Decoupling Land Values in Residential Property Prices: Smoothing Methods for Hedonic Imputed Price Indices

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Decoupling land values in residential property prices: smoothing methods for hedonic imputed price indices*

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Abstract

A property is a bundled good composed of an appreciating asset, land, and a depreciating asset, structure. The motivation to separate the value of the land and structure components of property prices includes a range of needs such as those of tax authorities, climate adaptation research, and national accounting. Recent approaches to disentangling these two assets have been based on employing non-linearities and a construction price index as an instrument. This study proposes to identify them by treating them as two unobserved components of the property price, each with a unique mapping to intrinsic hedonic characteristics and different dynamic behaviour. The estimation approach uses a modified form of the Kalman filter. One advantage of the model estimation strategy is that the estimated components and derived indices are less influenced by the composition of sales at any given period even when the model is estimated at higher frequencies (e.g. monthly instead of quarterly). The algorithm is simple to implement and we demonstrate, using three datasets representing different urban settings in two countries, that the method produces land value predictions comparable to the state valuer's assessments, and price indices comparable to recently published alternatives that rely on exogenous non-market information to disentangle land values. Our monthly indices are smoother than the quarterly counterparts produced by alternative methods.

Keywords: urban land prices, housing prices, state-space, unobserved components, Fisher price indices.

JEL: C53, C43, R31

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1 Introduction

A property is a bundled good composed of an appreciating asset, land, and a depreciating asset, structure. The importance of this distinction is increasingly recognized in both the real estate (see Bostic et al. (2009), Malpezzi et al. (1987)) and the price index construction literature (see European Comission et al. (2013), Chapter 13, Diewert et al. (2011, 2015) and Diewert and Shimizu (2013)). The motivations to separate the value of the land and structure components of property prices include the needs of tax authorities, climate adaptation research (Fletcher et al., 2013), and national accounting (Diewert et al., 2015). The literature review of Diewert et al. (2015) concludes that land and structure should be modelled as two additive components of the property value. This is the view we adopt as well.

Our paper provides an econometric approach to both obtaining the decomposition of the price of a residential property into land and structure components, and computing price indices for each component. The underlying economic model is a representation of the valuer's problem, whose task is to provide a valuation of either the property or the land. Econometrically, the identification makes use of the fact that each component (land/structure) can be mapped to unique characteristics with behaviours that differ dynamically. The economic arguments to justify the dynamic behaviour of each component are based on existing literature (see for example Bostic et al. (2009)). Due to the mobility of materials and labor, construction costs are generally uniform within a housing market and thus it must be the case that asymmetric appreciation across properties within a market arise from asymmetric exposure to common shocks to land values. At any point in time the value of the structure is its replacement cost less any accumulated depreciation. Thus, sufficiently large depreciation can result in the structure (improvements on the land) declining in value over time[1]. Using dynamic behaviour to separate unobserved components is an idea that dates back to Maravall and Aigner (1977) and has been used recently in a number of contexts (e.g. separating

[1]see Malpezzi et al. (1987) and Knight and Sirmans (1996) for an excellent discussion and treatment of modeling and accounting for depreciation and maintenance of the structure

measurement error, Komumjer and Ng (2014), inflation and output gap, Harvey (2011)). Our identification strategy is to use 'dynamic discount factors', popular in the Bayesian statistics literature dating back to the 1970s (see Koop and Korobilis (2013) for a recent application in econometrics). We write the model in state-space form, specify part of the covariance structure using discount factors and estimate in a classical framework. The estimation uses a modified form of the Kalman filter algorithm. The parameters associated with the covariance structure are estimated by evaluating the log-likelihood and those associated with the hedonic characteristics by running the modified Kalman filter equations. The method proposed here departs from recently proposed approaches to separate land and structure components, Diewert et al. (2015) and Färe et al. (2015), in that we do not rely on the use of exogenous price information as instruments to identify the components.

The use of hedonic characteristics that define the land and structure components is a feature of other recent works (structure size, distance to public transport, etc). In Diewert et al. (2011), the focus is on non-linear hedonic regressions with specifications that use piecewise or spline functions of the characteristics. However, the use of non-linearities is not sufficient to separate the two components. In their most recent work, Diewert et al. (2015), the model specification uses piecewise or spline functions of the characteristics together with exogenous price information in the form of a construction price index which is used as instrument to identify the land component (we refer to their approach as DdH). Diewert and Shimizu (2013) use a similar approach to compute components price indices for Tokyo. Färe et al. (2015) also use the construction price index as an instrument to develop further the DdH approach (although they refer to it as the DS method after the work of Diewert and Shimizu (2013)). Their estimator is based on a distance function approach; we will refer to their approach as FGSS. Färe et al. (2015) estimates econometric models with fixed parameters over the whole sample. This leads to index values that are functions of future market information. This is not a desirable characteristic. DdH use a period by period estimation of the model to construct the price indices, which might lead to excessive volatility due the varying composition of sales of the available sample in each time period. Our estimate of each component at any period, τ, is a weighted sum of information in periods $t = 1, \ldots, \tau - 1, \tau$. We construct two hedonic imputed Fisher price indices. They differ in the type of weighting used for transactions. When the weights are value shares (i.e. the weight of an individual transaction is the ratio of its sale price to the sum of prices of all properties sold in that time period) the index is labelled Fisher Plutocratic (FP). The index obtained by using equal weighting of each transaction is labelled Fisher Democratic (FD). The computed indices from DhH and FGSS are also Fisher type.

