



The Gender Earnings Rift Assessing Hourly Earnings Distributions of Males and Females using Structured Additive Distributional Regression

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Preliminary!

Abstract

This paper reconsiders the old issue of gender related discrimination with respect to labour earnings. Rather than employing a labour definition based on payment, we employ an activity based definition based on Margaret Reid's Third Party Criterion. Moreover, we assesses discrimination on the grounds of full conditional wage distributions using Structured Additive Distributional Regression instead of just their conditional means.

Examining earnings discrimination with respect to gender in Germany in 2013, we find that gender wage discrimination is greatly exacerbated if considered in such a framework, as women are faced not only with a lower expected pay, but also with a more unequal distribution as well as a higher workload of unpaid activities. Thus we find that pecuniary discrimination is not confined to a wage gap in the region of 21% as found using a conventional approach but rather constitutes an earnings rift between 46% and 61% depending on the degree of aversion of inequality considered.

JEL-Classification: C13, C21, J16, J31, J71

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1 Introduction

Analysing the discrepancies between the labour of men and women is arguably as old as economics itself. Clay tablets from a Mesopotamian city state yield accounts of factor inputs and grain outputs that consider “female labour days” hinting at an analytical discrepancy between the labour of men and women in an economic context as early as 2,200-2,100 B.C. (Nissen et al., 1993, p.54).

Since then economics has produced a vast literature on gender related questions. One of its most prominent branches has addressed the issue of pecuniary discrimination between the labour effort of men and women. Most of these analyses are focussed on the contemplation of wage discrimination caused by the “malfunctioning of the labour market” (Liu, 2015, p.2) in the sense that employers discriminate against women leading to a lower expected wage for women compared to equally qualified men. Therefore, most such wage gap analyses implicitly use a definition of work as those activities which are remunerated on the labour market.

Yet, this definition is problematic as it effectively imposes a dichotomy which includes some activities while excluding other activities depending on whether money was paid and not on the grounds of the nature of the activity. Given that many societies feature a division of labour that sees predominantly female labour discounted as “women’s work” that doesn’t need to be paid, this problem is particularly important for the assessment of gender discrimination. This paper thus proposes to use a broader work definition of work that is closer to the semantics of the origins of the word labour which implied toil and exertion.¹ Specifically, we categorise activities as work according to the Third Party Criterion proposed by the pioneering work of Margaret Reid (1934).

A second aspect which we address in this paper is that wage gap analyses are generally focussed on the difference in the expected outcome for men and women when controlling for some explanatory variables in a Mincer-type wage equation. While straight forward, such an approach has the drawback that it neglects the underlying conditional distributions. Yet as soon as the preferences are not indifferent to aspects of these distributions any assessment of the magnitude of discrimination should contemplate the distributional nature of earnings. This paper proposes structured additive distributional regression to estimate full conditional hourly earnings distributions. Using the conditional distributions, we are able to estimate welfare measures that account not only for differences in expected incomes but also for the differences in the degrees of inequality associated with the earnings distribution as in Sen (1976).

¹The etymology of the word labour connects it to the Middle English *labouren*, the Old French *laborer* and the Latin *laborare*. The word implies an activity inducing fatigue of the body or of the mind, either in doing or suffering. This stands in contrast to the Latin word *opera* that can also be translated as labour but which has a connotation towards work conceived as a creative process which we admire - and thus pay money for it to be done. It could thus be argued that conventional labour market analysis is focussed on an *operare* conception of work, while we base ours on a concept more akin to *laborare*.

The term we propose for the enhanced concept of gender discrimination incorporating an activity-based definition of work and a distribution-based earnings comparison is gender earnings rift. It should be pointed out at the outset of this paper, that the results presented are intended as a complement to and not a replacement for the conventional gender wage gap approach.

Thus, this paper contributes to the literature mainly in three ways: First, it introduces the measure of the gender earnings rift and contrasts it to the conventional concept of the gender wage gap. Following the suggestion of “broaden[ing] income measures to non-market activities” by Stiglitz et al. (2009, p.14), it thus broadens the central gender discrimination measure to include vital components of non-market activities considered laborious. Second, it proposes to use Structured Additive Distributional Regression to estimate full conditional earnings rate distributions in order to address the issue of potential differences in the earnings distribution beyond the mean. Third, it provides empirical results on the extent of the gender earnings rift in Germany in 2013. Using data from the German Socio-Economic Panel (SOEP), we find that pecuniary discrimination is not confined to a wage gap in the region of 21% as found using a conventional approach but rather an earnings rift between 46% and 61% depending on the degree of aversion of inequality considered. Additionally, we consider different sub-populations and find that the general pattern holds that discrimination is significantly greater for the gender earnings rift than for conventional gender wage gap measures.

The paper is structured as follows: In the following section, we portray the existent literature on gender discrimination and contrast the conventional concept of the gender wage gap with the concept of the gender earnings rift. In the subsequent section, we describe the data from the German Socio-Economic Panel that we used in this article for our analysis. In Section 4, we introduce structured additive distributional regression for estimating hourly earnings distributions conditioned on a set of covariates. The results are presented in Section 5. At last, Section 6 concludes.

2 The Conception of the Gender Earnings Rift

2.1 Existing literature

2.1.1 The gender wage gap concept

The conventional economic concept of wage discrimination between men and women is focussed on the differences in the expected wages offered (and eventually paid) by employers conditional on

a set of covariates. In a typical set-up, one would estimate the expected (log-)wage² given a set of covariates both for men and women in the following linear set-up (see Jenkins, 1994):

$$\log(Y_i) = \tilde{\mathbf{X}}_i' \boldsymbol{\beta}^M + \varepsilon_i^M \quad \forall i \in M, \quad (1)$$

$$\log(Y_i) = \tilde{\mathbf{X}}_i' \boldsymbol{\beta}^W + \varepsilon_i^W \quad \forall i \in W, \quad (2)$$

where M and W denote the set of individuals in the sample who are male and female respectively and Y_i denotes the wage paid to the i -th individual while $\tilde{\mathbf{X}}_i$ is the i -th row of the design matrix portraying the characteristics, \mathbf{X}_i . The coefficients $\boldsymbol{\beta}^M$ and $\boldsymbol{\beta}^W$ capture the rates of return to the various attributes captured in $\tilde{\mathbf{X}}_i$. Lastly, ε_i^M and ε_i^W denote the residual terms for men and women respectively.

In such a framework, wage differences between women and men can be divided up. Differences may arise from differences in the characteristics of men and women. As the considered characteristics are often thought to represent productivity potentials these differences are called “explained” differences in the literature (see Fortin et al., 2011). For example, we tend to include education as individuals with higher education levels are thought to pursue different (supposedly more difficult) tasks than less qualified individuals and are thus awarded higher wages. This kind of wage difference is consequently often considered non-discriminatory as they simply convey different levels of productivity of individuals. While it is generally acknowledged that the endowment of such characteristics may in itself be discriminatory (e.g. Neal and Johnson, 1996) and some approaches towards so called *detailed decomposition* have been made (e.g. Oaxaca and Ransom, 1999), the applied research on the gender wage gap has largely left aside this issue with the main thrust directed towards the following component of discrimination.

So called wage structure discrimination exists if and only if the coefficients $\boldsymbol{\beta}^M$ and $\boldsymbol{\beta}^W$ differ systematically inducing a different expected (log-)wage for a given set of covariates $\mathbf{X}_i = \mathbf{x}_i$. In lack of any observable difference between the men and women it is assumed that the difference must be considered discriminatory. In terms of the underlying economic model the source of such discrimination is generally associated with the demand side of the labour market, that is discrimination in the recruitment and payment of personnel which should be equally capable.

Lastly, differences in the wage structure of men and women may arise from differences in the residuals ε_i^M and ε_i^W . Some recent publications have gone beyond abstracting from the residual by focussing on the conditional expectation, $\mathbb{E}(\log(Y) | \mathbf{X} = \mathbf{x})$. For example, Christofides et al. (2013) apply quantile regression to assess wage differentials at different quantiles of the conditional wage

²Although the log-link allows for a nice interpretation in terms of elasticities, it also has some problems. For example, the nature of the log-link inhibits the inclusion of zero-wages. In addition, as Jenkins (1994) points out, everyday discussion are on the nature of wages and not log-wages.

distribution to construct counterfactual aggregate distributions. By contrast, van Kerm (2013) uses maximum likelihood to estimate conditional distributions to assess gender related conditional wage distribution differentials in Luxembourg. However, by and large the focus of applied research has been on the assessment of expected-outcome differences with the error term treated as residual matter which does not need to be focussed on. To date no measure of gender discrimination exists which explicitly contemplates distributional differences at a disaggregated level in the standard framework allowing for a multitude of (potentially continuous) variables.

2.1.2 The gender wage gap in Germany

The literature on the gender pay discrimination for Germany alone has become so large that we focus on a few recent selected studies. For an overview of the less recent literature see Hübler (2003) and Maier (2007).

Official publications by the German Statistical Office see the unadjusted gender wage gap largely unchanged from 2006 to 2013 at 22-23% with a significantly smaller gap in the East (Statistisches Bundesamt, 2014). The adjusted wage gap is not provided at a yearly basis but the most recent account from the year 2010 sees it at 7% (Statistisches Bundesamt, 2013).³

Arulampalam et al. (2007) inquire into the existence of sticky floors and glass ceilings in Germany and other European countries. They find that countries with a high work-life reconciliation policy and more family-friendly work policies generally feature lower wage gaps at the bottom of the wage distribution and wider wage gaps at the top. They also find that countries with a compressed wage distribution generally also feature show a greater gap at the lower wage spectrum and a smaller gap at the upper end. Concerning the impact of union coverage on sticky floors and glass ceilings they find a positive correlation which is insignificant though in both cases.

Hirsch et al. (2010) and Hirsch and Schnabel (2012) consider differences in the labour supply of men and women and find a systematically lower women's elasticity of supply for women. Due to this lower elasticity women are more prone to monopsony power and are thus paid lower wages. In another study that also follows a monopsonistic labour market model, Hirsch et al. (2013) find that the gender wage gap is wider in rural areas as more densely populated regional labour markets are more competitive and hence lower the scope for discrimination by employers.

Wolf et al. (2012) consider the gender wage gap in conjunction with nationality. They find that the gender pay gap is on average much greater than discrimination due to nationality. Furthermore they reflect the magnitude of both gender and nationality pay gaps and find that while the latter

³The adjusted wage gap mainly controls for differences in educational attainment, work experience, working hours as well as the occupational position and the occupational sector of employment between women and men. For a full account of the variables included see Statistisches Bundesamt (2010).

is smaller in business with a higher share of non-German employees the gender pay gap is even larger in enterprises with a higher share of female employees.

Ludsteck (2014) assesses the role of gender segregation on the gender wage gap. He finds that due to non-random sorting into jobs, establishment and occupation levels contributes around 8.2% to the gender wage gap.

Selezneva and van Kerm (2016) consider full gender related differences in conditional wage distributions. They find that women face wage distributions which not only have lower means but generally also feature higher levels of inequality as assessed by the Atkinson index. Applying the inequality-sensitive wage gap measures from van Kerm (2013), they find that the wage gap is exacerbated once inequality at the disaggregated level is considered. This paper will follow along the same lines of thought, proposing modifications in terms of the dependent variable of interest and the statistical technique employed.

2.2 Discrimination and the conception of the labour earnings rate

2.2.1 The conventional wage definition

One common thread running through the literature on the gender wage gap is an implicit definition of the labour earnings rate by the wage rate concept, i.e. the average rate of payment for time spent in a paid occupation. The wage rate Y_i paid to an individual i is normally defined as

$$Y_i = \frac{1}{|T_i|} \int_{t \in T_i} Y_{i,t} dt, \quad (3)$$

where $Y_{i,t}$ denotes the wage rate paid to the individual at time t . This wage rate is averaged over the timespan that we conceive to be working time, denoted by T_i . The cardinality of T_i is denoted by $|T_i|$. By this somewhat more complicated definition, we allow for the inclusion of various occupations which are potentially paid with different wage rates. Using standard data sources like the SOEP, Equation 3 simply means that we take the overall earnings for a given period's work, i.e. the monthly earnings, and divide this by the number of hours in that period, i.e. the monthly working hours which can be easily computed via the (actual or contractual) weekly working time.⁴

The implicit reasoning behind the conventional approach is that the gender wage gap analysis

⁴Some other studies consider the earnings and regress those on working hours and possibly some other attributes like employment status (e.g. Blau and Kahn, 1996) to compute a standardised monthly wage rate. Although this procedure produces slightly different wage rates as they cease to be proportional to working time once intercepts are non-zero, the employed implicit definition of working time also only encapsulates the time spent on labour in the conventional sense.

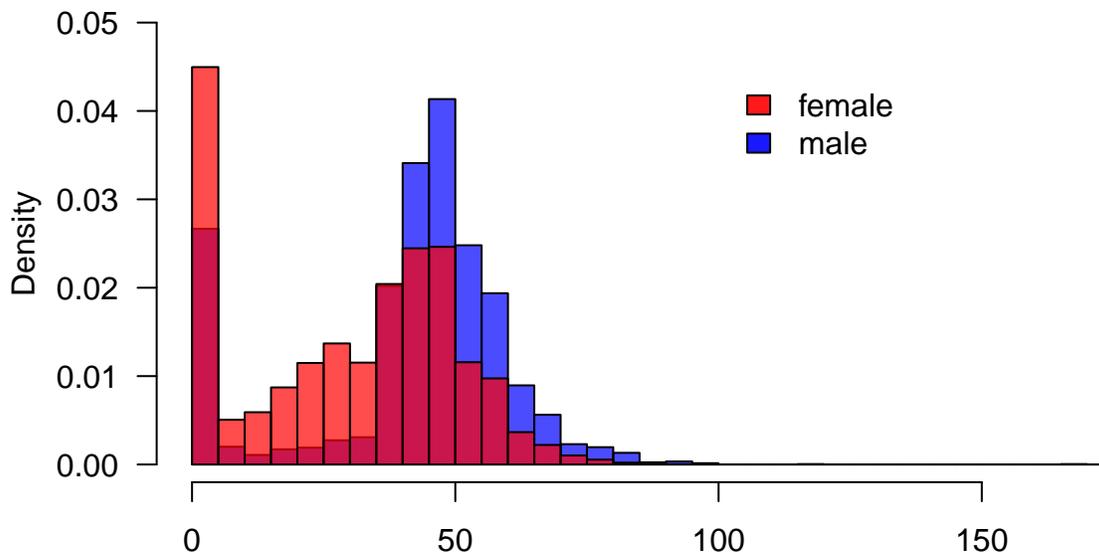


Figure 1: Histogram weekly working hours according to the conventional work definition.

concentrates on discrimination on the labour market in the sense that employers are liable to discriminatory practices, preferring men over women for a given job despite comparable qualifications. Thus the focus is on the demand side of the labour market and discriminatory distortions therein with discrimination thought to exist if and only if the offer of jobs/wages to an individual differ depending on the gender of that person. By contrast, the supply side of the labour market is supposedly largely down to individual choices and not societal circumstances. Individuals thus more or less freely choose whether to take up an occupation on the grounds of the offers at their disposal.

