to be non-negative, decreasing, and zero at or above the poverty line for subgroup i (denoted Z_i):

$$(8) \quad p_i'(x) \leq 0 \quad \text{for all } x \quad \text{and} \quad p_i(x) = 0 \quad \text{for } x \geq Z_i, \quad \text{for all } i = 1, \ldots, n.$$

As Atkinson (1992) shows, the class of poverty indices described by (7) and (8) includes many commonly used indices; for example, the headcount ratio, the poverty gap, the entire Foster *et al.* (1984) class, and the second class of measures proposed by Clarke *et al.* (1981).

There is, however, an important difference between Atkinson's (1992) formulation of the relationship between aggregate and subgroup poverty and our formulation.

Atkinson omits explicit mention of the population share terms, and assumes aggregate poverty is simply the unweighted sum of the poverty in each subgroup (i.e. $P_F = \sum_i P_{iF}$). Population share differences across groups are implicitly incorporated in the definitions of the subgroup income distribution functions. These are "normalized" so that $\sum_i F_i(A) = 1$: see Atkinson [1992, eqn (1)].[7]

We prefer our formulation because we believe there are advantages to be gained from explicitly separating income distribution changes from population composition changes (of which more below). Our subgroup $F_i(.)$ functions are conventionally-defined distribution functions, and it is now clear that the poverty contribution of each income unit receives an equal weight in the social aggregation process. Moreover our assumption is consistent with the now standard practice of *defining* subgroup-decomposable poverty indices to be those for which aggregate poverty is the population-share-weighted sum of poverty within each of the subgroups: see Foster *et al.* (1984).[8]

Now suppose that we can rank groups according to differences in the poverty reduction potential of a marginal increase in income:

$$(9) \qquad p_1'(x) \leq p_2'(x) \leq \ldots \leq p_n'(x) \leq 0 \quad \text{for all } x.$$

At each income level, the potential for poverty reduction is greatest for the neediest group, less for each successively less needy group, and lowest for the least needy group.

[6]Again the $\theta_i s$ can be subject to alternative interpretations (see fn. 4).

[7]By contrast, the definitions we use imply $\sum_i F_i(A) = n$ and $\sum_i \theta_{iF} F_i(A) = 1$.

[8]One of us has previously suggested that Atkinson's definition of aggregate poverty is also not invariant to replications of the population (Coulter *et al.*, 1992*a*, p. 115n). As long as the subgroup poverty indices are themselves replication-invariant, this claim is incorrect.

Notice that we have not assumed agreement about what the income levels corresponding to poverty lines $Z_1, \ldots, Z_n$ are. We are in effect making a weaker assumption: that more needy subgroups have higher poverty lines, i.e. $Z_1 \geq Z_2 \geq \ldots Z_n$, since at any given income level a higher poverty line provides greater scope for poverty reduction.

The extra assumptions embodied in (9) narrow the class of poverty indices. The headcount ratio is ruled out, for example, because the marginal poverty reduction may be zero for one group, infinite for another slightly more needy group, and then zero again for an even needier group.[9] On the other hand, if $p_i(x) = (Z_i - x)^\alpha/\alpha$, with $\alpha > 1$ or, alternatively if $p_i(x) = (Z_i^c - x^c)/c$ with $c \leq 1$, then the assumptions are satisfied (Atkinson, 1992).

Our second result can now be stated.

Theorem 2. $P_F \leq P_G$ for all $p_i(x)$, $1 \leq i \leq n$, satisfying (8) and (9) if

$$\sum_{i \leq j} [\theta_{iG} G_i(x) - \theta_{iF} F_i(x)] \geq 0 \quad \text{for all } x \leq Z_j \quad \text{and all } j = 1, \ldots, n.$$

The dominance conditions described by Theorem 2 are based on comparisons of differences in scaled distribution functions over restricted domains. The sequential procedure begins with the neediest subgroup, and comparisons are based on all incomes up to and including the relevant poverty line, Z_1. The next comparison is for the two neediest groups, and is based on incomes up to and including $Z_2 (\leq Z_1$, by assumption). Subsequent comparisons successively incorporate less and less needy groups, and use lower and lower poverty lines. At the last stage all groups are used and the upper bound of the income range used for comparisons is Z_n. Unambiguous poverty rankings are secured if the cumulated differences are non-negative at each stage.[10]

To check the conditions in practice we need to specify income levels for $Z_1, \ldots, Z_n$. The advantage of the framework set out is that there need not be consensus about what these income levels should be precisely.[11] By setting each Z_j at the *maximum* value we are prepared to contemplate, the dominance conditions require us to check all possible poverty lines within the range defined by these upper bounds (and the earlier assumption about rankings of subgroup poverty lines).

The dominance conditions in Theorem 2 can be written as

$$\sum_{i \leq j} \theta_{iG}[G_i(x) - F_i(x)] + \sum_{i \leq j} (\theta_{iG} - \theta_{iF}) F_i(x) \geq 0 \tag{10}$$

which shows clearly how poverty-reducing income distribution changes may be reinforced—or offset—by population composition changes.

[9] All that matters is whether incomes are above or below the poverty line, not the extent of the difference. See Atkinson (1992, p. 8) for further discussion.

[10] Differences in normalization aside, the dominance conditions in our Theorem 2 correspond to those in Atkinson's (1992) condition D1. Notice that his P is measured negatively (more poverty is a Bad Thing); his p_is are the negatives of ours.