We use three data sets to demonstrate and evaluate our proposed method as well as compare its performance against benchmarks. Two datasets from Queensland, Australia are used to illustrate the estimation algorithm with data at two different frequencies (monthly and annual) and compare the value of the predicted land component to that made by the state valuer's office. The third dataset from a Dutch city has been used by Diewert et al. (2015) and Färe et al. (2015) to construct indices of land and structure components and thus it provides the opportunity to compare the constructed hedonic imputed price indices from our method to theirs.

2 The Valuer's Model

In most developed countries there is the need to have a valuation for tax purposes of either the property or the land. In the empirical application used in this paper, data from two urban areas from the state of Queensland (QLD) in Australia are used. The state government has a statutory authority (the Valuer-General of QLD) which issues land valuations for all rateable properties in the state. These are then used by the local government to set 'rates' or the state to set land taxes. The valuations are typically carried out by expert assessors who use market sales reports and local knowledge to provide an estimate of the value of the property/land within a given property market[2]. The valuer's task is to provide the tax authorities and the rate payers with a valuation of their property or land. We write a simple model for the expected value of the property,

$$E_t(V_t) = E_t(L_t, S_t | \sum_{j=0}^{\tau} w_{t-j} \, [\text{market sales: } Property, Land]_{t-j}) \tag{1}$$

where,

V_t is the value of the property

L_t is the land component of the value

S_t is the structure component of the property value

w_t is a weight such that $w_{t-1} > w_{t-2} > w_{t-3} > ...$

The model simply states that the valuer uses all available information up to the current period on market trends, sales in similar locations, sales of similar dwellings, and sales of land and properties, and that transactions in more recent periods are more heavily weighted than those that have occurred further in the past. When land valuation is required, the value, L_t, has to be disentangled from V_t. This task requires the use of additional information that can assist in separating the trends in the value of each of the goods in this bundle (i.e., land and structure) and the assumption that the evolution of these two trends can be differentiated. This assumption is justified using the arguments made in the previous section; for the structure, the value is given by its construction cost minus depreciation. Therefore, the value of a given structure is likely to be mostly dominated by the trend in wages and materials costs in the local market and the depreciation due to age. For the land, on the other hand, appreciation in a given market is often related to the limited supply of urban land which results in common shocks to land values. For instance, there could be influx

[2] The state government's website makes the following statement regarding valuations: "*When determining statutory land values, valuers from our department: examine trends and sales information for each land use category (e.g. residential, commercial, industrial and rural); inspect vacant or lightly improved properties that have recently been sold; consider the land's present use and zoning under the relevant planning scheme; take into account physical attributes and constraints on use of the land*" (`http://www.dnrm.qld.gov.au/property/valuations/about/considerations`).

of population into an area which would assert an upward pressure on the price of land. In addition, location plays a big role as urban areas grow and expand. For instance, new infrastructure (transport, bridges, train lines, etc) can result in big changes in the demand for urban land in certain locations. It is obvious that the identification of the land value can be assisted by observations of sales of vacant land; however, in heavily urbanised areas, land sales are unusual. Using construction costs to identify the value of the structure is an alternative; however, such a value would be an approximation, which would require some adjustment for depreciation due to age as well as other intrinsic characteristics of the structure if available. In other words, the valuers adjust prices based on hedonic characteristics. In practice, valuers perform hedonic adjustment based on experience and data and it is unlikely that an explicit econometric model is used, although this is likely to be changing as relevant data become more readily available in electronic form and more research on statistically sound prediction models becomes available. To contribute to the latter, our aim in the next subsection is to set up an econometric specification that models the valuer's tasks of obtaining the required decomposition. The approach is based on an estimator that is a weighted sum of past information and has well defined statistical properties.

3 The Econometric Approach to the Decomposition and Construction of Price Indices

The aim of a our decomposition approach is to have a general model specification so that it works with most datasets and is simple enough to be easily implemented. In addition, we wish to obtain estimates that will not be subjected to frequent revisions as new data are gathered, because the components' estimates are also the inputs to the construction of separate price indices for land and structure. We start by writing the observed sale price as a sum of three additive components. This follows previous studies (Bostic et al. (2009) and Diewert et al. (2015)) where three additive components are defined, land (L), structure (S)

and noise.

$$y_t = L_t + S_t + \epsilon_t \tag{2}$$

where,

y_t is a vector ($N_t \times 1$) with the sale price of each property (or vacant land) sold in period t. That is, in each t, N_t properties in the sample are sold.