If we were to follow the definition and reasoning from above, the SOEP data discussed below would yield the two histograms of the weekly working hours displayed in Figure 1. Such a histogram connotes men as far more industries than women, with the fair gender more inclined to choose a life of leisure. Yet both in the literature and in the wider political debate there is a consensus that this discrepancy has less to do with women voluntarily choosing unemployment or underemployment, but rather that circumstantial factor inhibit or even prohibit a convergence of working hours. Arguably even more importantly, there is little outcry over this work-effort inequality as any casual observation of the female allocation of time portrays that they forfeit formal employment for other productive activities as Becker (1965) pointed out. The problem of the work depiction in Figure 1 is thus that it portrays a skewed picture of the work effort committed by men and women respectively. Indeed, Burda et al. (2013) show that people are generally aware of the fact that a solely market work based portrayal is not a fair account of the total labour effort committed by men and women, respectively.

One additional problem with the conventional perspective regards the treatment of those whose wage rate is not defined as the denominator becomes zero for those completely out of paid work. Much thought has been dedicated to the question of how to account for this problem with the seminal paper by Heckman (1979) the most prominent example. The underlying economic rationale of the now widely applied Heckman correction is that economic agents may choose in a non-random manner not to enter the sub-population of people with paid employment and thus a defined wage-rate. The sample of those in employment thus potentially suffers from sample selection biases which need to be corrected for. In case of the gender wage gap, it is assumed that predominantly women choose not to work (full-time) preferring the alternatives on disposal as they value the additional free time to pursue other unpaid activities more highly. By this correction, some individuals are included in the sample by means of an imputed counterfactual earnings rate based on the earnings of individuals with equivalent observable characteristics. However, for gender discrimination the conventional definition of work, the focus on the role of employers and the needed correction that it ensues is problematic. Numerous publications in the economic literature point out that the selection is not first and foremost based on truly free individual choices but that the choices themselves are circumstance-induced. As Manning (2011) points out, women usually report that they are constrained by their domestic commitments in the jobs they can accept. The overwhelmingly disproportionate allocation of domestic commitments has in turn been ascribed to social expectations by the economic literature (e.g. Fernandez et al., 2004; Fortin, 2005; Allmendinger, 2010).⁵ Hence, the literature indicates that women are disadvantaged by cultural institutions in their potential to receive payment for their work relative to men. Ideally, an account of discrimination should entail this disadvantage. Yet by nature of its conception the Heckman procedure corrects for such hypothetical differences using the counterfactual scenario of the women working at an imputed wage rather than the actual observation of the women not having any paid work at all. If this unemployment is due to discrimination, the neglect of demand-side discrimination thus is liable to provide biased estimates of the magnitude of discrimination on the labour market.

2.2.2 A new labour earnings rate definition

In contrast to the conventional approach of defining labour as the time spent “on the clock” of a paying employer, we will use an alternative definition which induces a conception of the earnings rate which differs from the conventional wage definition used. Specifically, we follow Margaret Reid’s Third Party Criterion, by which any activity should be considered work if the “activity is of such character that it might be delegated to a paid worker” (Reid, 1934, p.11). Thereby the

⁵See also Nussbaum (1999) for a more general discussion on women’s capabilities and their connection to the social circumstances.

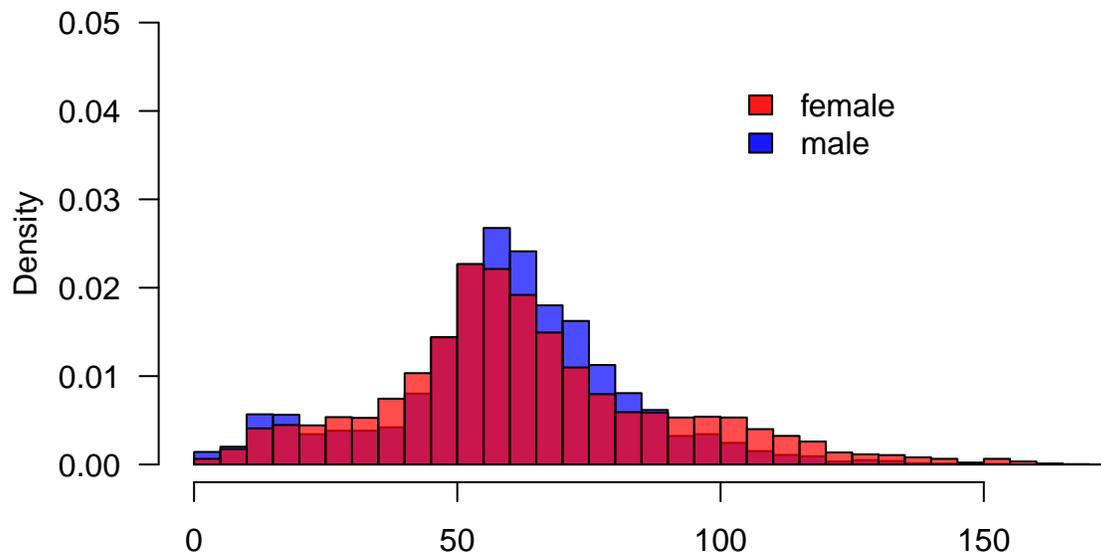


Figure 2: Histogram weekly working hours according to the Third Party Criterion.

following activities are additionally considered work even if they are not paid: errands, house work, child care, caring for adult persons as well as repair works - i.e. activities which could in principle be serviced by paid work.⁶

If we use such an activity based definition, the two unequal distributions of the workload displayed in Figure 1 quickly unravel. The distributions observed in Figure 2 not only display a much higher workload by both men and women on average but also show that the distributions are much more alike with women now actually carrying a slightly larger workload. The striking discrepancy between the two figures shows the major importance of unpaid labour and the major discrepancy in its distribution with women still doing the lion's share of it. If it is true that this division is largely down to cultural circumstances rather than free choice, the latter distribution appears to be a more appropriate account of the relation between men's and women's labour commitments.

For the analysis of the pecuniary rewards for this labour we thus propose to adapt the definition of the earnings rate accordingly. As variable of interest we thus consider the average labour earnings rate, i.e.

$$Y_i = \frac{1}{|\tilde{T}_i|} \int_{t \in \tilde{T}_i} Y_{i,t} dt, \quad (4)$$

which is equivalent to the wage rate in Equation 3 with the exception that we now consider not only the time spent by individual i in paid labour but in activities considered labour by the Third Party Definition, denoted by \tilde{T}_i . In contrast to the conventional wage-concept we conceive numerous activities disregarded by the former as labour activities remunerated with an earnings

⁶Our labour definition is thus similar to the total work concept from Burda et al. (2013).

rate, $Y_{i,t}$. In want of a better approach, we simply use an earnings rate of zero. The effect of this is twofold. As can be seen in Figure 2, this increases the denominator for those individuals who spend time on unpaid laborious activities. This lowers their earnings rate both in absolute terms and relative to those who devote less time to unpaid laborious activities, c.p. Secondly, this means that we do not constrain our analysis to those who are in paid employment but to the whole population who pursues any of the activities mentioned above for some time, which is practically the whole population.

Naturally, this concept has some inherent problems as well. Just as the dichotomic conventional definition of work for leaving out important work-like aspects can be criticised, the proposed concept can be criticised for leaving in activities which ought to be considered leisure. Following the definition put forward by Margaret Reid, it can be criticised that at least some of the activities of child care can hardly be seen as work as they cannot be fully outsourced to another person. After all, parental care is different to the care of another person. To address this problem it is naturally feasible to only account a given percentage of such work which is partly difficult to outsource as working time (e.g. 50%). However, such percentages are likely to be arbitrary. Following dialectical reasoning, we will consider the extreme case of considering all child care as work which is diametrical to the conventional reasoning of not accounting for child care at all. Additionally, it may be argued along the lines of Becker (1965) that the unpaid activities are productive consumption and first and foremost yield utility rather than disutility. To this, one could respond that while this is undoubtedly true, it is also true that many paid activities are intrinsically rewarding (see England et al., 2002) - at the very least for some individuals fortunate enough to be able to pursue academia for a living. In addition, it has been argued that a lot of the unpaid work goes towards things which can be considered public goods, like children or cleanliness, thus also benefiting the man (see Ponthieux and Meurs, 2015).

One may also argue along the lines of the unitary model of household behaviour, whereby one assumes that the unpaid work is rewarded indirectly as household members cooperate and divide the labour on a consensual basis (see Samuelson, 1956). Thus, the income is shared altruistically within the household, so that the potential utilities via consumptive capacity derived from the disutilities dispensed in paid work are shared equally within the household. However, this notion not only stands in contrast to the rationale of the principle of individualism underlying standard economic theory. The scarce empirical literature on the matter also shows that intra-household allocation is far from equitable but rather governed by the power structures based on the paid work's income (see Bourguignon et al., 1993). In any case, utility and disutility, transient and complex as they are in their nature, can hardly be operationalised for an empirical assessment of the gender earnings rift.

Lastly, one may criticise the valuation of unpaid work with zero earnings. In the literature two

other methods are dominant: The opportunity cost method would estimate what the person doing the housework would have received if the time were devoted to remunerated labour. The second concept of replacement cost would value the work at the estimated cost that a household would have to pay to obtain the same service on the market. In practice, one either uses the “specialist” concept imputing the hypothetical wage for the unpaid labour using a professional’s wage for the task at hand. Alternatively, one may use a ”generalist” concept whereby the wage of a generalist worker is used to impute the payment for the unpaid labour (see Eurostat, 2003). Despite the obvious advantages over the blunt approach of simply using a zero wage, we choose to do the latter. The reason for this is twofold.

First, as pointed out above, the opportunity cost approach may neglect the fact that circumstances inhibit or prohibit people in general and women in particular from realising the supposed opportunities on offer for them. Job offers may not be available by the hour (but only from a certain number of hours per week upwards) and thus people required to do unpaid labour cannot exchange the former for the latter. In addition, one may criticise this approach as it values activities differently depending on who pursues them. Lastly, the estimation process is full of technical difficulties most notably the difficulty of estimating opportunity costs for those who do not work at all. Landefeld and McCulla (2000) point to several problems with the replacement cost method. For example, the first approach cannot account for the fact that in the household several tasks can and are executed simultaneously. The second approach neglects the fact that several tasks cannot be executed by a generalist worker.

The second reason is more fundamental. All the imputation based approaches use assumably observed market prices for labour and extrapolate these to supposedly comparable activities which are not remunerated. However, as we discuss below, labour market remunerations show considerable variation even at the highly disaggregated level. The valuation of activities must thus be seen as stochastic, making extrapolation problematic. This problem is even enhanced by the possibility that valuation of unpaid activities is inherently different to that of paid activities. As the humorist Evan Esar pointedly remarked: “Housework is what a woman does that nobody notices unless she hasn’t done it.” (Esar, 1968, p.398). If it is true that much of the valuation of an activity is only realised in the explicit exchange for money and is undervalued, or even not valued at all, the extrapolations proposed above are not adequate here.

2.3 Wages and stochastics at the disaggregated level

It is evident that one does not observe one single equilibrium wage for workers with equivalent observable characteristics but rather a labour market with major deviations from the expected wage. This is equally true for the labour earnings rate introduced above as both monthly earnings

and work hours (paid and unpaid) vary substantially across people with equivalent observable characteristics. The reason for these deviations are manifold. They may be partly ascribed to innate abilities of the individual. They may in part be down to voluntary choices of the individual and in part to the societal circumstances facing the individual. In addition, one would probably need to agree with “the fact that luck played a role in the outcome” (Kahneman, 2011, pp.178-179). In light of the convoluted and complex nature and given the bounded ability of human beings (c.f. Selten and Gigerenzer, 2002) to disentangle and understand these deviations the knowledge of the true mechanism must be considered unknown. Thus, we believe it adequate to perceive that the individuals implicitly conduct an “axiomatic reduction from the notion of unknown to the notion of random” ascribing the deviations from the expected wage as chance.

However, this does not mean that women and men are indifferent to the nature of the stochastic variations. A broad empirical literature exists on how concerned people are with distributional aspects beyond the mean (see among others Ferrer-i Carbonell, 2005; Carlsson et al., 2005). For the assessment of the differences in the labour earnings rate we believe that the additional dimensions should be incorporated. Following the literature on Social Welfare (e.g. Sen, 1976; Yaari, 1988; Duclos et al., 2003), we use the traditional Gini social evaluation function, as well as additional social welfare functions analogously derived from measures of the generalised Gini coefficient⁷ (see Yitzhaki, 1979; Donaldson and Weymark, 1980) applied to the conditional labour earnings rate distribution. Therefore, we define an Equally Distributed Equivalent Labour Earnings Rate (EDELER) for males and females with equivalent observable characteristics as

$$\mathcal{W}_{\mathbf{x}}(\rho) = \int_0^{\infty} k(\rho, y) y dF_{\mathbf{x}}(y), \quad (5)$$

where $\mathcal{W}_{\mathbf{x}}(\rho)$ denotes the EDELER with $k(\rho, y)$ providing an ethical weight on an individual's earning which depends on the individual's rank in the group and on the parameter for inequality aversion ρ (see Yitzhaki, 1979). Using the generalised Gini coefficient this can also be expressed in the following straight forward form:

$$\mathcal{W}_{\mathbf{x}}(\rho) = \mu_{\mathbf{x}}(1 - G_{\mathbf{x}}(\rho)), \quad (6)$$

where $\mu_{\mathbf{x}}$ denotes the expected income for the conditional earnings distribution, while $G_{\mathbf{x}}(\rho)$ denotes its generalised Gini coefficient with inequality aversion parameter ρ .