[11] In intertemporal comparisons there are debates about whether poverty lines should change when average living standards change or remain fixed in real terms. This property goes some way towards taking account of both views.

The dominance criterion is more stringent the larger is each Z_j. If Z_n (the smallest poverty line) approaches the highest income present with non-zero frequency in either distribution, A^*, then

$$(11) \qquad \sum_{i \leq j} (\theta_{iG} - \theta_{iF}) \geq 0 \quad \text{for all } j$$

is a necessary condition. These restrictions on the differences in the population shares clearly constrain the scope of the criterion to address situations where the family composition distribution is not fixed and the poverty lines are not constrained, but they do not rule out such comparisons (as we shall see).

Notice that if the poverty dominance criterion is in fact satisfied when Z_t approaches A^*, then the welfare dominance criterion stated in Theorem 1 must also be satisfied.

III. Comparisons when the Income-receiving Unit is the Individual

In the earlier sections we assumed that the household was the income-receiving unit. Let us now consider the modifications to the analysis required when the individual is the income-receiving unit. It is important to do this because it is usually assumed that it is the economic well-being of individuals with which we are ultimately concerned.

The key practical problem which arises is how measures of personal well-being (which is unobserved) may be derived from the household information which is available. The usual way of solving this problem is to assume that incomes are equally distributed within households.[12]

With this assumption, the *money income* received by each person within a given household equals total household money income divided by the number of household members. This money income may not, of course, be a good measure of the person's economic welfare because it ignores the impact of differences in household needs. The standard way of meeting this point is to use some cardinal equivalence scale, and to deflate household income by the appropriate equivalence scale factor. Then each person is assumed to receive the equivalent income of the household to which he/she belongs.[13]

This strategy is not appropriate in the current context because we no longer assume there is a cardinal equivalence scale available. Instead assumptions about the impact of economies of size and differences in needs are incorporated via the $u_i(x)$ and $p_i(x)$ functions. Income x is unadjusted money income, not equivalent income.

Our proposals for adapting the conditions derived in Sections I and II to the case in which the individual rather than the household is the income unit also utilize the equal-sharing assumption. However, some other definitions and assumptions are modified.

For welfare comparisons of distributions for heterogeneous persons, we propose that each person be attributed an income equal to the total income of the household to which she belongs divided by the number of household members,

[12]See Jenkins (1991) for a review of alternative strategies.
[13]Danziger and Taussig (1979) and Cowell (1984) set out the rationale for this strategy.

i.e. household income per capita (and x is appropriately reinterpreted). The partitions of the population into the n subgroups are as before, but now the population share term θ_i is the total number of persons in subgroups i divided by the total number of persons. The distribution function F_i is now the distribution function for household income per capita for the persons in subgroup i.

The same modified interpretations of x, θ_i and F_i apply to poverty comparisons, but there is additional complication in this case: we have also to redefine the poverty line each person's income is compared with. Application of the same principles as above implies that the poverty line for a person from household type i should equal the household poverty line divided by the total number of household members, i.e. Z_i should be reinterpreted as a household poverty line per capita.

In Section II larger households were ranked as more needy than smaller households, *ceteris paribus*. An interesting question arises now as to whether *persons* from larger-sized households are ranked as more needy than *persons* from smaller-sized households. Does the household-based ranking of groups required by (3) and (8) coincide with the person-based ranking?

For welfare comparisons we cannot think of reasons why the ranking should be different. Indeed there is an additional reason for ranking a person from a larger household as needier than a person from a smaller household with the same (household per capita) income. The reason is that there is uncertainty about whether incomes are in fact equally shared within households, as assumed. Since uncertainty about whether a given person receives her household per capita income is likely to be greater the larger the household to which she belongs, and uncertainty is a welfare-reducing influence *ceteris paribus*, there is a case for reflecting this in higher marginal social valuations for persons from larger households.

For poverty comparisons, matters are more complicated. When the household is the income-receiving unit, poverty lines are typically increasing functions of household size. To see this notice that a common specification for the poverty line for a household of type i is $Z_i = Z_n M_i$, where Z_n is the poverty line for a single adult household and M_i is the equivalence scale rate for a household of type i, which is usually assumed to be increasing in household size. For example, according to the Buhmann *et al.* (1988) constant-elasticity scale, which provides a reasonable description of most of the equivalence scales in common use, $M_i = (N_i)^\rho$ where $0 \leq \rho \leq 1$ and N_i is the number of people comprising a household of type i. This suggests that one plausible *maximum* poverty line for use in empirical applications is $Z_n N_i$ (as in Atkinson, 1992).

If we apply the same thinking to the case where the individual is the income-receiving unit, then the appropriate personal poverty line for an individual from a household of type i is $(Z_n N_i)/N_i = Z_n$: the personal poverty line is the same for all persons, regardless of household type.

The argument concerning uncertainty and equal sharing might also be applied here; we might assume that the marginal poverty reduction generated by an income increase is greater in magnitude for persons from larger households than for persons from smaller households.

In sum, changing the definition of the income-receiving unit from the household to the individual raises complicated issues. The fundamental problem is the lack of information about how incomes are distributed within households.

IV. Changes in Poverty and Social Welfare: the U.K., 1981–86

We now use our theoretical results to analyse changes in poverty and social welfare between 1981 and 1986 for two-adult households in the U.K. We also comment on the empirical usefulness of the Theorems. Were the stringency of the sequential dominance criteria such that they were only very rarely satisfied, this would weaken our case for their use.