L_t is a vector ($N_t \times 1$) where each row is the value of the land component for the i^{th} property sold in period t

S_t is a vector ($N_t \times 1$) where each row is the value of the structure component for the i^{th} property sold in period t, and $S_{it} = 0$ if the sale is for vacant land

$\epsilon_t \sim N(0, H_t)$

In order to capture the trends in L_t and S_t, we must specify some form for these two components. We define functions of each component's unique hedonic characteristics and some unknown parameters so that the resulting estimates capture the underlying smooth trends in L_t and S_t.

$$L_t = f(X_t^L, \alpha_t^L) \tag{3}$$

$$S_t = g(X_t^S, \alpha_t^S) \tag{4}$$

where,

X_t^L is an $N_t \times k_l$ matrix of hedonic characteristics intrinsic to the land component, e.g. size of the lot, location

X_t^S is an $N_t \times k_s$ matrix of hedonic characteristics intrinsic to the structure component, e.g. age, size of the structure

α_t^c are vectors of unknown parameters, $c = L, S$

A simple, but very flexible, form of $f()$ and $g()$ is to use a linear combination of characteristics and time-varying parameters,

$$L_t = X_t^L \alpha_t^L \tag{5}$$

$$S_t = X_t^S \alpha_t^S \tag{6}$$

We propose to achieve identification by specifying the law of motion of α_t^L and α_t^S as driven by uncorrelated components' specific discount factors in the covariance structure.

$$\begin{aligned} \alpha_t^L &= \alpha_{t-1}^L + \eta_t^L \\ \alpha_t^s &= \alpha_{t-1}^s + \eta_t^s \end{aligned} \tag{7}$$

where,

α_t^L is a vector of time-varying parameters associated with land characteristics

α_t^S is a vector of time-varying parameters associated with structure characteristics

$\eta_t^L \sim N(0, Q_t^L)$

$\eta_t^S \sim N(0, Q_t^S)$

$E(\eta_t \eta_t') = Q_t = \begin{bmatrix} Q_t^L & 0 \\ 0 & Q_t^S \end{bmatrix}$; with $\eta_t = \begin{bmatrix} \eta_t^L & \eta_t^S \end{bmatrix}'$

$\alpha_t = \begin{bmatrix} \alpha_t^L & \alpha_t^S \end{bmatrix}'$ $t = 1, \ldots, T$

$\alpha_0 \sim N(a_0, P_0)$ is the initial condition.

We do not specify Q_t directly, instead we use what is known as a dynamic discounting approach that models α_t and its mean square error matrix, P_t, with a modified form of the Kalman filter. The approach originates in the engineering and statistics literature. There are alternative decompositions of property prices depending on the final objective. Property prices are well known to have trends, while X_t^S and X_t^L are unlikely to be trending variables.

The conventional decomposition is into an overall price trend and a quality adjustment linear combination of hedonic characteristics and parameters. This feature has been conventionally captured through the use of time dummy variables as well as the use of stochastic trends (see Rambaldi and Fletcher (2014) for a review). This conventional decomposition of the price is well suited to predict the bundle (structure and land); however, it cannot provide information on the differential movements in the prices of land and structure. The specification chosen here implies the time-varying parameters provide a venue to capture the time evolution of the marginal value of hedonic characteristics (e.g. the shadow price of land close to a Central Business District might be trending upwards at a much faster rate than the market valuation of an extra bathroom). The specification should be viewed as a device to obtain a decomposition that can identify the trends in the two components of the price. To achieve this, the hedonic characteristics used must be uniquely attached to one component, and this could not be achieved by having an intercept or standalone trend component.

3.1 Estimation of Components' Parameters

Writing (2) using expressions (5) and (6), and defining $Z_t = \begin{bmatrix} X_t^L & X_t^S \end{bmatrix}$, the vector of observed prices is linked to the characteristics of those properties sold at time t, and the vector $\alpha_t = \begin{bmatrix} \alpha_t^L & \alpha_t^S \end{bmatrix}'$,

$$y_t = Z_t \alpha_t + \epsilon_t \tag{8}$$

$$\alpha_t = \alpha_{t-1} + \eta_t \tag{9}$$

To estimate α_t in equations (8), we use a modified version of the Kalman filter which allows identification of α_t^c, $c = L, S$ through a restriction on the dynamic evolution of α_t. To show how this is achieved, we present the standard form of the Kalman filter algorithm and then show how it is modified when discount factors are used (West and Harrison (1999), Ch. 6 provides a detailed introduction to the use of dynamic discount factors in a Bayesian setting). Although the use of discount factors has been limited to applications in the Bayesian literature, the limiting behaviour of the mean square error of the Kalman filter when using discounting has been studied by Triantafyllopoulos (2007) who shows the covariance of $a_{t|t}$ converges to a finite non singular covariance matrix that does not depend on P_0.