As is standard in the literature, we consider the ratio between the variable of interest for women and men as a measure of the degree of discrimination. For a given level of inequality aversion and a given subpopulation selected by covariate combination $\mathbf{X} = \mathbf{x}$, we thus consider the ratio

⁷In the literature this is also known as the single-parameter Gini (S-Gini) or the extended Gini (E-Gini) (see Yitzhaki and Schlechtman, 2005).

between the EDELER for women and men, i.e.

$$\Delta_{\mathbf{x}} = \mathcal{W}_{\mathbf{x}}^W / \mathcal{W}_{\mathbf{x}}^M, \quad (7)$$

where $\mathcal{W}_{\mathbf{x}}^W$ is the EDELER for the women in the subpopulation while $\mathcal{W}_{\mathbf{x}}^M$ denotes that for the men. Hence, we represent the discrimination in the well known form of cents a women earns for every Euro that a comparable man gets but considering EDELERs instead of expected earnings.

The reasoning behind this approach is that individual's contemplate not only the absolute value of their earnings rate but that they assess this earnings rate against the backdrop of the observed earnings distribution in a reference group. For sake of simplicity, we simply assume that this reference group is all individuals of the same gender with the same covariate combination as the individual. For each group defined by covariate combination $\mathbf{X} = \mathbf{x}$, we base our assessment on the concept of rank order weighting proposed by Sen (1973). This concept stands in contrast to the 'optimal distribution' concept originally put forward by Samuelson whereby "the ethical worth of each person's marginal dollar" is kept equal (Samuelson, 1947, p.21) which may be used to justify the conventional approach of considering only the expected wage rate.

To give just one motivation of why this welfare approach is useful to the assessment of gender discrimination, let us consider the proposition put forward by Kathleen Gerson that women can either be a career women or a housewife (see Akerlof and Kranton, 2000, p.725). While taking this binary choice at face value is possibly somewhat exaggerated, it is nonetheless an undisputed fact that women may not be able to find the desired middle ground. It is thus feasible that women are likely to be in a position to feel unnerved by the discrepancy between their own occupational opportunities and the related earnings rate and the observed expected earnings rate constructed by averaging over both career women and housewives in their reference group. By contrast, men are unlikely to suffer from comparable life-career-related conundrums in a similar manner and more likely to find themselves close to the expected earnings rate. As a result male earning distributions can be expected to center more closely around their mean, while for women we would expect greater deviations from the mean. One effect of this distributional difference are phenomenons like the "pauperisation of motherhood" whereby women with children are faced with extremely low earning levels (see Folbre, 2006). To leave aside this structural difference in the level of inequality observed in the earning distributions in an assessment of the gender wage gap means to discard the aspect of societal discrimination described by Gerson facing many women.

2.4 Towards an alternative perspective on discrimination

To sum up, it must be conceded “it is painfully obvious that statistics on income [...] remain based on the unitary version of the household” (Ponthieux and Meurs, 2015, p. 999). This is particularly obvious in the blatant neglect of unpaid laborious activities in the assessment of pecuniary gender discrimination. To counter this and provide at least some empirical evidence, we pursue the following strategy. We simply assess empirically what is there to observe: the pecuniary payment awarded in a society to an individual for the spent labour, as defined by the Third Party Criterion. Whether this payment is justified or not, whether it is flanked by amenities (enjoyable nature of work, company car, etc.), whether it is ensued by transfers (from the family members or someone else) or whether it has a high esteem in society is beyond the scope of the gender earnings rift assessment - just as it is beyond the scope of the wage gap assessment.

In addition, we follow the remark of Dolton and Makepeace (1985, p.391) that “[in] principle, the amount of sex discrimination should be deduced from a comparison of the distribution of earnings”, by considering full earning rate distributions. As is standard, we will condition on some variables representing individual choices and circumstances independent of gender discrimination. With respect to variables which we consider to be down to or influenced by discriminatory circumstances, we will take the marginal perspective as we aim for a comprehensive measure on the extent of discrimination. For a more elaborate discussion on the reasoning underlying this choice see Section A.2 in the appendix. Concerning the sampling universe, we consider not only those in paid employment but also those out of paid employment.

Before we go on to putting such analysis to practice, it should be re-emphasised that our approach rests on some far reaching assumptions pointed out above. Yet, this is obviously also the case for the traditional gender wage gap concept. Therefore, we conceive the ensuing analysis as the dialectic complement to the wage gap concept - far from ideal but geared to eventually derive an improved synthesis.

3 The Data

In order to analyse nature and magnitude of distributional differences between male and female wages in Germany, we will use the German Socio-Economic Panel (SOEP) database (see SOEP, 2014) as our primary source of data. In addition, we use the German *Mikrozensus* (see Section A.1.3 in the appendix).

From the SOEP, we include persons aged between 21 and 60 years of age. In contrast to most other studies (e.g. Dustmann and Schönberg, 2009; van Kerm, 2013; Card et al., 2013), we explicitly

include both people who are not in (paid) work and those who are. For the latter, we include civil servants and the self-employed next to employees. Thus our sampling universe is the whole population in the 40 year age range during which the majority of the population is actively involved on the labour market. This yields 4,198 male observations and 4,984 females observations for 2013.

As dependent variable of interest, we consider the gross hourly wage, i.e. the gross earnings divided by the hours spent working, in the comprehensive sense discussed above. Following van Kerm (2013), it is computed by taking the gross monthly earning in the current job (including payments for overtime) and dividing it by the number of hours worked per week multiplied by 4.32 (for week per months). As pointed out above, we include not only actual working hours but also hours spent on making errands, house work, child care, caring for adult persons as well as repair works for the latter - i.e. activities which could in principle and are often, at least partially, serviced by paid work and are thus considered work according to the Third Party Criterion. In the appendix, we also provide gross hourly wages when only considering paid work as working time.

Concerning our explanatory variables, we consider the age and the education as is standard in a Mincer type wage equation. As noted by Morduch and Sicular (2002) the coarse discretisation of continuous variables such as age which is commonly applied in the literature (e.g. Dustmann and Schönberg, 2009; Selezneva and van Kerm, 2016) can pose problems. Hence, we consider age in a continuous manner. With respect to education, we follow van Kerm (2013) and consider 4 levels of education which are constructed as follows: the first level entails all persons who only have general elementary education or less (i.e. those who fall under the ISCED97 categories 0-2 according to the SOEP). The second level incorporates the persons with completed secondary education, (ISCED level 3). The third level incorporates those with vocational training and Abitur as well as those with a higher vocational qualification (ISCED levels 4 and 5). The last level entails all those with completed higher education (ISCED level 6).

In addition, we consider the nationality of the person as a binary variable differentiating only whether the person is German or not. Concerning the household characteristics of the persons, we consider a binary variable on whether the person has children who are not yet economically independent in the sense that the household still receives child support⁸ for them. Lastly, we consider the federal state of residence.

We do not include aspects like job characteristics or the industry of employment, as these are typically strongly related to gender (see among others Folbre and Nelson, 2000; Charles and Grusky, 2004; Lillemeier, 2014). As Richard Anker (1998, p.3) points out: “Occupational segregation by sex is extensive and pervasive and is one of the most important and enduring aspects of labor markets around the world.” Conditioning on industry of employment or type of occupation would

⁸In Germany parents got between 184€ and 215€ per month for each child in 2013.

hence capture much of the variation between males and females which we consider discriminatory, as was already pointed out by Oaxaca (1973).⁹ Equally, we do not condition on actual employment experience as the standard literature generally does if the data is available. Analogue to the argument by Neal and Johnson (1996) for racial discrimination, we argue that there is discrimination inherent in women’s lower levels of employment experience. Women more frequently interrupt their careers than men for vital societal activities like child care. These interruptions are likely to see many women gulped by the snakes or at least unable to advance the ladders in the game-like mechanism of the labour market put forward by Manning (2003) thus leading to earning differentials. Therefore, we take a marginal perspective rather than a conditional perspective with respect to these latter variables. For the same reason, we also do not include individual specific effects.

On a more general level, it is clear that the distinction between circumstance and choice constitutes a major obstacle for the assessment of the degree of discrimination. Given the huge complexity of the nature of human choices and the differences in circumstances the individuals are faced with (as well as their interdependencies), any attempt to truly disentangle and identify choice-induced and circumstance-induced differences in earnings must be considered futile. In light of this, we will therefore follow dialectic reasoning and take the opposite view, i.e. that most choices taken by individuals are preconditioned on the circumstances facing an individual unless there is evidence to suppose otherwise.

Overall, we thus consider one continuous variable, two binary variables, one categorical variables with four levels and one with 16 levels. Even if we use a rather coarse discretisation of the continuous variable age into four ten-year periods this would yield 2048 combinations if we would fully interact these variables. Given the samples sizes of the SOEP, it becomes quickly apparent that regularisation is required to provide stable estimates. This regularisation is provided by the estimation strategy, which we turn to next.

4 Methodology

For the estimation we employ the fundamental idea underlying all regression techniques that individuals close to one another in the covariate space should have a similar earnings distribution. In other words, if the covariate vector \mathbf{x}_1 of one individual does not deviate much from the covariate vector of a second individual \mathbf{x}_2 , then the two individuals’ earnings distributions $\mathcal{D}_{\mathbf{x}_1}$ and $\mathcal{D}_{\mathbf{x}_2}$

⁹Naturally, a relationship of these covariates with characteristics we consider, like education, may well distort our results. Yet, as Blau and Kahn (2016) point out, at least education and experience seem to work primarily within the industry of employment and type of occupation so that exclusion of the latter hardly distorts the results of the former.

should adhere to a similar form. In order to make an estimation feasible, we use the well established idea of imposing an additive structure onto the covariate effects.

In order to allow for potential differences in the wage impact of the covariates on the wage distributions for males and females, we will run two separate regressions for males and females, with each regression specified as follows.

4.1 A generic representation for the predictors

As a general framework we consider structured additive distributional regression (SADR) (Klein et al., 2015) whereby the distribution of wages, $\mathcal{D}_{\mathbf{x}}$, is conditioned on a set of covariates, \mathbf{x} . The conditional distribution is assumed to follow a parametric form. Thus, the conditional distribution can be written in the form $\mathcal{D}(\theta_1(\mathbf{x}), \dots, \theta_K(\mathbf{x}))$, where $\theta_k(\mathbf{x})$ is the k -th parameter in the parametric distribution and is conditioned on the covariate combination of the specific stratum. For notational brevity we will drop the suffix (\mathbf{x}) in the following. Additionally, we define $\boldsymbol{\theta} = \theta_1, \dots, \theta_K$.

In this paper, we use the following generic representation for every parameter of the distribution: Each parameter θ_k can be linked to a structured additive predictor η^{θ_k} via a suitably specified link function, g_k , mapping the predictor into the parameter space such that $\theta_k = g_k^{-1}(\eta^{\theta_k})$. The predictor η^{θ_k} can be specified in the following form:

$$\eta^{\theta_k} = \beta_0^{\theta_k} + f_1^{\theta_k}(\mathbf{x}) + \dots + f_{J_k}^{\theta_k}(\mathbf{x}), \quad (8)$$

where $\beta_0^{\theta_k}$ represents the intercept of the predictor and the functions $f_j^{\theta_k}(\mathbf{x})$, $j = 1, \dots, J_k$, can capture both linear and non-linear effects of single or multiple elements of the covariate vector \mathbf{x} . The latter is done by means of representing the function by a suitable linear combination of basis functions which are generally penalised such that the non-parametric estimate adheres to the required smoothness (see Fahrmeir et al., 2013).

In our application we will use the following regression set-up for all the predictors:

$$\begin{aligned} \eta^{\theta_k} = & \beta_0^{\theta_k} + \beta_1^{\theta_k} kids + \beta_2^{\theta_k} nat + \beta_3^{\theta_k} educ_2 + \beta_4^{\theta_k} educ_3 + \beta_5^{\theta_k} educ_4 \\ & + f_1^{\theta_k}(age) + heduc \cdot f_2^{\theta_k}(age) + f_{spat}^{\theta_k}(region), \end{aligned} \quad (9)$$

with *kids* and *nat* binary, effect-coded variables set to unity if the person has at least one child and has German nationality, respectively. We use education-specific intercepts by three effect-coded variables $educ_e$, where the subscript e denotes the education level, with the first education level taken as the base. As mentioned above *age* is a continuous variable expressed in years. In the

standard Mincer wage equation, age, as a proxy for potential experience, is incorporated by a polynomial of degree two (see Lemieux, 2006). While this linearity in parameters has proven to perform well for the expected (log-)income, a more flexible effect of age seemed desirable for the possibly non-linear nature of the effect of age on the various parameters of the whole conditional wage distribution.¹⁰ Hence, we use a flexible smooth function, based on P-splines (see Eilers and Marx, 1996; Brezger and Lang, 2006), to model the effect of age. Thereby, f generally consists of a number of basis functions allowing for a high degree of flexibility and a penalisation term ensuring the desired degree of smoothness adhered to by the function. To account for different developments over the life-span depending whether the person has enjoyed higher education and interaction with the effect-coded variable *heduc* which is unity if the person has a degree in higher education, i.e. if the ISCED level is 6 according to the SOEP.

In order to capture differences between the economic dynamism of different federal states in Germany, we include a hierarchical spatial effect, such that:

$$f_{spat}^{\theta_k}(region) = \beta_6 east + \gamma_{region}, \quad (10)$$

where *east* is an effect-coded binary variable that is unity if the federal state is situated in the East, thus capturing the difference between the former Federal Republic of Germany and the German Democratic Republic. The state-specific random effect is denoted γ_{region} and accounts for variations across individual states. For the random effect we impose a Gaussian prior centred around zero. By incorporating this region specific term we capture the effect of contemporary region specific effects such that past-unemployment does not induce an effect via the contemporary economic background facing workers.

In our application, we thus condition on a handful of aspects which we believe to be identifiable and independent from gender-related circumstances. This entails two variables which we regard as choices that are by and large independent of gender discrimination in contemporary Germany: the individual's choice of whether to have children and which federal state to live in. In addition, we condition on two variables which we consider as circumstances that are equally independent of gender discrimination: whether the individual has German nationality as well as the individual's age. Lastly, we consider a hybrid of the two - education. We conceive education free of discrimination in Germany such that dependent on his/her aptitude (which we consider a non-discriminatory circumstance) the individual is free to choose the education irrespective of the gender.

Naturally, this leaves out many important variables which, be they based on choices or circumstances, will affect the individual's wages independently of the gender. Next to the issues of data

¹⁰Using the DIC as a model selection criterion we show the non-linear approach to be superior over the linear approach - see Section Section A.11 in the appendix.

availability and model stability, we do not include further variables on the grounds that the marginal perspective which we take with respect to the left-out variables emulates the individual’s perspective, who we conceive to see the world through the eyes of a person with bounded cognitive abilities. To such a person many influences are likely to go unnoticed and much of the variation in wages which are explained by a score of different factors will seem as random. The distributions we thus model are conditioned on variables such that they aim to represent the perceived wage distributions for men and women with limited information at their disposal.