We focus attention on a relatively homogeneous section of the total population, *viz.* two-adult non-elderly households, classified into four subgroups according to number of dependent children (0, 1, 2 or 3+ children).[14] The rationale for focussing on two-adult non-elderly households is that there is probably widespread agreement about how households within this group should be ranked according to needs: households with three or more children are the neediest; households with two children, the second neediest; households with one child, the third neediest; and childless households the least needy. Assuming consensus about the needs ranking enables us to concentrate on the other issues which arise when checking dominance conditions.[15]

Our data sources are household micro-data from the 1981 and 1986 UK Family Expenditure Surveys. The definition of income used is household disposable income per week. Disposable income comprises market income from all sources plus cash social assistance and social security benefits less direct taxes (income tax and employee National Insurance contribution).[16] To take account of inflation, all incomes were adjusted to December 1986 prices using the monthly all-items Retail Prices Index.[17]

Some summary statistics are provided in Table 1. In the top panel of the table, the household is assumed to be the income-receiving unit; in the bottom panel, the individual. Following the discussion of Section III, income is defined as (unadjusted) household income in the former case, and as household income per capita in the latter one.

Between 1981 and 1986, there was a clear trend towards smaller-sized households. For example, the proportion in the neediest group fell from 13.4 percent in 1981 to 11.3 percent in 1986 according to the household data, and from 21.0 percent to 19.7 percent according to the person data. Average income for each subgroup rose, whether measured using the mean or the median. Income growth for the lowest decile point was less than the growth for the average, however, and in fact zero or negative for households with fewer than two children. Income

[14]We define non-elderly households to be those with the household head aged less than 65 years. A dependent child is either (i) aged under 16 years, or (ii) aged more than 16 years and less than 19 years, unmarried, and in full-time non-advanced education, and living with his/her parents.

[15]If there is not unanimity about how subgroups should be ranked, then the dominance criteria need to be checked for each set of rankings which is proposed (Atkinson, 1992).

[16]We use specially-created income and demographic variables to ensure definitions are the same in each year: published variables are not consistently defined. Full details of how our data were created are given by Coulter (1991). Coulter *et al.* (1994) demonstrate the importance of using consistently defined data, and provide detailed comparisons of changes in incomes for different household and family types, using both household and family income distributions.

[17]A more extensive analysis would require some analysis of the sensitivity of results to the choice of the price index. Similarly, cross-national studies need to investigate the sensitivity of results to the choice of purchasing power parities.

TABLE 1

U.K. NON-ELDERLY TWO-ADULT HOUSEHOLDS, 1981 AND 1986: SUBGROUP SHARES AND INCOMES

	θ_i		Median		Mean		Coeff. Var.		10%ile		90%ile	
	1981	1986	1981	1986	1981	1986	1981	1986	1981	1986	1981	1986
Household data†												
2 adults, 0 children	41.0	45.0	180	193	195	217	0.50	0.61	94	93	307	351
2 adults, 1 child	18.3	18.3	168	192	186	211	0.49	0.65	97	97	291	338
2 adults, 2 children	28.2	25.4	182	211	194	230	0.44	0.48	103	113	303	364
2 adults, 3+ children	13.4	11.3	179	195	204	216	0.57	0.48	103	118	317	349
All	100.0	100.0	178	197	194	219	0.49	0.57	98	102	304	353
Person data‡												
2 adults, 0 children	26.0	29.4	90	97	98	108	0.50	0.61	47	47	153	176
2 adults, 1 child	17.4	17.9	56	64	62	70	0.49	0.65	32	32	97	113
2 adults, 2 children	35.7	33.1	45	53	49	57	0.44	0.48	26	28	76	91
2 adults, 3+ children	21.0	19.7	33	36	38	40	0.59	0.50	19	21	63	66
All	100.0	100.0	52	59	61	71	0.64	0.72	25	27	111	128

Source: Authors' calculations from the U.K. Family Expenditure Survey.

Notes: θ_i = number of income units in subgroup ÷ total number of income units. Incomes are in pounds per week (December 1986 prices). † income = household disposable income. ‡ income = household disposable income per capita. Coeff. Var. = coefficient of variation. 10%ile = bottom decile point. 90%ile = top decile point.

inequality grew for each subgroup, with the notable exception of the neediest subgroup, for whom it fell. On the one hand, the stagnant income growth for the poorest in each subgroup, and the inequality growth, each suggest the dominance criteria may not be satisfied. On the other hand, the growth in average incomes, and the decline in relative size of the neediest groups, suggest the 1986 distribution might be preferred to the 1981 one.

We have to check our dominance criteria to test these conjectures rigorously. First, however, we need to specify maximum values for subgroup poverty lines. We intentionally chose high maxima to see whether this would affect our results. Our household subgroup maximum poverty lines are: two-adult household with 0 children, £440; two-adult household with 1 child, £660; two-adult household with 3 children, £880; two-adult household with 3+ children, £1,320. We calibrated these noting that mean income in 1986 was about £220 per week, took this as a (very generous) poverty line for a single person, and multiplied £220 by household size for each of the two-adult subgroups (assuming six persons in two-adult households with three or more children). For the person data, the maximum poverty line is the same for all subgroups. We used £100 per week, a round number well above the mean per capita income in 1986 (£71).