Denote the filter estimate of $\alpha_{t-1|t-1}$ by $a_{t-1|t-1}$. Given the assumptions about $\epsilon_t, \eta_t, \alpha_0$, we can write its distribution $a_{t-1|t-1} \sim N(\alpha_{t-1}, P_{t-1|t-1})$. Given the form of the transition equations (9), the prediction and updating equations in the standard Kalman filter are given by the expressions

$$\hat{y}_{t|t-1} = Z_t a_{t-1|t-1} \tag{10}$$

$$\begin{aligned} a_{t|t} &= a_{t-1|t-1} + K_t \nu_t \\ \nu_t &= y_t - Z_t a_{t|t-1} \\ K_t &= P_{t|t-1} Z_t' F_t^{-1} \\ P_{t|t} &= P_{t|t-1} - K_t F_t K_t' \\ F_t &= Z_t P_{t|t-1} Z_t' + H_t \end{aligned} \tag{11}$$

$$P_{t|t-1} = P_{t-1|t-1} + Q_t \tag{12}$$

The distribution of the one-step ahead prediction of y_t is $\hat{y}_{t|t-1} \sim N(y_{t|t-1}, F_t)$. To estimate the covariances Q_t and H_t in a classical estimation context, the log-likelihood function is written in prediction error form using ν_t, the one-step ahead prediction error, and its variance-covariance, F_t, which are obtained from the above Kalman filter recursions. The mean square error of $a_{t|t-1}$ (in this case $a_{t|t-1} = a_{t-1|t-1}$) is associated with $P_{t|t-1}$, and that of $a_{t-1|t-1}$ is $P_{t-1|t-1}$. However, $P_{t-1|t-1}$ is the mean square error of $a_{t|t-1}$ if there is no stochastic change in the state vectors from period $t-1$ to period t ($Q_t = 0$). This is the global model (converging state); however, locally the dynamics are better captured by $Q_t \neq 0$. In our case these are the dynamics that will allow us to extract the underlying trends in the components. The dynamic discounting literature specifies $P_{t|t-1}$ as a discounted $P_{t-1|t-1}$ by a proportion. As our model has two components, we specify the problem with two discount factors, each associated with one of the components. We write equation (12) as follows

$$P_{t|t-1} \;\; = \;\; diag\left\{\left(\delta_L^{-1} P_{t-1|t-1,[1:k_L,1:k_L]}\right), \left(\delta_S^{-1} P_{t-1|t-1,[(k_L+1):(k_L+k_S),(k_L+1):(k_L+k_S)]}\right)\right\} \qquad (13)$$

where $P_{t|t-1}$ is a partitioned diagonal matrix with sub-matrices $P_{[a,b]}$ corresponding to the land and structure components. Each partition is a function of a discount factor, $0 < \delta_L, \delta_S \leq 1$, and $P_{t-1|t-1}$ associated with the movements in the land or the structure components in the model in period $t-1$. It is easy to verify that the estimates of the state, $a_{t|t}$, for one component are still functions of both the land and the structure characteristics, X_t^L and X_t^S, since F_t is a function of Z_t.

Using (13) and (12), it is easy to derive an explicit form of Q_t which is equal to zero if the discount factors are equal to 1. The conventional approach was to set the discount factors to a value, see West and Harrison (1999), usually in the range between 0.85 and 0.99 for models with components such as trend, seasonal and cyclical. Recently Koop and Korobilis (2013), in the context of large time-varying parameter vector autoregressive models, proposed to estimate discount factors, which they refer to as *forgetting factors*, through a Bayesian model averaging approach. Here we combine a grid search and an evaluation of the likelihood function in an iterative procedure (the full algorithm is described in the next subsection).

Based on the arguments made in the literature, land and structure require different discount factors that obey the following restriction: $\delta_L < \delta_S < 1$. Urban land destined for residential housing is an appreciating scarce asset, and thus shocks in the market due to changes in population trends as well as overall macroeconomic conditions are expected to drive the trend in land prices. The trend in the value of the structure is expected to be in line with movements in construction costs, which are likely to closely follow changes in wages and more generally CPI, as well as a more homogeneous evolution across markets (such as major urban cities within a state or country). Individual structures will be subjected to depreciation which would be a function of the age of the structure. This point is important as it provides a strong argument for the need to include a measure of the age of the structure in X_t^S.