4.2 Parameter estimation

Estimation is performed in a Bayesian framework using Markov Chain Monte Carlo (MCMC) techniques implemented in the software BayesX (Belitz et al., 2015). See Klein et al. (2014) for details on the estimation procedure. Concerning the coefficients’ prior distributions, we employ non-informative flat priors for the linear effects and multivariate normal distributions for the basis function coefficients of the smooth effects which are in turn scaled by inverse gamma hyperpriors aimed at providing a data-driven degree of smoothness. Concerning the MCMC algorithm, we use 200,000 MCMC realisations for burn-in and thin out the following 800,000 MCMC realisations by a factor of 800. Thus, we use 1,000 MCMC realisations for each predictor η^{θ_k} to construct the posterior of the predictor, which is then transformed by the corresponding link function g_k into the posterior distribution of the parameter of interest. These distributions are proper under mild conditions (see Klein et al., 2015). For our inferential purposes we use the median from the posterior as point estimate for the parameters in order to provide estimates for the resultant full conditional distribution for the desired covariate combination. Using the MCMC realisations, we also provide point-wise credible intervals giving a notion of uncertainty attached to the estimators.

For the estimation of the conditional hourly earnings distribution, we will use a Type II Dagum distribution, which entails one parameter for the point mass at zero and follows a Type I Dagum distribution for positive values.

4.2.1 Estimating the probability of zero wages

In order to incorporate zero-wages, we follow a two-stage strategy. In the first stage, we estimate the probability mass of zero-wage and positive wages respectively. This is done by simple logistic regression. In a subsequent estimation step we estimate the conditional distribution of wages greater than zero.

First, we thus estimate the probability that a person is without any remunerated employment, π_0 , either because of lack of work or the unpaid nature of the work they do. In both cases the wage

rate is considered zero, yielding the probability of receiving a zero-wage $\pi_0 = P(y = 0 \mid \mathbf{X} = \mathbf{x})$:

$$g_1(\pi_0) = \eta^{\pi_0}, \quad (11)$$

where g_1 is a logit-link and η^{π_0} is the predictor as specified in Equation (9) for a given covariate combination.

4.2.2 Estimating the density of positive earnings

As pointed out above, we use the Type I Dagum distribution which has a track record of performing very well for modelling positive earnings at the aggregate level (e.g. Kleiber and Kotz, 2003; Chotikapanich, 2008).¹¹ A natural alternative which is the work-horse distribution in the literature is the log-normal distribution (see Flabbi, 2010), yet preliminary studies have found this distribution to have problems in modelling conditional earning distributions (Sohn et al., 2015). Although much more research must be done on the nature of conditional wage distributions, we consider the Dagum distribution to be a good starting point as it allows for great flexibility in the modelling of wage distributions.

The density of the Type I Dagum distribution is given by

$$p_+(y \mid a, b, c) = \frac{acy^{ac-1}}{b^{ac}(1 + (y/b)^a)^{p+1}}, \quad a \in \mathbb{R}_{>0}, \quad b \in \mathbb{R}_{>0}, \quad c \in \mathbb{R}_{>0}. \quad (12)$$

The estimation of the three parameters a , b and c is done using the following generic predictor set-up as discussed above:

$$g_2(a) = \eta^a, \quad (13)$$

$$g_3(b) = \eta^b, \quad (14)$$

$$g_4(c) = \eta^c, \quad (15)$$

where all three link functions are log-link functions ensuring a positive support for the parameters and inducing a multiplicative connection between the covariates.

¹¹This distribution is very similar to the Singh-Maddala distribution which has been used by Biewen and Jenkins (2005) for conditional earning distributions and by van Kerm (2013) for modelling wage rates. Yet, Kleiber and Kotz (2003) remark that the Dagum distribution generally performs slightly better than the Singh-Maddala. It should be noted that other more complex distributions, like the Generalised Beta of Second Kind, have been suggested in the literature for aggregate income distributions which outperform the Dagum distribution. However, these more complex distributions have so far proven to be too complex for stable estimation. A comparative study on the performance of the fit of these various parametric alternatives would be needed though to provide more profound assessment on this issue.

Overall, the estimation procedure gives us four parameters to estimate over the covariates space. Using this parametrisation, the density of the conditional wage distribution can be expressed as a mixture of a point-mass at zero and a continuous distribution thereafter:

$$p(y \mid \pi_0, a, b, c) = \pi_0 \mathbb{1}_{\{y=0\}} + (1 - \pi_0)p_+(y \mid a, b, c), \quad (16)$$

where p denotes the probability mass or probability density for a given wage y . For the point mass of zero wages $\mathbb{1}_{\{y=0\}}$ denotes an indicator function which is unity for a wage of zero and thus gives a probability mass of π_0 . For earnings greater than zero, we obtain the density as specified by the Type I Dagum distribution.

It should be noted that currently a lot of work is being done on various other statistical approaches allowing for the estimation of conditional distributions. For a discussion of other estimation strategies, see the Section A.4 in the appendix.

5 Results

In this section we consider the differences between wages for males and females in a distributional perspective as discussed in Section 2.

5.1 Interpreting the distributional effects at the disaggregated level

As is well-known from the literature on generalised linear models, the use of link functions in Equations (11)-(15) implies that the impact of explanatory variables varies across the covariate space (see among others Nelder and Wedderburn, 1972). Following Fox (1987) we employ effect displays for three different covariate combinations - see Figure 3. Subsequently we analyse some distribution measures for these distributions.

5.1.1 Visual analysis of conditional wage distributions

First, we display the estimated wage distribution for men (D_{1M}) and women (D_{1W}) with 30 years of age, no formal education beyond primary school, no children and no German citizenship who live in Mecklenburg-Western Pomerania in Eastern Germany, i.e. persons who we would typically expect to be economically disadvantaged. As can be seen both for men and women there is a considerable proportion without employment (45% and 56% for men and women respectively). Considering only those in employment, we can observe a distribution for men which interestingly

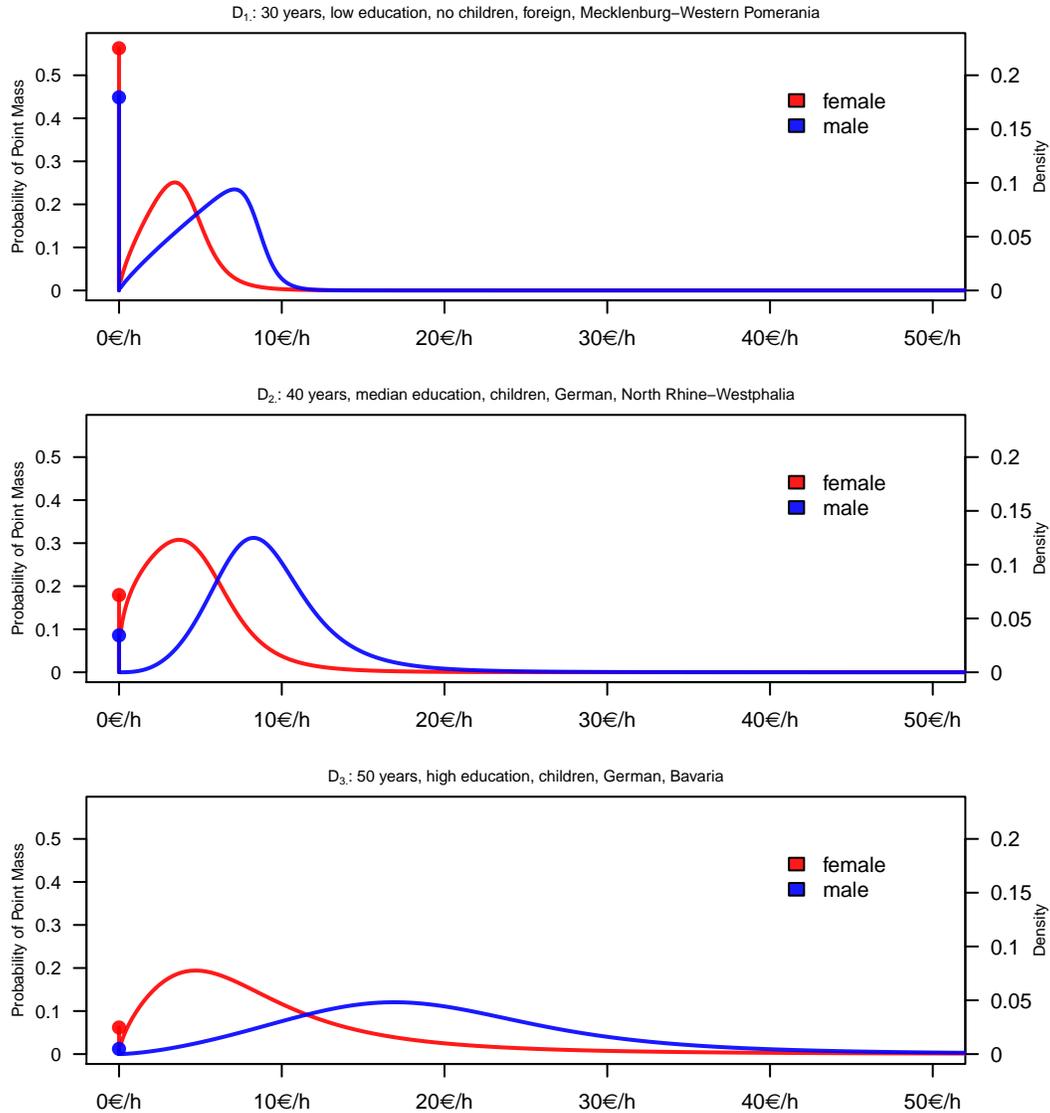


Figure 3: Conditional wage distribution estimates of males (blue) and females (red).

does not adhere to the standard positive skew observed in most income related distributions but is negatively skewed. As we would expect we also see that men’s wages are higher and slightly more dispersed than those of women.

In the centre of the figure, we display the estimated wage distribution for men (D_{2M}) and women (D_{2W}) who are forty years of age, have a completed education yielding an ISCED classification 3 or 4, have at least one child, have German citizenship and live in the populous state North Rhine-Westphalia in the west of Germany, i.e. persons who can be considered the "average Joe/Jane". As we can see the wage distributions differ substantially between men and women, not only in their first two moments but also in the shape of the distribution. As expected, women are much more likely to be receiving no earnings at all than men. If they are in employment their earnings

rate distribution is also shifted to the left of the men’s distribution, with the female distribution also portraying a higher skewness, implying that women are much more likely to find themselves below the expectation of their distribution.

The third set of distributions displays the wages of those who are generally associated with the upper strata of society - 50-year-olds, with higher education, children and living in Bavaria located in the affluent south of Germany (D_{3M} and D_{3W}). It shows a much smaller probability mass for those without employment and a much wider distribution for those in employment. Again women generally earn a lower wage than men, with the shape differences similar to the one above, albeit a much higher expectation and dispersion both for men and women.

One of the problems with the direct interpretation of income distributions is that they are naturally very complex and cannot be as easily grasped as scalar distribution measures. In the following, we will thus consider a handful of measures which may facilitate the interpretation of the distributional differences between men and women.

5.1.2 Distributional measures of conditional wage distributions

	D_{1M}	D_{1W}	D_{2M}	D_{2W}	D_{3M}	D_{3W}
μ	3.17[2.52;3.91]	1.53[1.11;2.10]	8.45[7.55;9.41]	3.72[3.25;4.17]	20.13[17.69;22.97]	9.92[8.28;12.55]
G($\rho=2$)	0.22[0.16;0.30]	0.29[0.23;0.36]	0.21[0.18;0.25]	0.34[0.31;0.38]	0.27[0.23; 0.32]	0.45[0.39; 0.52]
G($\rho=3$)	0.79[0.73;0.83]	0.86[0.81;0.88]	0.42[0.37;0.46]	0.66[0.60;0.71]	0.40[0.34; 0.46]	0.64[0.58; 0.70]
G($\rho=4$)	0.84[0.82;0.86]	0.84[0.80;0.86]	0.51[0.46;0.56]	0.76[0.71;0.81]	0.47[0.40; 0.53]	0.72[0.66; 0.77]
$\mathcal{W}(\rho=2)$	2.47[1.87;3.12]	1.08[0.78;1.53]	6.68[5.93;7.49]	2.43[2.09;2.82]	14.66[12.65;16.90]	5.40[4.57; 6.45]
$\mathcal{W}(\rho=3)$	0.68[0.45;1.00]	0.22[0.14;0.37]	4.94[4.28;5.66]	1.27[0.97;1.64]	12.12[10.21;14.18]	3.59[2.91; 4.30]
$\mathcal{W}(\rho=4)$	0.51[0.38;0.70]	0.25[0.20;0.32]	4.17[3.51;4.88]	0.88[0.63;1.19]	10.71[8.85;12.68]	2.78[2.20; 3.43]

Table 1: Some distribution measures for 3 conditional wage distributions

In the first row of Table 1, we display the expected values (μ) for the six underlying wage distributions under consideration. This would be the measure of interest if there were no aversion against inequality, i.e. $\mathcal{W}(\rho=0)$, and which is used in conventional analysis. In the squared brackets we display the 95% credible intervals. Little surprisingly, we can observe that for all three groups the expected wage is lower for females. The literature is full of such analyses and generally arrives at the same result - that there is wage discrimination between women and men. In terms of magnitude, the results in the literature generally find a smaller degree of discrimination, due to the fact that they neither consider those without any employment nor the work done outside employment on the labour market.

As we discussed in Section 2.3, it is important to go beyond the “one-dimensional straightjacket” (Cowell, 2011) of expected wage analysis and consider wages’ stochastic nature. Here, we will focus on one additional distributional aspect in particular - the inequality associated with the

distribution.¹² To this end, we consider the Gini coefficient (G), which is generally the most widely used inequality measure (see Cowell, 2000). Rather than just using the conventional Gini coefficient, we display the Gini in its extended form of the generalised Gini coefficient, where the parameter ρ yields the level of inequality aversion. While $\rho = 2$ yields the conventional Gini coefficient, higher values of ρ reflect rising inequality aversion such that $\rho \rightarrow \infty$ would yield the attitude of a Max-Min decision maker. By contrast, $\rho \rightarrow 1$ would reflect an attitude of somebody who is indifferent to inequality, which is equivalent to considering the expected earnings. In addition to the conventional Gini coefficient, we also display the coefficient for two higher levels of inequality aversion ($\rho = 3$ and $\rho = 4$) as findings from Ebert and Welsch (2009) indicate that inequality aversion is higher than implied by the standard Gini coefficient with $\rho = 2$.