Our first checks for poverty dominance are illustrated in Figure 1, which uses the household data and shows information for the neediest group. For poverty in 1986 to be judged unambiguously lower than poverty in 1981 according to all poverty measures satisfying (9), the curve for 1986 should lie everywhere on or below the curve for 1981. The graph on the left gives the picture for all incomes up to the maximum poverty line, £1,320, and suggests the dominance condition is satisfied for this subgroup. Clearly the use of a very generous poverty line does not affect this conclusion.

However, what does affect the conclusion is how one assesses income changes at the very bottom of the income distribution.[18] The right-hand picture focusses on incomes below £80 and shows there is in fact a triple crossing of the curves. A strict interpretation of the dominance criterion implies that we cannot derive an unambiguous poverty ranking of the 1981 and 1986 distributions.

We get similar results for the welfare dominance check for the neediest group: see Figure 2. For social welfare in 1986 to be judged unambiguously lower than social welfare in 1981 according to all measures satisfying (3), the curve for 1986 should lie everywhere on or below the curve for 1981. The left-hand picture showing the curves throughout the income range suggests dominance but the right-hand picture, which focusses on the lowest incomes, shows a single crossing at £60. A strict interpretation of the dominance criterion implies that we cannot derive an unambiguous social welfare ranking of the 1981 and 1986 distributions.

These conclusions about whether we can make unambiguous poverty and welfare rankings are robust to the definition of the income-receiving unit. Figures 3 and 4, based on the person distributions, also shown that we should reject poverty and welfare dominance if we interpret the conditions strictly.

However, should we interpret the conditions as strictly as we have? One argument as to why we should not is that the differences between the curves are

[18]For reference, note that the bottom percentile income in 1986 is £45.