Denoting by a_t the estimate of α_t and by P_t the mean squared error matrix of a_t, conditional on all information up to period t, the Kalman filter is a form of nowcasting and provides estimates such that a_t is a weighted sum of current and past information,

$$a_t = \sum_{j=1}^{t} w_j(a_{t|t}) y_j \tag{14}$$

Explicit forms of the weights, $w_j(a_{t|t})$, have been derived by Koopman and Harvey (2003) based on the recursions of the Kalman filter. This work, as well as that presented in Harvey and Koopman (2000), show the relationship between implied weighting patterns obtained using the recursions of the Kalman filter, as used in this study, and those obtained using kernels in the non-parametric literature. The weights are functions of current and past values of the one-step ahead prediction error $\nu_t = y_t - Z_t a_{t|t-1}$ and its mean squared error matrix, F_t, which is a function of present and past values of H_t and $P_{t|t-1}$ in our modified filter. The type of pattern of $w_j(a_{t|t})$ is such that, at any point t, a_t is a function of current sales data and a decreasing function of past sale prices, with weight, $w_t(a_{t|t})$, determined by current and past X_t^c $(c = L, S)$, discount factors and H_t. Assuming X_t^L and X_t^S capture the characteristics of the land (including location) and the structure, it is reasonable to specify the distribution of the noise, ϵ_t, as spherical. Therefore, $H_t = \sigma_\epsilon^2 I_{N_t}$ with N_t the number of transactions in period t.

3.2 Estimation of covariance parameters

All parameters associated with the functions (5) and (6), are in the state vector; thus, once values of σ_ϵ^2, δ_L, and δ_S are available, it is clear from the recursions of the Kalman filter that the estimates of the state, α_t, and its mean square error matrix, P_t, can be computed. To estimate $\psi = [\sigma^2_\epsilon, \delta_L, \delta_S]$ we combine a grid search and an evaluation of the likelihood function in an iterative procedure. We first obtain an initial estimate of σ^2_ϵ by running a conventional regression with intercept time dummies. The algorithm is as follows:

Estimation Algorithm:

Initial Estimates of ψ, $\hat{\psi}$: For σ^2_ϵ we run least squares on a model with fixed time dummies and all regressors, X^S and X^L, and compute $\hat{\sigma}^2_\epsilon$ from the OLS residuals. Given this estimate, we run the Kalman filter equations over a grid of pairs of values $[\delta_L,\ \delta_s]$ and compute the squared correlation between the actual price and the predicted price of each property (constructed as the sum of the predicted land and structure components). This is a general R^2 measure. The pair of discount factors that provides the highest generalised R^2 is taken as the initial estimates, $[\hat{\delta}_L, \hat{\delta}_S]$.

Maximum Likelihood estimate of σ^2_ϵ: Given $[\hat{\delta}_L, \hat{\delta}_S]$, a Newton-Raphson algorithm is used to find the value, $\tilde{\sigma}^2_\epsilon$, that maximises the log-likelihood function, using $\hat{\sigma}^2_\epsilon$ as initial value.

Final Estimates of δ_L and δ_s: Given $\tilde{\sigma}^2_\epsilon$, we run the Kalman filter equations over a grid of pairs of values $[\delta_L,\ \delta_s]$ again and compute the squared correlation between the actual price and the predicted price of each property. The value pair that provides the highest squared correlation are the final estimates $\tilde{\delta}_L$ and $\tilde{\delta}_S$.

The final estimates $\tilde{\sigma}^2_\epsilon$, $\tilde{\delta}_L$ and $\tilde{\delta}_S$ are used in the filtering equations (10-13) to obtain the estimates of α^S_t and α^L_t which are used to compute predicted prices of the components for each property and time period using equations (5 and 6) with α^c_t replaced by $a^c_{t|t}$, $c = L, S$.

If the dataset contains a reasonable number of transactions of land only sales, we compute both an overall correlation between each sale and the model's prediction, but also compute the generalised R^2 to assess the accuracy of the model's predictions of sales of land only.

The set of discount factors found to give the maximum generalised R^2 for land sales is used. The algorithm, coded in Matlab, is fully automated and runs in seconds for the datasets used in this study. The grid search for the discount factor pairs, δ_L δ_S, covers 19 possible combinations in the ranges $d_L = 0.7 - 0.95$ and $d_S = 0.8 - 0.99$, and so that $d_L < d_S$ to reflect the underlying restriction, $\delta_L < \delta_S < 1$.

In Section 4.1 we present an empirical implementation of the modelling and estimation using two datasets. These two datasets represent two different urban settings and two time-series frequencies. The first is an expanding urban area (monthly observations), while the second is an old and well established suburb very close to the city's central business district (annual observations).

3.3 Hedonic Imputed Price Indices

Diewert et al. (2015) construct Fisher type indices for the price of the land component. The component is identified econometrically through the use of exogenous information in the form of a construction price index produced by a statistical agency. Färe et al. (2015) construct Fisher type indices for the land and the structure. Their identification of the land component also uses the exogenous construction price index. To identify a construction price index, they use their constructed land price index as the exogenous instrument.