As expected, we observe in Table 1 that the inequality according to the Gini coefficient is greater among wage distributions of women than among that of men. This difference is consistent across all inequality aversion parameters and significant¹³ at the 5% level in most cases (with the major exception being the first case). If we regard inequality in the conditional wage distribution as undesirable, which despite some heterogeneity can generally be assumed according to the literature (e.g. Bellemare et al., 2008), this further disadvantage for women should be considered in an analysis of discrimination.

To this end, we use the standard social welfare concepts and display the Equally Distributed Equivalent Labour Earnings Rate (\mathcal{W}) discussed in Section 2.3, which takes into account both the expected level and the inequality attached of the earnings distribution under consideration. While the differences between males and females of this measure are treated in more detail below, a quick glance at the table already shows that the ratios between the selected males and females are generally greater than for the simple mean.

5.1.3 Assessing the magnitude of discrimination

As portrayed in Section 2.3, we use the ratio of both expected earning rates as well as the welfare measures incorporating inequality to get an impression of the magnitude of discrimination for the three groups under consideration. As Table 2 shows, the discrepancy is larger once we consider an activity-based work definition, include those out of employment and use the EDELER.

Focussing on the group with the average men and women (D_2), the discrimination measure is 75 cents to the Euro if we consider a conventional work definition and exclude all zero-incomes (μ_0^{conv}). The discrepancy widens to around 49 cents to the Euro once considering work definition based on

¹²Note that it is of course also possible to consider additional aspects like polarisation - see Section A.7.

¹³We use the term “significant” in the sense that according to the posterior belief the chance of the specified alternative is assessed to be below a level of α .

	D_1	D_2	D_3
$\Delta_{\mathbf{x}}$ for μ_0^{conv}	0.94[0.76;1.18]	0.75[0.67;0.84]	0.70[0.59;0.82]
$\Delta_{\mathbf{x}}$ for μ_0	0.61[0.47;0.81]	0.49[0.42;0.57]	0.52[0.42;0.68]
$\Delta_{\mathbf{x}}$ for μ	0.48[0.33;0.71]	0.44[0.37;0.52]	0.49[0.39;0.65]
$\Delta_{\mathbf{x}}$ for $\mathcal{W}(\rho=2)$	0.44[0.29;0.68]	0.36[0.30;0.44]	0.37[0.30;0.46]
$\Delta_{\mathbf{x}}$ for $\mathcal{W}(\rho=3)$	0.33[0.18;0.62]	0.26[0.19;0.34]	0.29[0.23;0.38]
$\Delta_{\mathbf{x}}$ for $\mathcal{W}(\rho=4)$	0.48[0.33;0.72]	0.21[0.15;0.30]	0.26[0.19;0.35]

Table 2: Ratios of male and female wage distribution measures

the third party criterion as described above (μ_0). The gap widens further once the full population is considered and not just those in employment. Considering the expected income (μ), which is equivalent to the social welfare without inequality aversion (i.e. $\mathcal{W}(\rho=1)$), we see a relation of 44 cents to the Euro. Once one includes inequality aversion in the assessment, we observe that women have an EDELER estimate of only 36 cents to the male Euro when inequality is assessed using the standard Gini coefficient ($\mathcal{W}(\rho=2)$). For higher degrees of inequality aversion, this estimate falls to just 21 cents, meaning that the EDELER of the average male is fivefold that of their female counterparts.

Concerning the other two groups, the picture is analogue by and large. However, it may be interesting to note that the relation between the level of inequality aversion (ρ) and the magnitude of the resultant welfare ratio relation ($\Delta_{\mathbf{x}}$) is not necessarily monotonic as the example of the first group shows (which is also pointed out by Selezneva and van Kerm, 2016). Here, the discrepancy decreases again from $\rho = 3$ to $\rho = 4$. The reason for this is probably largely found in the complex structure of the earnings distribution (see Section 5.1.1). This highlights the complexities in the underlying distributions which deserve further attention and may yield interesting insights in the future. Additionally, one may observe that the effect of the various inequality aversion parameters differs largely for the three different groups, which is hardly surprising. Thus rather than solely contemplating only one (or three) selected group(s), we will give an aggregate picture in the following.

5.2 An aggregate perspective on discrimination

In order to get an aggregate discrimination measure (Δ_a) we simply integrate up the conditioning argument.¹⁴ Applied to a given discrimination measure ($\Delta_{\mathbf{x}}$) from Equation (7), we integrate over

¹⁴Note that this integration naturally constitutes a major reduction of potentially interesting and important information - a problem which has been pointed out by Jenkins (1994).

the covariate space (Ξ), i.e.

$$\Delta_a = \int_{\mathbf{x} \in \Xi} \Delta_{\mathbf{x}} dF_{\mathbf{x}}(\mathbf{x}) \approx \sum_{\tilde{\mathbf{x}} \in \tilde{\Xi}} \Delta_{\tilde{\mathbf{x}}} s(\tilde{\mathbf{x}}), \quad (17)$$

where $F_{\mathbf{x}}(\mathbf{x})$ denotes the distribution function of the covariates and $s(\tilde{\mathbf{x}})$ denotes the share of individuals in the population (both of men and women) who display the covariate combination $\tilde{\mathbf{x}}$ in an adequately discretised covariate space ($\tilde{\Xi}$).

In our application, we use data from the German *Mikrozensus* (see Section A.1.3 in the appendix) to obtain the shares in our discretised covariate space. We consider age as a finely discretised variable on a yearly basis and the other variables in accordance with the categorisation discussed in Section 3. See Section A.1.3 for further elaboration on the covariate distribution.

The discretised covariate space yields 20,480 covariate combinations for which we estimate the conditional wage distribution for both men and women. From these we can derive discrimination measures, such as the ratios presented in Table 2. The ratios can then be aggregated in accordance with Equation (17) yielding an average ratio presented below:

	μ_0^{conv}	μ_0	μ	$\mathcal{W}(\rho=2)$	$\mathcal{W}(\rho=3)$	$\mathcal{W}(\rho=4)$
Δ_a	0.79[0.75;0.82]	0.59[0.54;0.64]	0.54[0.49;0.59]	0.49[0.45;0.55]	0.42[0.37;0.48]	0.39[0.34;0.45]

Table 3: Average ratios of male and female wage distribution measures

As can be observed, estimates using the conventional approach yields an “adjusted” wage gap between male and female earnings of around 21%.¹⁵ Yet this discrepancy is significantly enlarged once considering a third party criterion based definition of work. Depending on the parameter selected for inequality aversion the discrepancy between women and mean amounts to between 46% and 61%.¹⁶ Indeed, based on the findings by Ebert and Welsch (2009) on inequality aversion levels in Europe, it is the latter of the two figures which must be deemed the more appropriate of the two. In other words the arguably still bridgeable wage gap of 21% pointed to by conventional analysis quickly widens into a major rift two to three times the size when the underlying foundations of a conventional work definition and a mean focussed analysis are not underpinning the assessment.

¹⁵This is only slightly lower than the official unadjusted pay gap of 22% (Statistisches Bundesamt, 2014) and considerably higher than the official adjusted pay gap of 7% (taking the latest available figure from 2010 -Statistisches Bundesamt (see 2013)). The main reason for this later discrepancy is that we account for fewer aspects than the statistical office, like occupation and industry. In addition it should be noted that for the official pay gap, *geringfügig Beschäftigte* are not considered, which who we do consider in the analysis. This group is predominantly female and lowly paid and thus likely to increase the pay gap. Other selection aspects, like age restrictions, are also likely to contribute to the difference.

¹⁶Further inquiry into this much larger discrepancy found by the gender earnings rift may shed some light on the paradox of declining female happiness found by Stevenson and Wolfers (2009) whereby female happiness has declined relatively to that of men despite a narrowing gender wage gap.

5.3 Discrimination for sub-populations

In the following, we attempt to shed some light on the underlying mechanics of the results displayed above, we will consider the discrimination measures for some subgroups of the population.

In Table 4 we show the distribution at the margin of four age groups. The four age groups we consider are 21-30 years of age, 31-40 years of age, 41-50 years of age and 51-60 years of age. As before, we still integrate over the rest of the covariate space as well as across age within the indicated interval for each group. As one can see in the table, we generally observe the same pattern with

	μ_0^{conv}	μ_0	μ	$\mathcal{W}(\rho=2)$	$\mathcal{W}(\rho=3)$	$\mathcal{W}(\rho=4)$
Δ_{21-30}	0.89[0.84;0.95]	0.72[0.64;0.80]	0.69[0.61;0.78]	0.70[0.61;0.81]	0.73[0.62;0.87]	0.69[0.59;0.84]
Δ_{31-40}	0.80[0.77;0.83]	0.59[0.54;0.64]	0.51[0.46;0.56]	0.44[0.40;0.49]	0.31[0.27;0.36]	0.27[0.24;0.31]
Δ_{41-50}	0.74[0.71;0.78]	0.56[0.52;0.61]	0.53[0.48;0.58]	0.47[0.43;0.51]	0.40[0.36;0.45]	0.37[0.33;0.42]
Δ_{51-60}	0.75[0.72;0.79]	0.53[0.49;0.58]	0.47[0.43;0.51]	0.41[0.37;0.45]	0.31[0.28;0.36]	0.29[0.26;0.33]

Table 4: Average ratios of male and female wage distribution measures

the magnitude of discrimination increasing from left (conventional wage gap) to right (EDELER with high inequality aversion). Concerning the differences between the age groups, it can generally be observed that discrimination measured by conventional measures generally increase as the age group increases. This is to be expected, supposing that men are in a better position to advance their career and thus extend the wage gap. One notable and interesting variation is that the EDELER wage difference show higher differences for the age range between 31-40 than for 41-50. Indeed for the highest degree of inequality aversion, i.e. $\rho=4$), the discrepancy is even largest for the former age range. The most probable reason for this is that during the former age span the time required for nurturing both children and career is at its height. This aspect is discussed in more detail below. Moreover, while between 41-50 and 51-60 there is no significant difference in the conventional measure indicating stagnation, considering the EDELER, we observe significant differences for $\rho=4$. This indicates that while differences in average wages are stagnating in the later stages of professional life, discrepancies in terms of inequality are still widening. One possibly explanation for this finding is that many of those who have been out of paid work for long tracts of time, mostly women, fail to revitalise their careers while others succeed. As a result inequalities between those who go through long-term unemployment and/or very lowly-paid jobs and those who do manage to obtain decent wages are generally more pronounced among women than among men.

Secondly, we differentiate with respect to whether the person has a child currently eligible to child-benefits, again integrating over the remaining covariate space. The corresponding discrimination measures are displayed in Table 5. Little surprisingly, the gender discrepancy widens with children, as one would expect given the evidence from the literature on the wage gap resulting out of motherhood (see Ponthieux and Meurs, 2015). However, as above it may be noted that the gap

	μ_0^{conv}	μ_0	μ	$\mathcal{W}(\rho=2)$	$\mathcal{W}(\rho=3)$	$\mathcal{W}(\rho=4)$
$\Delta_{no\ kids}$	0.81[0.78;0.85]	0.65[0.59;0.71]	0.61[0.55;0.67]	0.57[0.51;0.63]	0.51[0.45;0.58]	0.47[0.42;0.54]
Δ_{kids}	0.72[0.69;0.75]	0.42[0.40;0.46]	0.35[0.31;0.38]	0.28[0.25;0.31]	0.17[0.14;0.20]	0.15[0.13;0.18]

Table 5: Average ratios of male and female wage distribution measures

is even more pronounced for the EDELER, especially when considering inequality. While for the expectation, μ , we see a shift from 61 cents to the Euro down to 35 cents to the Euro, i.e. roughly halving the ratio, the EDELER with the highest inequality aversion, $\mathcal{W}(\rho = 4)$, yields a change from 47 cents to the Euro down to only 15 cents to the Euro, such that the measure is even slashed by two thirds. The reason for this heightened level of inequality among women can be mostly ascribed to a large fraction of women completely out of employment or in very lowly-paid part-time occupations due to the time requirements of child care. In conjunction with those women who are able to earn a relatively high income despite having children, a higher inequality, as measured by the Gini coefficient, is generated.

In addition, we also consider differentiations with respect to education (higher education vs. no higher education) and region (former West vs. East Germany). The results are displayed in Section A.9 in the appendix. As for above, these measures abide with the observation that the gender wage rift generally points to a larger magnitude of discrimination than the conventional gender wage gap concept.

5.4 Testing for parametric misspecification

Lastly, we consider a standard Kolomogorov-Smirnov type test for the parametric specification used for our modelling approach. Using an adaptation of the test proposed by Rothe and Wied (2013), we obtain a p-value of 0.18 for men and a p-value of 0.62 for women using 1,000 bootstrap repetition. These high p-values imply that the hypothesis that our parametric fit is adequate to describing the nature of the conditional distributions is not rejected at any of the usual levels. Hence, we consider our specification of the conditional earning distributions to be adequate. For further details see Section A.10 in the appendix.

6 Conclusion

This article argued for the use of a more comprehensive conception of earnings discrimination between males and females to complement the traditional concept of the gender wage gap. Looking at gender related earnings discrimination in Germany, we show possible inroads to such a

comprehensive earnings discrimination analysis by considering an activity-based definition of work (rather than a pay-based definition) as well as basing the analysis on full conditional earnings distributions (rather than just their expectation) using structured additive distributional regression. We find that this change of view fundamentally alters the magnitude of pecuniary discrimination of labour with respect to gender. Rather than an arguably still bridgeable wage gap of around 21%, we find a considerably wider earnings rift of between 46% and 61%, i.e. a discrepancy two to three times the size. This analysis thus shows that much more work is left to do than is indicated by common labour market analysis until women's labour receives the same pecuniary valuation as that of their male compatriots and ascriptions like the one from ancient Mesopotamia truly become obsolete.

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A Appendix

A.1 Data

As primary source for our data we use the SOEP database (SOEP, 2014). We use all available waves in 2013, which provides us with 10 waves (A to J). Only taking those values for which we have the full set-of variables, as described below, we obtain 9,182 observations.

A.1.1 Variables used

In order to obtain our dependent variable of the gross hourly wage we use the variable BDP7701 from the individual questionnaire (DIW Berlin, 2014a) to obtain the monthly earnings of the individual.¹⁷ This is divided by 4.32 times the number of hours worked per week. As discussed in Section 2.2, we do not only consider the hours of paid work but the hours spent on activities which we consider work. For the former we use the the variable BDP66 , which is the actual number of hours worked last week. The hours spent on housework, errands, care work and repair works are taken from the answers from BDP10\$\$\$. These variables give the number of hours spent on each type of activity for a regular weekday, a Saturday and a Sunday, which we simply sum up to get the number of hours per week. It must be noted that these variables are likely to be rather imprecise such that our data is affected by some considerable data related uncertainty.