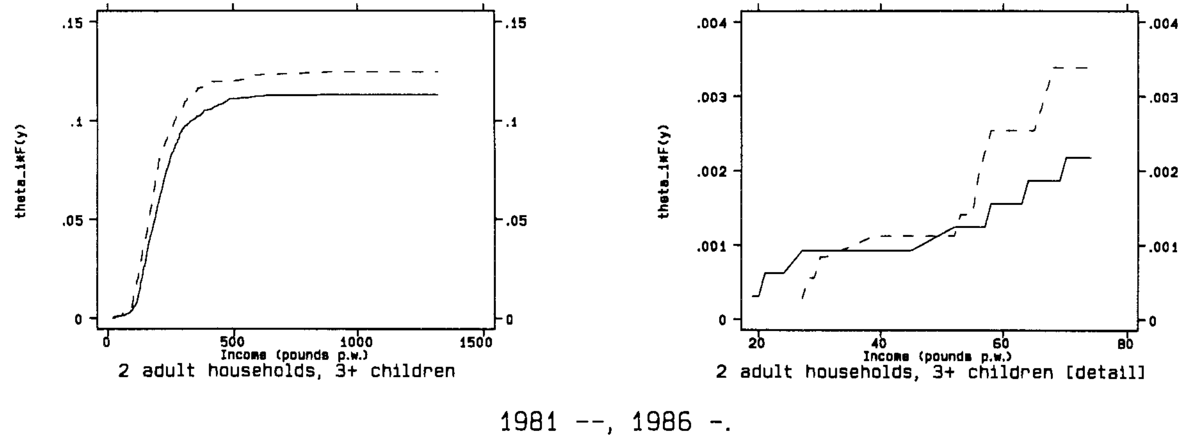

Figure 1. Poverty Comparisons (Households), 1981 and 1986

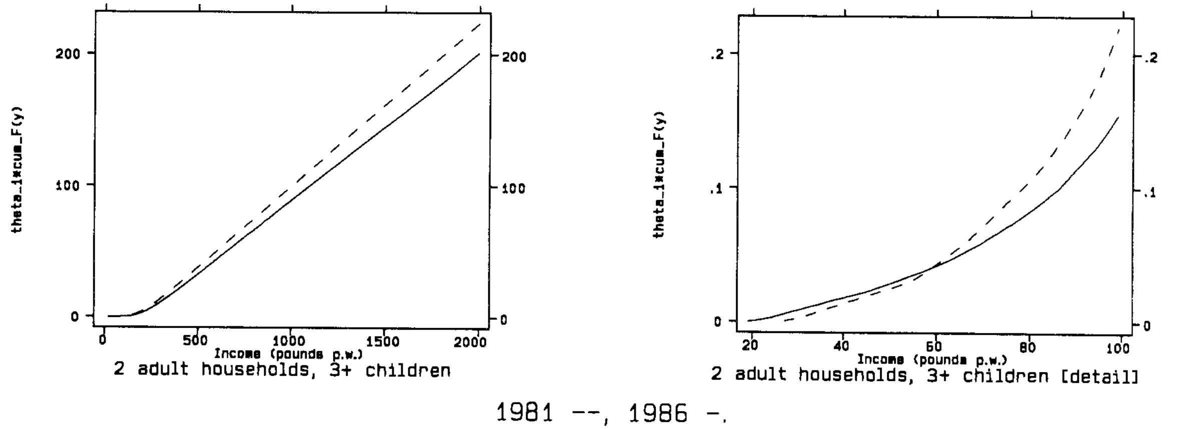

Figure 2. Welfare Comparisons (Households), 1981 and 1986

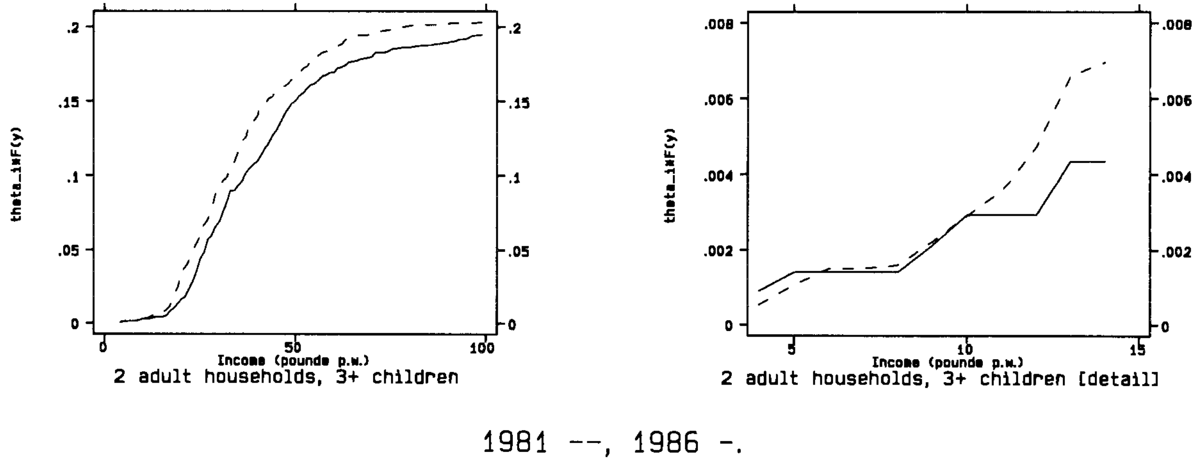

Figure 3. Poverty Comparisons (Persons), 1981 and 1986

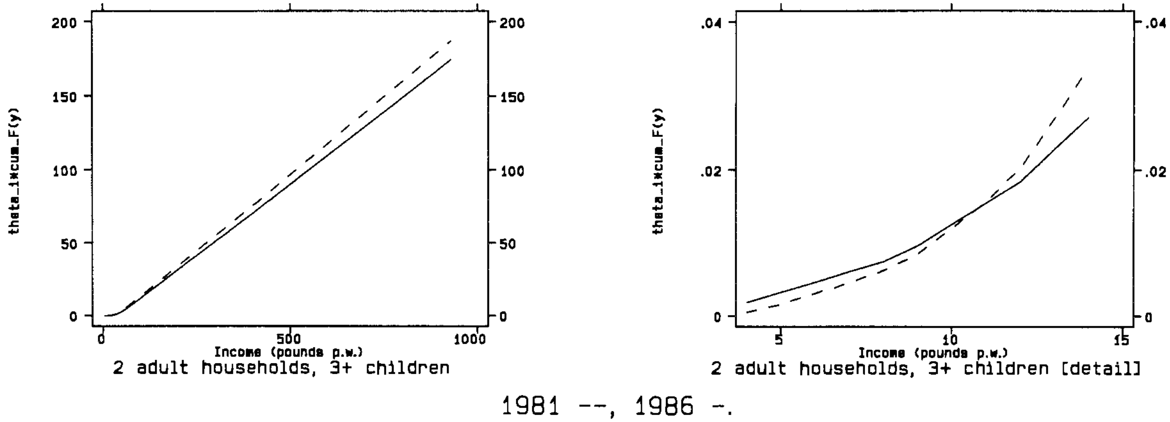

Figure 4. Welfare Comparisons (Persons), 1981 and 1986

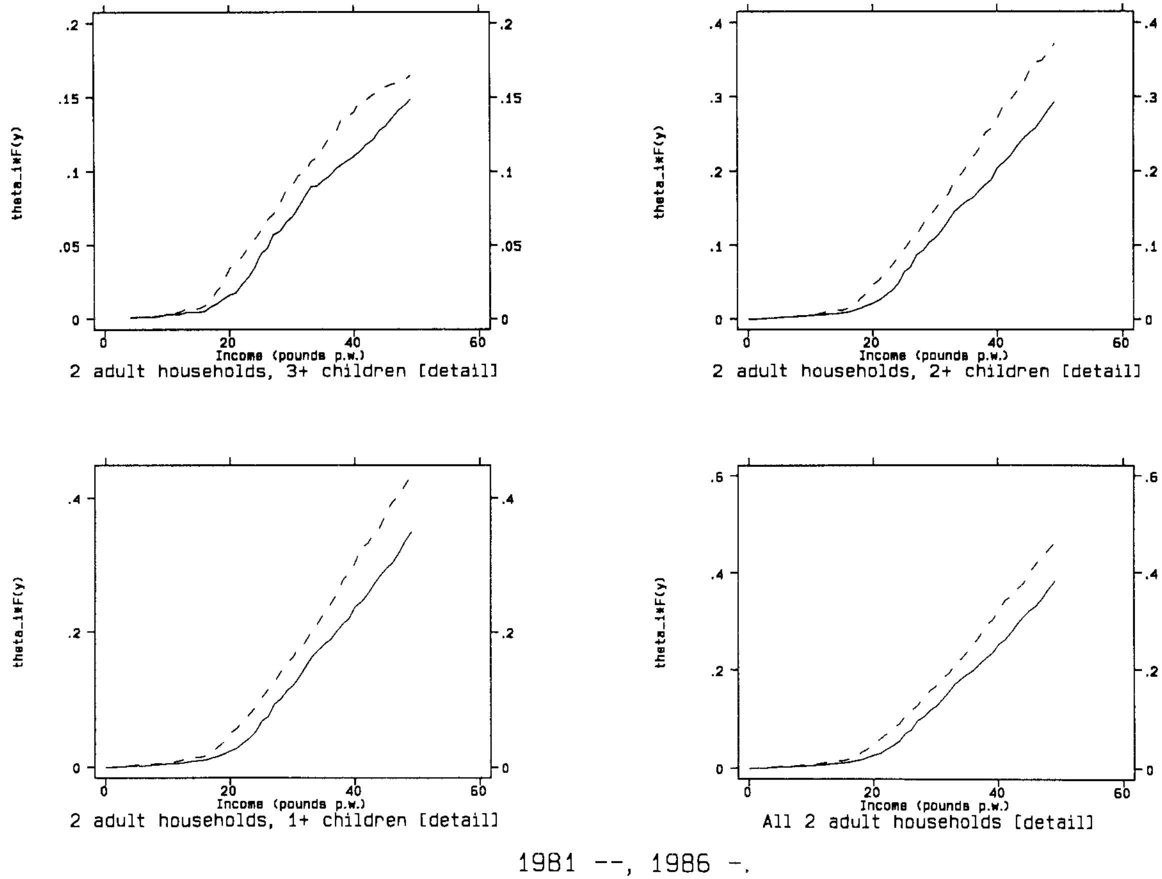

Figure 5. Poverty Comparisons (Persons), 1981 and 1986

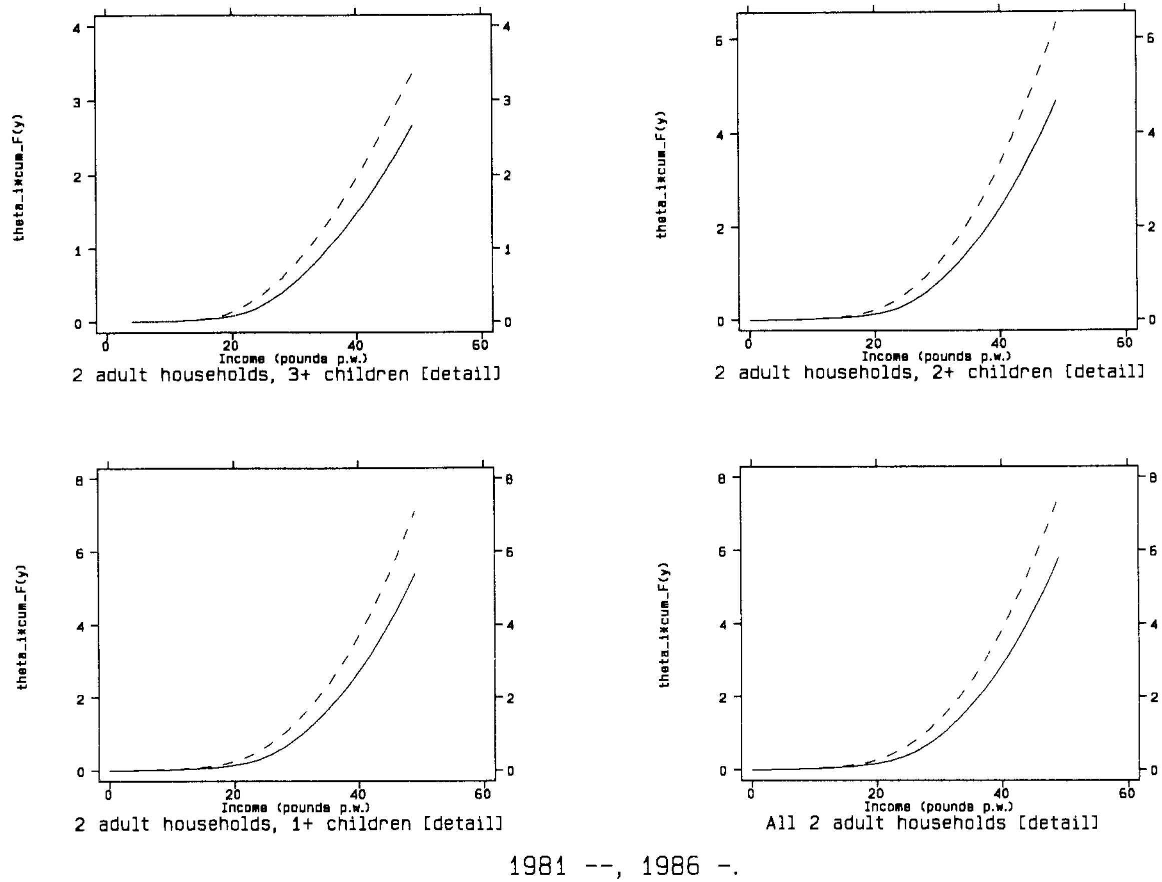

Figure 6. Welfare Comparisons (Persons), 1981 and 1986

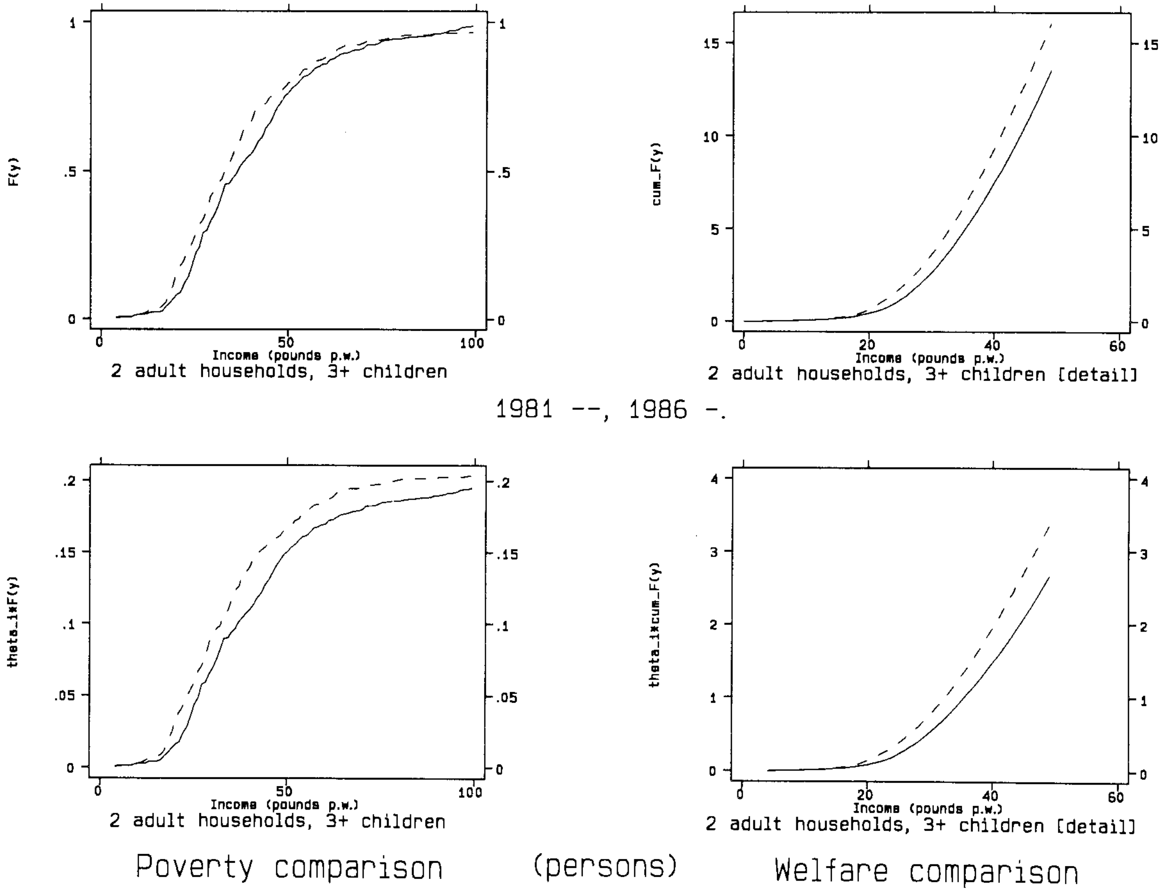

Figure 7. Favourable Population Composition Changes: An Example

relatively small and so likely to be within the bounds of sampling error. This argument cannot be fully assessed since sampling error is an issue that we, like others who have derived sequential dominance criteria, have not addressed.[19] Our conjecture is that the crossings shown are not statistically significant (especially since there may be greater measurement errors for very low incomes)—but this is only a conjecture.