We compute Fisher price indices for the land and structure components estimated by our approach. Two alternative Fisher indices are computed by using a different weighting scheme, they are labeled Fisher Plutocratic (FP) and Fisher Democratic (FD), respectively. The plutocratic version uses weights that are functions of value shares, and it is defined as:

$$F^P_{(t-s),t} = \sqrt{L^P_{(t-s),t}\, P^P_{(t-s),t}} \tag{15}$$

where $L^p_{t-s,t}$ and $P^p_{t-s,t}$ are respectively the Laspeyres and Paasche index numbers,

For the land component the indices are given by

$$L^p_{(t-s),t} \;=\; \sum_{h=1}^{N_s} w^h_{(t-s)} \left(\frac{\hat{P}^L_t(x^L_{(t-s)})}{\hat{P}^L_{(t-s)}(x^L_{(t-s)})} \right); \quad P^p_{(t-s),t} \;=\; \left[\sum_{h=1}^{N_t} w^h_t \left(\frac{\hat{P}^L_{(t-s)}(x^L_t)}{\hat{P}^L_t(x^L_t)} \right) \right]^{-1}$$

where,

$\hat{P}^L_{(t-s)}(x^j_t)$ for $s \geq 0$ is a prediction of the value of the land component of property h, sold at time t with characteristics X^j_t (j = Land, Structure) using a vector of shadow prices for time period $t-s$ and the value shares defined as in (16).

For time t, the weight of property h sold in that period, w^h_t, is given by:

$$w^h_t = \frac{P^h_t}{\sum_{n=1}^{N_t} P^n_t} \tag{16}$$

where,

P^h_t is the observed sale price of property/land h and N_t is the number of sales in period t. Similarly, using the prediction of the structure component of property h, $\hat{P}^S_{(t-s)}(x^j_t)$ the structure price index is computed.

The Fisher democratic version of the index, $F^D_{(t-s),t}$, is computed by replacing the weights, $w^h_t(w^h_{t-s})$, by $N^{-1}_t(N^{-1}_{t-s})$, where $N_t(N_{t-s})$ are the number of observations at time $t(t-s)$. In this case each transaction within a given time period is given an equal weight.

Section 4.2 presents monthly hedonic imputed Fisher indices using data from a Dutch city. The dataset has been labelled "Town of A" in previous studies (European Comission et al. (2013), Diewert et al. (2011, 2015) and Färe et al. (2015)). These are compared to the quarterly price indices produced by DdH and FGSS.

4 Empirical Implementation

In this section our estimation approach, the land/structure decomposition, and the construction of price indices for the components are illustrated using our three datasets. Each set provides an opportunity to evaluate our method against comparative benchmarks. The two datasets from Queensland, Australia are used to show the workings of the estimation algorithm with data at two different frequencies (monthly and annual), and compare the value of the predicted land component to that made by the state valuer's office. The discussion is in Section 4.1. The third dataset is that used by Diewert et al. (2015) and Färe et al. (2015) and thus it provides the opportunity to compare the constructed hedonic imputed price indices from this method to theirs. The discussion is presented in Section 4.2.

4.1 The Decomposition into Land and Structure

In this section we illustrate the model estimation and decomposition with two datasets. The Bay side data contains monthly observations from an urban area north of the city of Brisbane. What is sometimes labelled as the "greater Brisbane area" has expanded rapidly as the South East of the state of Queensland (SEQ) underwent a large influx of population in the last two to three decades. These data are from Fletcher et al. (2011). The BrisSub dataset is for a single suburb in the inner city of Brisbane. The suburb is within five km of the central business district (CBD) and one of the oldest and most well established in the city. These data are annual and have been sourced from Rambaldi et al. (2013). Descriptive statistics for these datasets appear in Appendix A.

4.1.1 Monthly Data - Bay Side

The data area is composed of several urban centres surrounding the Bay which act as satellite suburbs to Brisbane. A substantial proportion of the population travel into Brisbane every day to reach their employment location. The market in the Brisbane and "greater Brisbane" region went through a boom period between 2001 and 2005 and this is reflected in the number of transactions per month for that period. The market was more stable between 2005 and 2008. Figure 1 (top panel) shows the number and type of transactions per month over the sample period which covers 1991:5 to 2010:9 (233 months). There are transactions of properties (land and structure) as well as land only. The 2008 global financial crisis is noticeable in that the number of transactions drops to levels similar to the early 1990s and remained volatile for the rest of the sample. It is widely accepted that the influx of population into the region, together with a lag in the release of new urban land, drove land prices up substantially. Figure 2 shows the median price of land sales. In this data set there are a total of 13088 transactions, 9774 are of properties and the rest are land. The sale of land in this area is due to urban sprawling in the area known as 'South East Queensland' (SEQ). Having observations of both types of transactions is unusual, and thus this data set provides a unique opportunity to test the method. In addition, a large number of hedonic characteristics of the structure and land are available for each transaction. The available data contain sale price, land characteristics (size and location) and structure characteristics (size, age, number of bedrooms and bathrooms), see Table 3 in Appendix A. In the notation of equations (5) and (6), these characteristics fit into X_t^L or X_t^S. Land area, structure footprint and age entered as a quadratic function to capture the wide range of the data[3].