For the explanatory variable age we simply use the birthyear (GEBJAHR) from the individual questionnaire and subtract it from 2013, while the sex is determined by the variable BDSEX. The binary variable of whether a person has children that is still dependent on the household is based on the variable BDH503 from the household questionnaire (DIW Berlin, 2014b). The variable *nat* is constructed on grounds of variable BDP143 in the individual questionnaire (DIW Berlin, 2014a).

The education level is taken on grounds of the variable ISCED13¹⁸ from the PGEN variables. All observations equal or lower than 2 (general elementary and below) are put in the first education category, with all observations in category 3 (middle vocational) put in the second category. All observations with ISCED13 values 4 and 5 (vocational with Abitur and higher vocational) are in the third group, while the highest group consists only of individuals with a value 6 (higher education).

For the spatial effect we use the variable BDBULA with the variable *east* set to unity for all federal

¹⁷We thus neglect aspects any transfer payments, like the *Elterngeld*. While this is obviously problematic in some cases it seems much more appropriate than the consideration of net-incomes, i.e. including all transfer payments (like *Elterngeld*), as we want to consider the remuneration of activities by a society prior to welfare considerations as is standard in the discrimination literature.

¹⁸It should be noted that the SOEP ISCED levels are not a 100% equivalent to the ISCED levels elsewhere.

states formerly belonging to the German Democratic Republic, including the whole of Berlin, which in the SOEP is equivalent to states 1-11. West Berlin is not accounted for separately.

All observations are weighted using the variable BDPHRF.

A.1.2 Observations dropped

Concerning the wage rate, we only consider those persons who for who we have a value greater or equal to zero, i.e. all observations that are not missing, not applicable or highly improbable according to the SOEP.

Concerning the age, we only consider person who are between 21 and 60 years of age and for whom we have age observations.¹⁹

Concerning education, we all observations who have a value greater or equal to zero, i.e. all observations that are not missing, not applicable or highly improbable according to the SOEP.

Concerning time spent, we drop observations who state more than 168 hours for activities per week.

Concerning the federal state of residence, we use all observations.

Concerning the nationality, we use all observations.

Concerning the children, we use all observations.

Concerning the weight, we use all observations with a positive weight.

A.1.3 Construction of covariate distribution

For the construction of the covariate distribution in the discretised covariate space, we use data from the German *Mikrozensus* provided upon request from the German Statistical Office. The *Mikrozensus* entails information from around 370,000 household with 830,000 persons equivalent to roughly 1% of the population. It thus allows for a relatively accurate account of the number of persons in each strata as defined by our covariates.

We use the variables in the *Mikrozensus* to categorise our population strata emulating the categorisation based on the information (and variables) from the SOEP as closely as possible. To this end, we proceed as follows:

¹⁹We explicitly do not exclude students or other persons currently in the process of enhancing their capital stock to further incomes later on. As long as such education breaks from employment are roughly equivalently distributed between males and females, which they are in the case of tertiary education (Statistisches Bundesamt, 2013), this should not distort the discrimination estimates as we define it.

For age we have yearly age brackets at our disposal allowing us a one-to-one comparison with the variables from the SOEP. The same is possible for the categorisation into Germans and persons of foreign nationality which is also recorded in the *Mikrozensus* equivalently to the SOEP. Concerning the question of whether the individual has dependent children, we use information from the *Mikrozensus* on whether the individual has under age children. This is not 100% equivalent to the definition we use based on the SOEP, but should by and large give us similar results. Lastly for education we use information on the education level with regard to job qualification (*höchster beruflicher Ausbildungsabschluss*) available. The four education levels we use are constructed as follows:

- educ*₁ The first education level comprises all individuals who have no formal job-qualification (*ohne berufsqualifizierenden Ausbildungsabschluss*) and those on whom we have no information (*ohne Angabe zur Art des Abschlusses*).
- educ*₂ The second education level comprises all individuals who have the standard job qualification, which in Germany is generally a completed apprenticeship (*Abschluss einer Lehre oder gleichwertiger Berufsfachabschluss*).
- educ*₃ The third education level comprises all individuals who have a higher technical technical qualification (*Meister-/Techniker oder gleichwertiger Fachschulabschluss* and *Fachschulabschluss der DDR*)
- educ*₄ The fourth education level comprises all individuals with higher education (*Fachhochschulabschluss* and *Hochschulabschluss*).

Using this categorisation, we construct the covariate distribution yielding population shares for all covariate combinations in $\tilde{\Xi}$ as used in Equation 17.

A.2 Wages, Choices and Circumstances

Let $Y_{i,t}$ be a random variable of the earnings-rate offered and/or paid to individual i at time t . This earnings-rate is thought to be derived from the individual's capability to pursue a set of activities required for some specific work at a given time and the surrounding circumstances of society via some function w , i.e.

$$Y_{i,t} = w(\mathbf{A}_{i,t}, \mathbf{S}_{i,t}), \quad (18)$$

where $\mathbf{A}_{i,t}$ is a vector denoting the set of activities which is selected from a space of possible human activities \mathcal{A} , while vector $\mathbf{S}_{i,t}$ denotes the circumstances in a society currently facing the individual stemming from the space of conceivable circumstance-scenarios \mathcal{S} . The circumstances comprise aspects like demand and supply for an activity, legislation, social norms, etc. and are thought of

as stochastic but possibly dependent on the gender of the individual. The ability of the individual to perform an activity is thought to be dependent on the circumstances as well as on individual choices, both past and present. Formally, we can write

$$\mathbf{A}_{i,t} = a(\mathbf{S}_{i,t}, \mathbf{C}_{i,t}), \quad (19)$$

where a is a vector valued function which determines the ability to perform a set of activities in $\mathbf{A}_{i,t}$ based on the set of societal circumstances $\mathbf{S}_{i,t}$ and the individual's choices up to and including time t , denoted $\mathbf{C}_{i,t}$.²⁰ The choices $\mathbf{C}_{i,t}$ are also thought of as a stochastic vectorial variable from choice-space \mathcal{C} , which entails all possible choice combinations, including aspects like effort, having children, etc. As a function of two random variables, $\mathbf{A}_{i,t}$ can also be seen as random unless we fully condition on both $\mathbf{S}_{i,t}$ and $\mathbf{C}_{i,t}$. For a schematic representation see Figure 4.

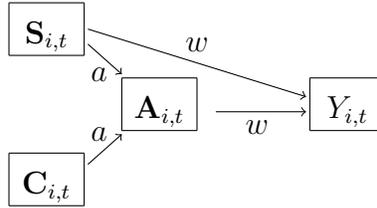


Figure 4: Relation of earnings, choices and circumstances.

In this framework, a gender earnings rift is thought to exist, if and only if the circumstances facing men and women respectively induce structurally different resultant hourly earnings distributions. These differences can stem from Equation (18), i.e. direct discrimination, where despite the same ability to perform the necessary activities women are paid differently to men. However, in addition the differences may be the product of discrepancies of activities pursued by men and women. If these differences cannot be fully attributed to free choice but are at least in part due to differences in circumstances, this would constitute discrimination stemming from Equation (19). A concept which captures discrimination from both equations may be referred to as comprehensive earnings discrimination. It is this concept that we focus on in this article.

A.3 A Remark on the Inclusion of those out of Formal Work

In this paper, we consider the whole population with the wage rate that is actually earned, including zero wage rates. This is converse to the standard approach in the literature which concentrates on those earning a positive wage and includes those with zero wages along the lines of the seminal paper by Heckman (1979) whereby labour market participation is seen as a sample selection problem.

²⁰Here we simply suppose that a is defined in such a way that $\mathbf{A}_{i,t}$ is theoretically identifiable in Equation (18).

The underlying economic concept of the Heckman approach is that individuals choose to work or not to work based on some (presumably utility-based) criteria. In order to account for this self-selection Heckman proposes a correction which would emulate the situation whereby those persons who choose not to participate are paid the wage they would have been paid had they chosen to work for the offered wage.

Yet for our assessment of the magnitude of discrimination this approach would lead a skewed assessment, as it only corrects for (some) self-selection of those who fully opt out of working for money. By contrast, those who choose (or are forced by circumstances) to work-part time (and most likely accept a lower wage) would be included in no other way than those working full-time. Equally those who choose one more lowly paid job over another for non-pecuniary reasons would also induce a problem of selection. Or to put it in the framework presented in Section A.2, the problem is that the Heckman correction only selectively accounts for individual's choices and circumstances in Equation 19.

As mentioned before, it remains a simple (sad) fact that economic regressions and the empirical data available are far from allowing us to get anywhere near to identifying individual's choices and societal circumstances. Thus, rather than applying a counter-factual correction to a subset of the population we opt for what we consider the more consistent approach - consider the actual wages paid.

A.4 Estimation Methodology

A.4.1 GAMLSS and SADR

For our estimation we use structured additive distributional regression in order to estimate conditional distributions. It should be noted that this methodology is closely related to the concept of Generalised Additive Models of Location, Scale and Shape (GAMLSS) first proposed by Stasinopoulos and Rigby (2007) in a frequentist framework and by Klein et al. (2014) in a Bayesian setting. Yet to the frequentist framework and the associated R package `gamlss` is still very limited with regard to the flexibility of the effects. Especially the ability of fitting non-linear effects in a reliable and stable manner which are well suited to modelling the age effect in a flexible manner constitutes a major advantage of structured additive distributional regression and its implementation in BayesX. Following Klein et al. (2015) we use the phrase structured additive distributional regression as the aim of our estimation methodology is not so much simply extending the number of distributional aspects, like the moments, beyond just one particular aspect, like the mean. Rather the methodology's primary focus is to model the whole conditional distribution.

Pursuing the same aim along a different path several publications have recently appeared which could in principle be used to estimate the conditional earnings distributions.

A.4.2 Direct Maximum Likelihood Estimation

In his paper, van Kerm (2013) proposes a simple maximum likelihood based estimation procedure employing a Singh-Maddala distribution with each of the three parameters allowed to vary along a coarsely discretised covariate space. Yet this approach generally becomes infeasible as the number of covariate combinations on the covariate space becomes larger. Given the large number conceivably influential covariates (and their possibly continuous nature) a regression based approach like ours seems more appropriate than a direct estimation of the distributions for many subsets of the sample.

A.4.3 Distributional Regression and Conditional Transformation Models

Another approach, which has recently received considerable attention is the proposition to estimate conditional distributions by discretising its domain into small ranges and estimating the corresponding density functions of those domains using the well established statistical machinery from survival analysis. In the econometric literature this is generally called distributional regression (Chernozhukov et al., 2013) while in the statistical literature refers to such approaches as conditional transformation models (Hothorn et al., 2014).

While this approach allows for great flexibility, this comes at the cost of high instability of the estimation. Given the rather limited number of observations with respect to the size of the covariate space this alternative distribution methodology is not applicable to our research question.

A.4.4 Quantile Regression

An obvious alternative to estimating conditional distributions is by the use of quantile regression, which has been used for the estimation of conditional income distributions by Machado and Mata (2005) among others. Again this methodology is arguably more flexible than our approach as it does not require a possibly erroneous assumption about the parametric nature of the distribution. However, again this flexibility comes at the cost of estimation stability which given the limited sample size does constitute a problem. Another particular problem with quantile regression is the fact that due to the inclusion of unemployed persons, we have a high point mass which is known to cause problems with quantile crossing, especially if the effects are simply linear (see Klein et al., 2015).

While there are alternative estimation strategies, we thus opt for structured additive distributional regression as this methodology yields a good balance between modelling flexibility and estimation stability due to its parsimonious parametric formulation. We explicitly want to stress that our approach, while having some advantages over the other approaches presented here, naturally also has disadvantages. It is up to the specific problem at hand to choose the appropriate methodology. Further studies comparing the performances of the different approaches would greatly aide the choice but are to our knowledge not yet available.

A.5 Effects on the Parameters

In Table 6 we display the estimates for the parameters of the three conditional wage distribution considered in Section 5.1.2.

	D_{1m}	D_{1f}	D_{2m}	D_{2f}	D_{3m}	D_{3f}
a	14.55[8.79;24.63]	5.92[3.68;10.28]	5.38[4.40; 6.59]	4.57[3.77;5.69]	4.36[3.48; 5.58]	2.37[1.93; 2.94]
b	8.65[7.05;10.74]	5.01[3.51; 7.05]	9.70[8.40;11.15]	6.73[5.78;7.62]	22.48[18.94;26.53]	9.42[7.72;11.34]
c	0.13[0.07; 0.26]	0.29[0.12; 0.60]	0.69[0.50; 0.97]	0.30[0.22;0.40]	0.58[0.40; 0.85]	0.70[0.49; 0.98]
π_0	0.45[0.39; 0.51]	0.56[0.47; 0.65]	0.09[0.07; 0.11]	0.18[0.13;0.24]	0.01[0.01; 0.02]	0.06[0.04; 0.09]

Table 6: Distribution parameters for 3 conditional wage distributions

As one can observe there is substantial variation between the parameters beyond the parameter b , which is a scale parameter and thus most comparable to the parameter μ in case of the log-normal distribution. This shows that there is considerable variation of additional aspects of the conditional distributions in the covariate space which conventional analysis is liable to neglect.

A.6 Effects on the Wage for other Covariates

In Figure 5, we display the resultant estimates from our model for different combinations of age and two education levels (the second and the fourth). The other covariates are fixed as follows: German Nationality, with at least one child and living in West Germany.

The upper graph showing the expected earnings, shows the usual pattern of concavely increasing wages with age, while higher education also increases the expected wage with women generally paid less, c.p. Only towards the lower end of the spectrum, women with higher education are able to surpass the earnings of more lowly qualified men. One additional aspect which may be noted is the wide credible intervals, especially for women of higher ages. This is due to the fact that we have very few women with higher education of a higher age.