There is also the issue of whether we are imposing a stricter interpretation than most authors (including ourselves) have done in previous empirical work. Our comparisons are made at *every* income within the relevant range (as the theory requires), which is in contrast to most previous comparisons: income distributions are usually compared at five, ten or maybe twenty points maximum (e.g. Lorenz curves are derived at quintiles, deciles or, rarely, vingtiles). Reducing the number of comparison points reduces the potential for detecting crossings of curves.

Let us see what happens if we are a little less stringent in our application of the dominance conditions. We continue to compare incomes at every point, but change the range of incomes for which we present information. For the poverty comparisons based on the person distributions, we now plot the distribution functions for all incomes below £60 per week, a value roughly equal to the median income in 1986. This is a compromise between the maximum income used in the original comparison (£100, Figure 3 left-hand picture), and that used in the corresponding scaled-up picture (£15, Figure 3 right-hand picture). What we are doing is looking at the distribution in more detail than is usually done, but are nonetheless choosing a degree of magnification such that "very small" crossings of curves are not seen.

Figures 5 and 6 show the results of implementing this methodology. Now all the sequential poverty and welfare dominance criteria are apparently satisfied, since each 1986 curve visibly lies on or below the corresponding 1981 curve. Since we already know that the curves do not cross at incomes above the maxima pictured, the poverty ranking conclusion is a powerful one when this perspective is employed: we have dominance whatever the poverty line chosen, and so poverty dominance also implies welfare dominance.

The distinction between "pure" income distribution changes and population composition changes which we made in (6) and (10) is illustrated in Figure 7. This is based on the person distributions for the neediest group, and compares the income distribution component (top two graphs) with the overall change (bottom two graphs). The graphs on the left summarise the poverty comparison; those on the right summarise the social welfare comparison. Recall that the population share of persons in the "2 adult households with 3+ children" group fell from 21 percent in 1981 to 20 percent in 1986 (Table 1); this had both welfare-improving and poverty-reducing influences, illustrated by the greater gaps between corresponding pairs of curves. The poverty-reducing effect is especially noticeable: the population composition change "removes" a crossing of curves between £90 and £100.