The estimates of the land and structure components, $\hat{L}_t$ and $\hat{S}_t$, are converted to proportions for presentation. These estimates are presented in Figure 1 (bottom panel), and Table 1 presents the estimates of the discount factors and the overall noise variance (σ_ϵ^2) at each step of the algorithm. We note that the initial estimate of σ_ϵ^2, which is obtained from a time-dummy regression, is very large compared to the maximum likelihood estimate of the time-varying components model. This is a good indication that the model has been able to capture most of the dynamics of the land and structure components.

The left bottom panel in Figure 1 shows the land value as a proportion of the property transaction sale value. The proportion was around 60% over the period 2002 - 2007. The global financial crises downturn is visible, and the proportion due to land is estimated a bit lower for the last part of the sample. Looking at Figure 1's top panel, it is clear that sales of land only dried up over the period of late 2007 and 2008, and the recovery was slow over the 2009-2010 period.

Concentrating on the sales of properties (i.e. ignoring land only transactions, bottom

[3]Plots of some of the coefficient estimates are available from the conference paper version, the rest are available upon request

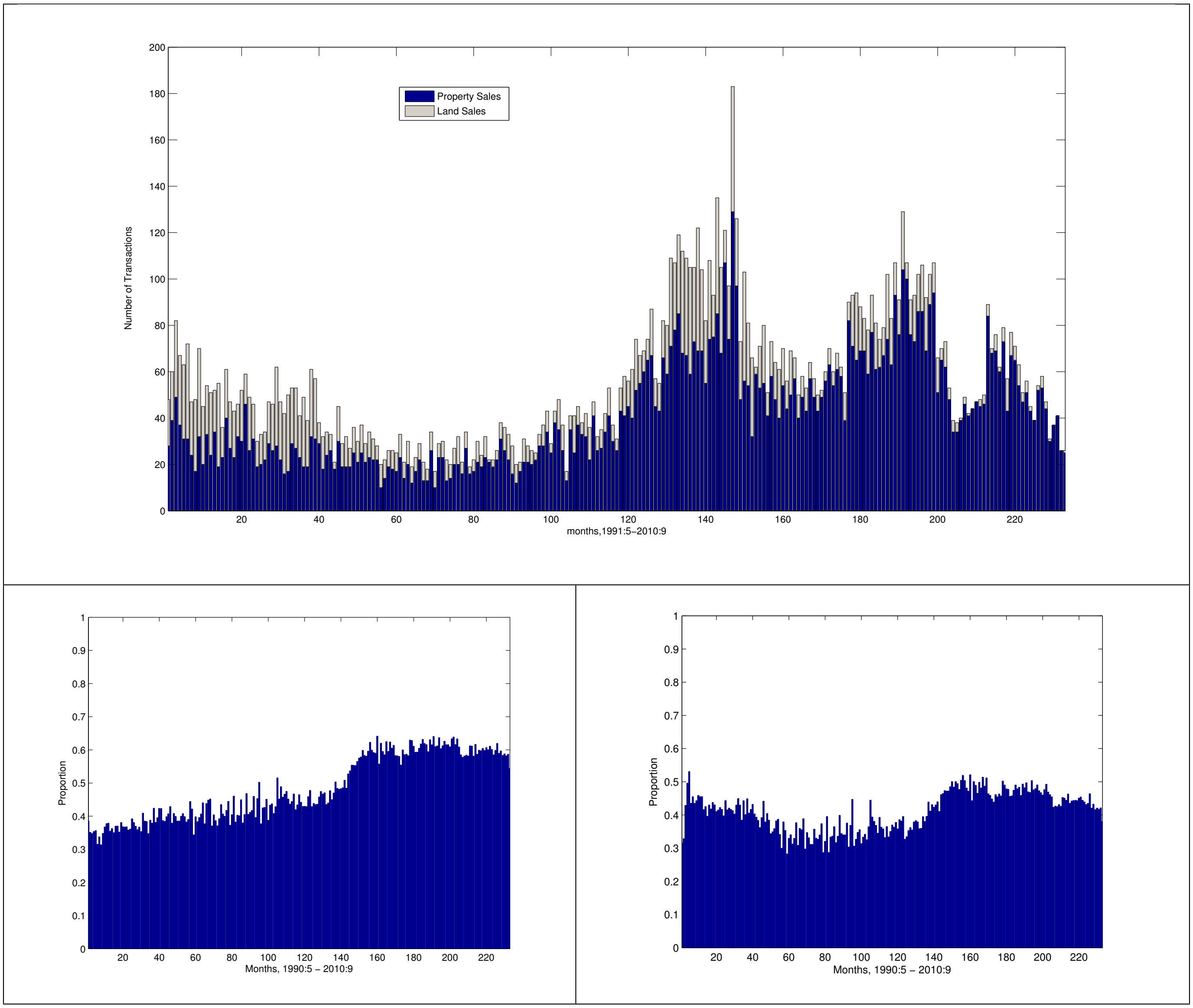


Figure 1: Bay side - Number of (monthly) Transactions (top); Median of Estimated Proportion of Land Component - all sales used in the estimation (bottom left) and Vacant land sales ignored in the estimation (bottom right)

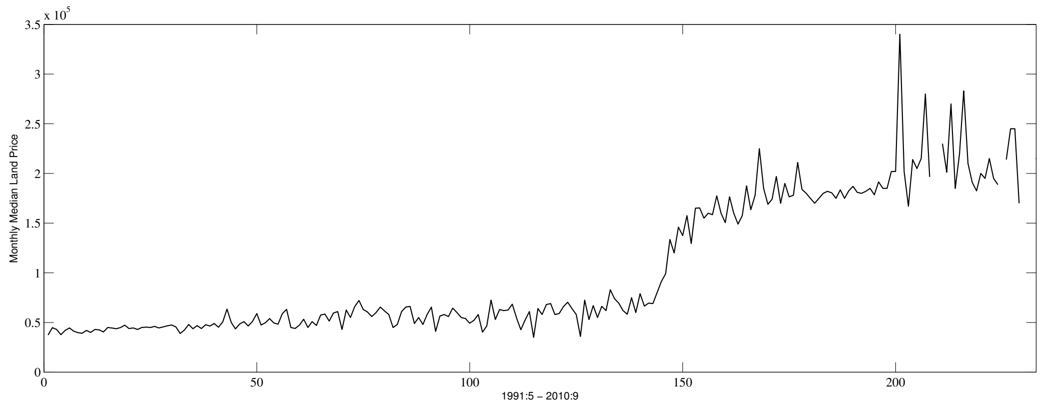


Figure 2: Bay side -Median of (monthly) Observed Vacant Land Prices

right in Figure 1), indicates that the value of the land component for existing properties in this area was around 45-50% over the 00's decade and after the global financial crisis the land component accounted for between 40% and 45%. This would seem reasonable for an area that was expanding and where property would not be sold for the purpose of replacing a single detached dwelling by townhouses or terraces, as is the case of the suburb in inner Brisbane studied in the next section.