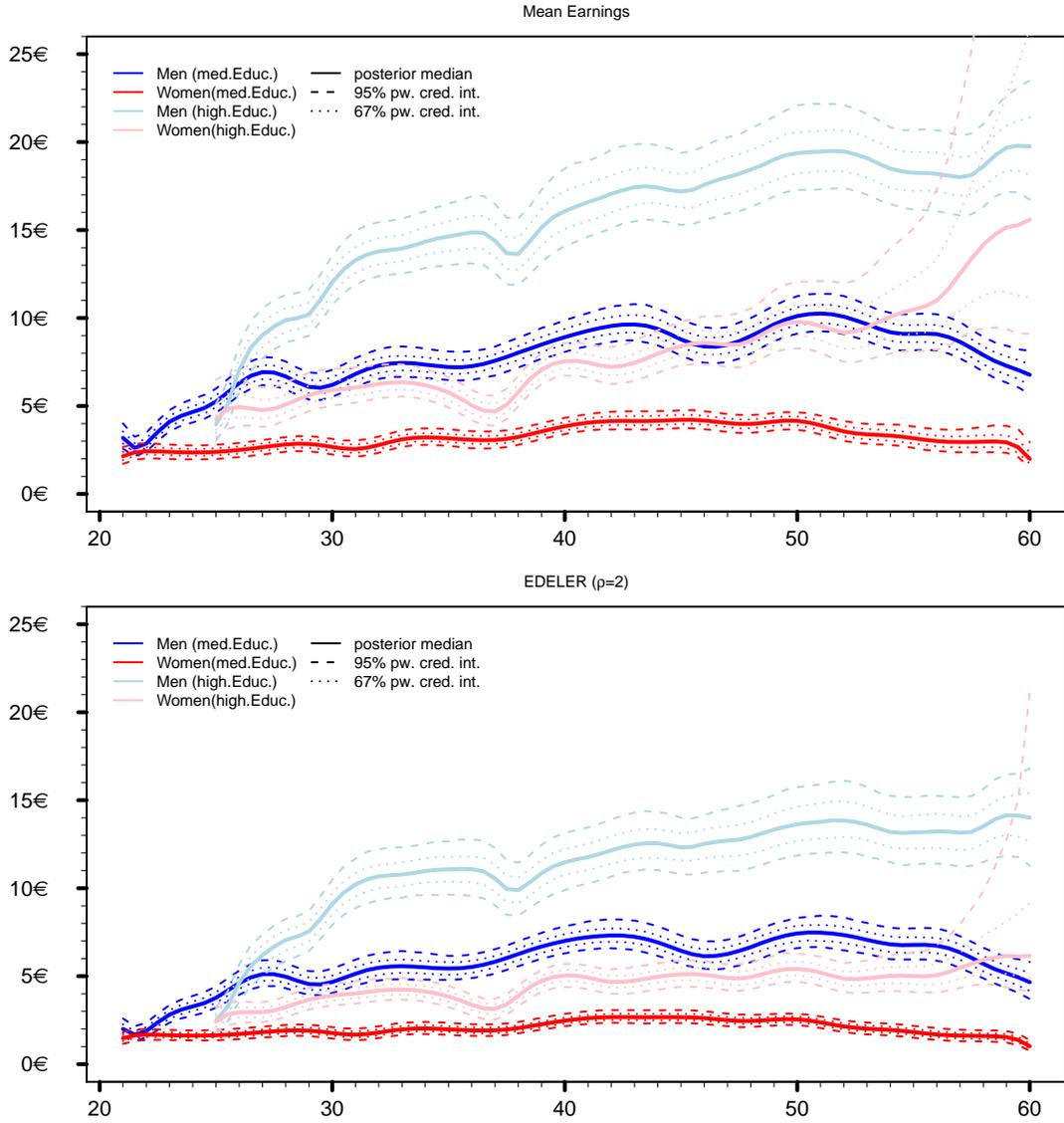


Figure 5: Expected Wage (upper) and Certainty Equivalent Wage (lower) w.r.t. age and education.

In the lower graph, we display the EDEE for using the standard gini specification (i.e. $\rho = 2$). Here, we observe a similar yet more pronounced difference between men and women, as we would expect. This is most visible in the case of older women with higher education, who despite their much higher expected earning (in comparison to men without higher education) also have much higher inequality in the earnings distribution such that the differences in the EDEE are much smaller and even reversed.

Lastly, it has to be conceded that the age-earnings relationship portrayed is not very smooth and does show considerable curvature for parts of the covariate space considered. Given the fact that we do not smooth over the earnings but rather over the individual parameters this is hardly surprising. Nonetheless, this is not ideal and further research in the direction of higher dimensional

smoothing using tensor splines or the like would allow for an improved smoothing procedure.

A.7 Additional Inequality and Polarisation measures

In Table 7 we display two additional measures of economic interest - the Atkinson index for three levels of inequality aversion as well as one polarisation measure.

	D_{1M}	D_{1W}	D_{2M}	D_{2W}	D_{3M}	D_{3W}
AI($\epsilon=0.5$)	0.44[0.39;0.50]	0.54[0.46;0.61]	0.11[0.09;0.14]	0.25[0.20;0.30]	0.07[0.05; 0.10]	0.22[0.18; 0.28]
AI($\epsilon=1.0$)	0.90[0.87;0.93]	0.92[0.89;0.94]	0.43[0.37;0.50]	0.67[0.58;0.75]	0.19[0.15; 0.24]	0.52[0.44; 0.60]
AI($\epsilon=1.5$)	0.99[0.98;0.99]	0.98[0.98;0.98]	0.91[0.88;0.94]	0.95[0.92;0.96]	0.63[0.54; 0.71]	0.90[0.85; 0.94]
P($\alpha=\beta=1$)	0.30[0.02;0.58]		1.07[0.77;1.33]		2.31[1.35;3.26]	

Table 7: Additional Inequality and Polarisation Measures

As a second inequality measure, we consider the Atkinson inequality index (AI) proposed in Atkinson (1970), for which we consider three levels of inequality aversion ($\epsilon = \{0.5, 1.0, 1.5\}$).²¹ As can be observed the general pattern is equivalent to the Gini, although it should be noted that the Atkinson index is much more sensitive to zero incomes, especially for its higher coefficients of inequality aversion.

Polarisation measures have seen increasing attention in the literature in recent years following the publications from Esteban and Ray (1994) and Wolfson (1994). Here, we employ a polarisation measure (P) proposed in Esteban et al. (2007) where α is the sensitivity to polarisation between groups and β indicates the weight assigned to inequality within the wage distribution.

Following Gradín (2000), we apply this measure to the wage distributions of the corresponding wage distributions of males and females using $\alpha = 1$ and $\beta = 1$. This thus gives us an indication on the degree of polarisation of wages paid to men and women respectively. As we can see, this is hardly the case for the first group, which as we observed in Figure 3 strongly overlapped, while it is more pronounced for the second and the third group, where a lot of wages earned by men are practically out of reach for women with equivalent characteristics \mathbf{x} .

A.8 Estimates for the Conventional Wage Concept

A.8.1 Analysis of Conditional Wage Distributions

As in Section 5, we consider the wage distributions of the same three covariate combinations by visual inspection. In contrast to the graphs in Figure 3, we can observe that the wage distributions

²¹Note that the limiting form for $\epsilon \rightarrow 1$ involves the natural logarithm being applied to the wage. To obtain finite values, we replace zero-wages with very small wages (0.01€) as is standard in the literature.

of men and women overlap to a much greater degree, if one only considers the ordinary work-time concept. Naturally, the wages are also a lot higher as the working time according to the introduced concept is a superset of the conventional definition of working time.

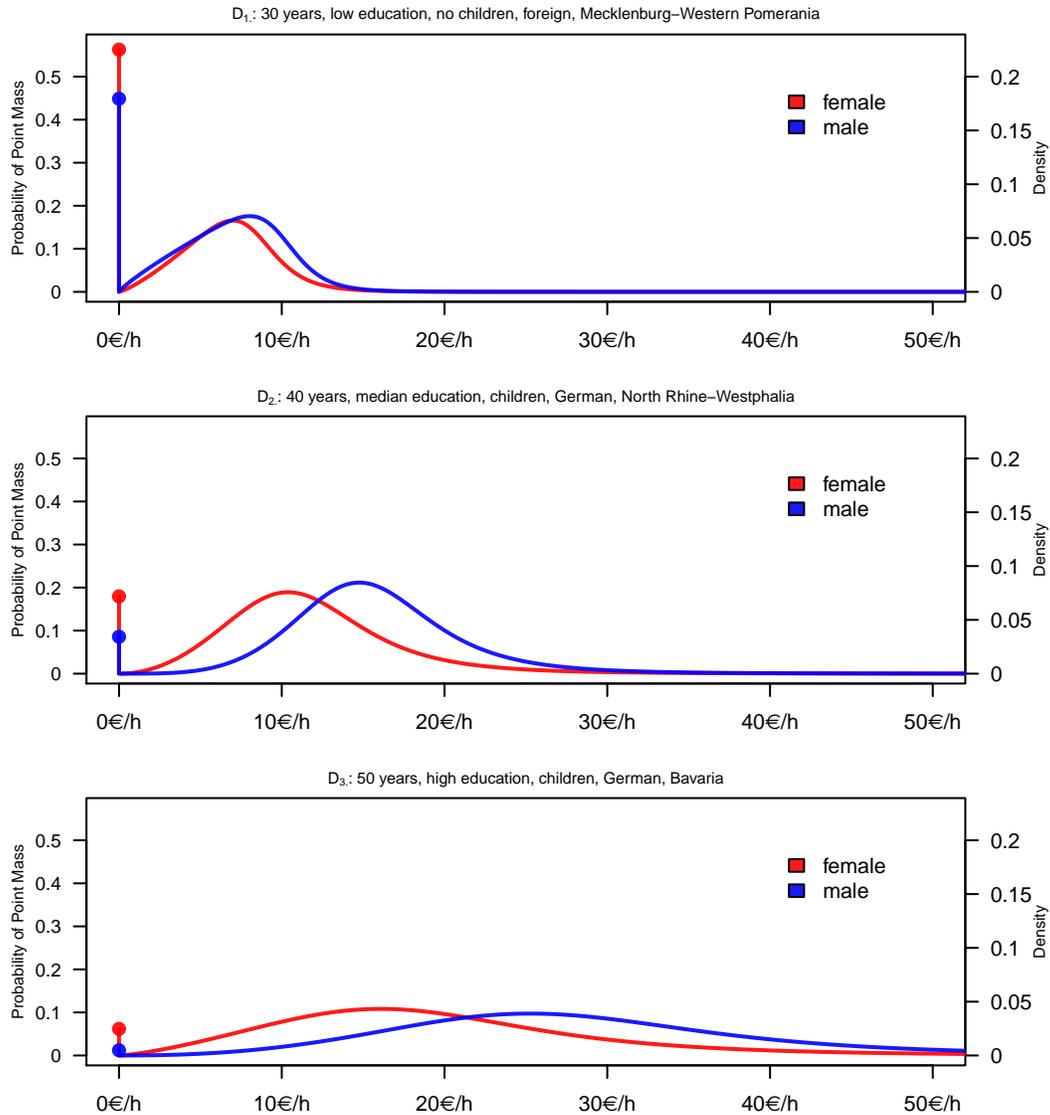


Figure 6: Conditional wage distribution estimates of males (blue) and females (red).

For the first graph, we see that the graphs are again not adhering to the standard positive skew normally found in most income related distributions. However, the skew is lower than in the case in Figure 6. Nonetheless, also with these graphs it is observable that there are differences beyond location and scale between the wage distribution of men and women.

This is also true for the second wage distribution, where women show a higher skewness as well as a higher coefficient of variation. Additionally we can observe that women are much more likely to be receiving no wages at all than men.

The third set of distributions displays a much wider distribution than originally. Especially for women, we observe a much lower skewness, as for a lot of the women in this subgroup the working time is significantly reduced if aspects like care work are not considered.

In Table 8 we display the equivalent measures to those displayed in Table 1 in the article.

	D_{1m}	D_{1f}	D_{2m}	D_{2f}	D_{3m}	D_{3f}
μ	3.95[3.23;4.85]	2.94[2.20;3.82]	14.62[13.23;15.97]	9.80[8.79;10.86]	29.01[26.05;32.39]	19.14[17.09;21.56]
$G(\rho=2)$	0.24[0.18;0.31]	0.23[0.18;0.29]	0.18[0.15; 0.21]	0.24[0.21; 0.27]	0.23[0.20; 0.27]	0.30[0.26; 0.34]
$G(\rho=3)$	0.79[0.74;0.83]	0.84[0.79;0.86]	0.38[0.34; 0.42]	0.56[0.49; 0.62]	0.35[0.30; 0.40]	0.48[0.44; 0.54]
$G(\rho=4)$	0.84[0.82;0.86]	0.83[0.79;0.84]	0.47[0.42; 0.52]	0.67[0.60; 0.73]	0.41[0.36; 0.47]	0.57[0.52; 0.63]
$\mathcal{W}(\rho=2)$	3.00[2.33;3.74]	2.27[1.64;3.00]	12.01[10.85;13.17]	7.43[6.62; 8.31]	22.19[19.66;24.94]	13.45[11.84;15.18]
$\mathcal{W}(\rho=3)$	0.83[0.56;1.19]	0.49[0.31;0.78]	9.08[8.02;10.22]	4.36[3.38; 5.33]	18.92[16.53;21.47]	9.89[8.44;11.48]
$\mathcal{W}(\rho=4)$	0.63[0.48;0.84]	0.51[0.43;0.65]	7.78[6.74; 8.94]	3.28[2.40; 4.22]	17.07[14.69;19.51]	8.21[6.79; 9.69]

Table 8: Some distribution measures for 3 conditional wage distributions

The Table also shows that the expected wages are much higher both for men and women with the discrepancy between wages of men and women generally reduced in contrast to the comprehensive wage definition used in the article. For the first group, it may even be noted that the credible intervals for males and females overlap, although this is more likely down to the relatively high uncertainty of the estimates in structured additive distributional regression than an actual equivalence in the expected wages.

Concerning the Gini coefficient (G), we find generally find slightly reduced measures, although this trend is not true for all six distributions and mostly non-significant.

Concerning the Equally Distributed Equivalent Earning (\mathcal{W}), we find that it is generally higher due to the higher expected income.

	D_1	D_2	D_3
$\Delta_{\mathbf{x}}$ for μ	0.74[0.53;1.04]	0.67[0.58;0.77]	0.66[0.56;0.78]
$\Delta_{\mathbf{x}}$ for $\mathcal{W}(\rho=2)$	0.76[0.52;1.11]	0.62[0.53;0.72]	0.61[0.51;0.72]
$\Delta_{\mathbf{x}}$ for $\mathcal{W}(\rho=3)$	0.59[0.33;1.09]	0.48[0.36;0.61]	0.52[0.42;0.64]
$\Delta_{\mathbf{x}}$ for $\mathcal{W}(\rho=4)$	0.81[0.58;1.16]	0.42[0.30;0.57]	0.48[0.38;0.61]

Table 9: Ratios of male and female wage distribution measures

To get a measure of discrimination, we use the ratios between male and female wage measures, as displayed in Table 9. As would be expected the measures decrease relative to the measures we find in Table 2 using the comprehensive wage definition. Nonetheless, we still the same general relationship that the EDEE induce a higher degree of discrimination than would be portrayed by simple comparison of expected incomes.