[19]For a survey of statistical inference for Lorenz and Generalized Lorenz dominance comparisons in the homogenous population case, see Bishop *et al.* (1989). In practice, even these standard error formulae are typically applied at the decile points only: see *inter alia* Bishop *et al.* (1991).

V. Concluding Comments

Having an equivalence scale is very convenient for empirical work, but any particular scale incorporates cardinal assumptions about which there may not be widespread agreement. We have considered methods for ranking income distributions when the cardinality assumptions are dropped.

Our theoretical contribution is a set of dominance conditions for poverty and social welfare comparisons which may be applied in situations with different distributions of needs, e.g. intertemporally or cross-nationally. In addition, we have shown how population composition changes may offset or reinforce pure income distribution changes, and we have developed the analytical modifications required when the individual, not the household, is assumed to be the income-receiving unit.

Our empirical illustration shows that the dominance conditions are useful in practice. Although the criteria are demanding, they are not so strict as to rule out clearcut rankings altogether. The empirical results suggest that amongst non-elderly two-adult households in the U.K., poverty fell and social welfare rose between 1981 and 1986. Of course, regardless of whether the dominance criteria were actually satisfied, the checking of them provides detailed information about where the crucial changes in the distributions of income and needs are. For our sub-sample of U.K. households we have shown that income changes at the very bottom of the income distribution are particularly important for assessments. Moreover the trend towards smaller households between 1981 and 1986 had poverty-reducing and social welfare-improving influences.

Our illustration also shows that empirical applications introduce new complications, notably judgements about the level of detail at which comparisons should be undertaken and about statistical significance. These issues are of course common to all empirical income distribution comparisons, not only rankings for heterogeneous populations. They have received relatively little attention to date, and deserve more attention in future research.

What should one do if the dominance conditions are not satisfied? To get unambiguous distributional rankings in this case, stronger assumptions about the relationships between income and needs have to be introduced. Comparisons using equivalence scales will be necessary and, as recommended by Coulter *et al.* (1992*a*, 1992*b*), these should cover the full range of plausible income-needs relativities.[20]

Appendix: Proofs of Theorems 1 and 2

We first state and prove Abel's Lemma:

Abel's Lemma. If $v_1 \geq v_2 \geq \ldots \geq v_n \geq 0$, a sufficient condition for $\sum_i v_i w_i \geq 0$ is $\sum_{i \leq j} w_i \geq 0$ for each j. If $v_1 \leq v_2 \leq \ldots \leq v_n \leq 0$, the same condition is sufficient for $\sum_i v_i w_i \leq 0$.

[20] It is important to consider relativities throughout the full range, not just at the extremes, because the relationship between distributional indices and equivalence scale generosity is not monotonic.

Proof.

$$\sum_i v_i w_i = v_n \sum_{i\leq n} w_i + (v_{n-1} - v_n) \sum_{i\leq n-1} w_i + (v_{n-2} - v_{n-1}) \sum_{i<n-2} w_i$$
$$+ \ldots + (v_1 - v_2) w_1. \qquad \text{QED.}$$

Now, for *Theorem 1*, consider the difference between W_F and W_G using the definition in (1), and integrate by parts to obtain

$$W_F - W_G = \sum_{i=1}^{n} \int_0^A u_i'(x)[\theta_{iG} G_i(x) - \theta_{iF} F_i(x)]\, dx + \sum_{i=1}^{n} [\theta_{iF} - \theta_{iG}] u_i(A).$$

Integrating by parts again, and making the assumption that $u_1(A) = u_2(A) = \ldots u_n(A)$, $W_F - W_G$ reduces to

$$W_F - W_G = \int_0^A \sum_{i=1}^{n} [-u_i''(x)] S_i(x)\, dx + \sum_{i=1}^{n} u_i'(A) S_i(A)$$

where $S_i(x) = \int_0^x [\theta_{iG} G_i(y) - \theta_{iF} F_i(y)]\, dy$. (See Lambert, 1993, pp. 62–64, for an extensive discussion of the properties of the function $S(x)$ in the homogeneous population case.) By (3) and Abel's Lemma it is sufficient for $W_F \geq W_G$ that $\sum_{i\leq j} S_i(x) \geq 0$ for all x and all $j = 1, \ldots, n$. This is what is claimed in Theorem 1. QED.

For *Theorem 2*, consider the difference between P_F and P_G, and integrate by parts to obtain

$$P_F - P_G = \sum_{i=1}^{n} \int_0^{Z_i} p_i'(x)[\theta_{iG} G_i(x) - \theta_{iF} F_i(x)]\, dx$$
$$= \int_0^A \sum_{i=1}^{n} p_i'(x)[\theta_{iG} G_i(x) - \theta_{iF} F_i(x)]\, dx$$

since $p_i(x) = 0$ for $x \geq Z_i$, for all i. From (8) and Abel's Lemma, it is sufficient for $P_F \leq P_G$ that $\sum_{i\leq j} [\theta_{iG} G_i(x) - \theta_{iF} F_i(x)] \geq 0$ for all $x \leq Z_1$ $(= \max\{Z_1, \ldots, Z_j\})$, and all $j = 1, \ldots, n$. In fact, since the weights $p_i'(x)$ are zero for $x > Z_j$, it is enough that these inequalities hold for all $x \leq Z_j$, $j = 1, \ldots, n$, as stated in Theorem 2. QED.

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