Using the conventional concept, we also show the results if we aggregate the various subgroups:

	μ	$\mathcal{W}(\rho=2)$	$\mathcal{W}(\rho=3)$	$\mathcal{W}(\rho=4)$
Δ_a	0.69[0.66;0.72]	0.62[0.59;0.64]	0.47[0.45;0.50]	0.44[0.41;0.47]

Table 10: Average ratios of male and female wage distribution measures

Analogously to above, we find that although the magnitude is smaller, the general finding that the inclusion of risk in the assessment of gender discrimination generally increases the magnitude estimates is robust to the definition of work.

A.9 Estimates for other sub-populations

In Table 11, we display the ratio between female and male pay for two subgroups differentiated by the region of residence, divided into the federal states belonging to former West Germany (excluding Berlin) and former East Germany (including Berlin).

	μ_0^{conv}	μ_0	μ	$\mathcal{W}(\rho=2)$	$\mathcal{W}(\rho=3)$	$\mathcal{W}(\rho=4)$
Δ_{West}	0.77[0.74;0.81]	0.57[0.53;0.63]	0.52[0.46;0.57]	0.47[0.42;0.52]	0.38[0.32;0.45]	0.36[0.30;0.42]
Δ_{East}	0.87[0.80;0.93]	0.68[0.60;0.76]	0.68[0.60;0.76]	0.63[0.55;0.71]	0.62[0.52;0.73]	0.56[0.47;0.67]

Table 11: Average ratios of male and female wage distribution measures

Our findings echo those of the literature with the gender pay gap generally higher in the West than in the East, regardless of the measure used. Again it should be noted that the discrepancy is higher for the EDELER than for the conventional measure, both for the West and the East.

The same measure is displayed for a division along education levels. Here we solely differentiate in a binary manner between those who have enjoyed higher education and those who have not. The results are displayed in Table 12.

	μ_0^{conv}	μ_0	μ	$\mathcal{W}(\rho=2)$	$\mathcal{W}(\rho=3)$	$\mathcal{W}(\rho=4)$
$\Delta_{lower\ educ.}$	0.79[0.76;0.83]	0.59[0.54;0.65]	0.54[0.49;0.59]	0.49[0.45;0.55]	0.42[0.37;0.48]	0.39[0.35;0.45]
$\Delta_{higher\ educ.}$	0.76[0.72;0.80]	0.58[0.54;0.63]	0.53[0.49;0.58]	0.49[0.45;0.54]	0.41[0.37;0.47]	0.37[0.32;0.43]

Table 12: Average ratios of male and female wage distribution measures

Again we observe the same pattern of the measure increasing from left to right. Concerning the difference between the two it is noteworthy that indeed there is very little discrepancy between the two subgroups for any of the measures considered. While more interactive differentiation (especially including age) may show some ‘education effect’, a simple analysis with the other variables integrated out, does not.

A.10 Misspecification Testing

A.10.1 Hypotheses and Algorithm

One quintessential assumption made for our estimation approach is the use of an adequate parametric form for the conditional wage distributions. This assumption is tested by considering the following null-hypothesis \mathcal{H}_0 against the alternative hypothesis \mathcal{H}_1 :

\mathcal{H}_0 : The conditional wage distributions can be modelled by our parametric form, $p(y \mid \boldsymbol{\theta})$, for all observed values of y and some values of $\boldsymbol{\theta}$ derived for the corresponding covariates, \mathbf{x} .

vs.

\mathcal{H}_1 : The conditional wage distributions cannot be modelled by our parametric form, $p(y \mid \boldsymbol{\theta})$, for all observed values of y and any values of $\boldsymbol{\theta}$ derived for the corresponding covariates, \mathbf{x} .

In order to test these hypotheses we will use an adaptation of the Kolmogorov-Smirnov test, which is arguably the most renowned and most widely used test for distributional assumptions. Our adaptation is based on the work of Andrews (1997) and Rothe and Wied (2013) who proposed a frequentist framework for the testing of conditional distributions. Using the idea to transform the conditional moment restrictions imposed by the parametric specification of our structured additive distributional regression model into unconditional ones (see Rothe and Wied, 2013), we are able to specify the test statistic T_n as

$$T_n = \sqrt{n} \sup_{(y, \mathbf{x})} | \hat{H}_n(y, \mathbf{x}) - \hat{H}_n^0(y, \mathbf{x}) |, \quad (20)$$

where $\hat{H}_n(y, \mathbf{x})$ and $\hat{H}_n^0(y, \mathbf{x})$ constitute estimates of the joint cumulative distribution function of both dependent and independent variable for n observations integrated up with respect to the marginal distribution of the conditioning variables:

$$\hat{H}_n(y, \mathbf{x}) = n^{-1} \sum \mathbb{1}_{\{Y_i \leq y\}} \mathbb{1}_{\{\mathbf{X}_i \leq \mathbf{x}\}}$$

and

$$\hat{H}_n^0(y, \mathbf{x}) = n^{-1} \sum \hat{P}_n \mathbb{1}_{\{\mathbf{X}_i \leq \mathbf{x}\}},$$

where \hat{P}_n denotes the estimated cumulative density function based on n samples using structured additive regression while $\mathbb{1}$ again denotes an indicator function.

As the asymptotic distribution of T_n under the null hypothesis depends on the data-generating

process in a complex fashion we use a bootstrap procedure to simulate it. In order to incorporate the uncertainty attached to the parameter estimates we use draws from the MCMC realisations and contrast it with simulated realisations of Y for a set of randomly selected covariate combinations of \mathbf{X} . Our bootstrap algorithm thus as follows:

- Step 1 Draw a bootstrap sample of covariates $\{\mathbf{X}_{b,i}; 1 \leq i \leq n\}$ with replacement from the obtained values in the sample $\{\mathbf{X}_i; 1 \leq i \leq n\}$.
- Step 2 Randomly select the \mathbf{m} -th MCMC draw from set \mathcal{M} for the parameter estimates, yielding $\theta_{b,1}^{(\mathbf{m})}(\mathbf{x}), \dots, \theta_{b,K}^{(\mathbf{m})}(\mathbf{x})$.
- Step 3 Use $\{\theta_{b,k}(\mathbf{x}); 1 \leq k \leq K\}$ and $\{\mathbf{X}_{b,i}; 1 \leq i \leq n\}$ to simulate $\{Y_{b,i}; 1 \leq i \leq n\}$ in accordance with the parametrically specified conditional distributions.
- Step 4 Use bootstrapped data $\{Y_{b,i}; 1 \leq i \leq n\}$, $\{\mathbf{X}_{b,i}; 1 \leq i \leq n\}$ and $\{\theta_{b,k}(\mathbf{x}); 1 \leq k \leq K\}$ to compute estimates $\hat{H}_{b,n}$ and $\hat{H}_{b,n}^0$ yielding the bootstrap realisation of the test statistic:
$$T_{b,n} = \sqrt{n} \sup_{(y,\mathbf{x})} | \hat{H}_{b,n}(y, \mathbf{x}) - \hat{H}_{b,n}^0(y, \mathbf{x}) |.$$

Using the simulated distribution of T_n we can then derive the corresponding p-value and or critical values to assess the test statistic.

A.10.2 Use of MCMC samples for the misspecification test

To implement the test, we use 10,000 bootstrap draws from the algorithm discussed in Section A.10.1. For our wage distributions have two independent MCMC samples, one for π_0 and one for a, b and c . Hence, we have a two dimensional sample set $\mathcal{M} = (1, \dots, M_1) \times (1, \dots, M_2)$, with $M_1 = M_2 = 1000$. A given random bootstrap sample b thus takes a random set of realisations from the MCMC output $\{m_1; m_2\} = \mathbf{m} \in \mathcal{M}$ to yield $\pi_b^{m_1}$, $a_b^{m_2}$, $b_b^{m_2}$ and $c_b^{m_2}$.

A.10.3 Simulation study

We briefly assess the misspecification test from Section A.10.1 by means of three simple simulation studies in order to validate its performance. In each simulation study, we consider a simple framework with one explanatory variable which has a linear effect on all the predictors of the Dagum distribution, i.e.

$$g(\eta^{\theta_k}) = \beta_0^{\theta_k} + \beta_1^{\theta_k} x, \tag{21}$$

where x is an integer from the interval $[1, 10]$ and g is the log-link.

For the simulations we use 1,000 observations and 1000 bootstrap repetitions and contrast the

results for a true specification, as specified above, with a misspecified parametric model. For the misspecification we use a log-normal distribution with mean and coefficient of variation equivalent to that of the Dagum specification.

Sim.Run	Sim. Study 1		Sim. Study 2		Sim. Study 3	
	\mathcal{H}_0 TRUE	\mathcal{H}_0 FALSE	\mathcal{H}_0 TRUE	\mathcal{H}_0 FALSE	\mathcal{H}_0 TRUE	\mathcal{H}_0 FALSE
1	0.30	0.00	0.02	0.00	0.76	0.12
2	0.58	0.00	0.60	0.00	0.66	0.09
3	0.40	0.00	0.60	0.00	0.40	0.03
4	0.21	0.00	0.82	0.00	0.84	0.07
5	0.71	0.00	0.64	0.00	0.45	0.08
6	0.40	0.00	0.92	0.00	0.63	0.07
7	0.07	0.00	0.05	0.00	0.95	0.10
8	0.92	0.00	0.98	0.00	0.81	0.09
9	0.28	0.00	0.45	0.00	0.54	0.10
10	0.10	0.00	0.80	0.00	0.92	0.08
11	0.91	0.00	0.57	0.00	0.62	0.09
12	0.21	0.00	0.50	0.00	0.59	0.09
13	0.47	0.00	0.00	0.00	0.90	0.09
14	0.29	0.00	0.71	0.00	0.37	0.12
15	0.66	0.00	0.15	0.00	0.72	0.10
16	0.15	0.00	0.13	0.00	0.48	0.09
17	0.92	0.00	0.15	0.00	0.48	0.10
18	0.73	0.00	0.23	0.00	0.92	0.12
19	0.49	0.00	0.41	0.00	0.46	0.10
20	0.98	0.00	0.21	0.00	0.67	0.09
21	0.06	0.00	0.31	0.00	0.80	0.09
22	0.29	0.00	0.68	0.00	0.82	0.10
23	0.26	0.00	0.73	0.00	0.84	0.11
24	0.49	0.00	0.82	0.00	0.97	0.10
25	0.39	0.00	0.57	0.00	0.57	0.11
26	0.79	0.00	0.40	0.00	0.60	0.11
27	0.91	0.00	0.06	0.00	0.80	0.10
28	0.20	0.00	0.54	0.00	0.44	0.10
29	0.35	0.00	0.92	0.00	0.56	0.09
30	0.43	0.00	0.05	0.00	0.42	0.16
μ	0.47	0.00	0.47	0.00	0.67	0.10
σ	0.28	0.00	0.31	0.00	0.18	0.02
Q_1	0.26	0.00	0.17	0.00	0.50	0.09
Q_2	0.40	0.00	0.52	0.00	0.64	0.10
Q_3	0.70	0.00	0.70	0.00	0.82	0.10

Table 13: Results from Simulation Studies for Misspecification Test

The result from our simulation studies are displayed in Table 13. The p-value for each simulation run as well as the mean of the p-values of all simulation runs (μ), their standard deviation (σ) as well as the three quartiles (Q_1, Q_2, Q_3).

In the first two columns, we display the results for the a scenario where we have negligible parameter uncertainty such that the standard deviation of the posterior distribution is 1% of its expectation. As can be observed the p-values in the first column roughly follow a uniform distribution, as we

would expect, while the second column repeatedly rejects the null.

In the third and fourth column, we display a scenario with moderate parameter uncertainty such that the standard deviation of the posterior distribution is 5% of its expectation. As for the first two columns we see that the results are able to clearly distinguish between the correct and the false specification.

In the last two columns, we assume considerable parameter uncertainty with the standard deviation of the posterior distribution is 50% of its expectation. Given this large uncertainty, the model specification test is less likely to reject the false hypothesis. With higher parameter uncertainty, the test is thus conservative.

Overall, the simulation study indicates that the test generally works although its power is mitigated by large parameter uncertainty.

A.11 Model Selection

Subsequently to asking whether one parametric model fits at all let us look at a handful of parametric models and consider their fit. Following our estimation approach we will consider the modelling of the point mass for zero wages and positive wages separately. Here we will consider only one generic predictor for all parameters. While it would in theory be possible to specify the predictor for each parameter individually, this would yield a high dimensional model selection problem, which given the lack of automated routines is not feasible.

A.11.1 Variable selection for the logit model

For the logit model we simply consider three alternatives:

M_1^0 As the first model, we use the model used in the paper.

M_2^0 For comparison we will consider the model, using a linear approach to modelling the impact of age, i.e. use a parabola ($age + age^2$), with the other covariates specified as before.

M_3^0 Lastly, we will consider a model which only considers two education levels only differentiating between those with and without university education.

For comparison of the models we will use the DIC, which has been shown to be an adequate model selection criterion in Klein et al. (2015). The results are displayed in Table 14.

	males	females
M_1^0	301850	413997
M_2^0	337540	445537
M_3^0	336439	451336

Table 14: DIC for zero wages model approaches

A.11.2 Variable selection for the Dagum model

For the modelling of positive earnings we consider the the following five models:

M_1^+ As the first model, we use the model used in the paper.

M_2^+ For comparison we will consider a model, using a linear approach to modelling the impact of age as above for all three parameters of the Dagum distribution.

M_3^+ Thirdly, we will consider a model, using only two education levels as above for all three parameters of the Dagum distribution.

In contrast to the Dagum distribution we will also consider the log-normal distribution.

M_4^+ As a first log-normal specification, we will use the same specification as above for both parameters of the log-normal.

M_5^+ As a first log-normal specification, we will use the same specification as above for μ , while considering σ as a nuisance parameter and keeping it constant, as is standard in the conventional literature.

The results are displayed in Table 15.

	males	females
M_1^+	22316.8	20687.2
M_2^+	23632.2	21864.9
M_3^+	23723.4	22946.8
M_4^+	24418.4	20931.5
M_5^+	24474.4	21239.3

Table 15: DIC for zero wages model approaches