Table 1: Estimates of $\psi = \sigma^2_\epsilon, \delta_L, \delta_S$ - Bayside Data

	σ^2_ϵ	δ_L	δ_S	Highest Gen R^2 Land	Highest Gen R^2 All
Model:	Property and Land Sales ($N = 13088$)				
Initial Estimates	2.61e+03	0.7[a]	0.95[a]	0.768[b]	0.866[c]
Final Estimates (MLE/Highest Gen R^2)	46.734	0.7[a]	0.95[a]	0.768[b]	0.866[c]
Model:	Property Sales ($N = 9774$)				
Initial OLS/Highest R^2	3.07e+03	0.7	0.90		0.857
Final Estimates (MLE/Generalised R^2)	233.17	0.7	0.90		0.857

[a]Chosen using Gen R^2 from Land transactions; [b]this value is equal for $d_L, d_S = [0.7, 0.95]$ and $[0.7, 0.99]$; [c]this value is for $d_L, d_S = [0.7, 0.90]$

4.1.2 Annual Data - BrisSub

The suburb (neighbourhood) is in the inner city of Brisbane. The dataset covers 41 years from 1970 to 2010[4] (descriptive statistics are presented in Table 4 in Appendix A); however, the number of transactions is very small and dispersed over the time period covered compared to the previous dataset. For this reason the estimation frequency is annual. Nevertheless, our method performs quite well when asked to predict the proportion of the sale value due to the land component. We provide the comparison of the model's predictions of land values to those made by the state valuer's office in the year 2009.

There are very few transactions of land only (about 2% of transactions) and they appear in the latter part of the sample (Figure 3 top panel). This is consistent with sales of blocks that would have housed a single dwelling (smaller and with a backyard) which are then developed into townhouses or terrace type structures as the area undergoes a process of infill due to its location and the substantial growth of the city of Brisbane.

The estimates of the covariance discount factor parameters are similar to those obtained for the Bay side data set with $\hat{\delta}_L = 0.7$ and $\hat{\delta}_S = 0.9$. This is expected as both sub-markets are part of SEQ, which has experienced massive increases in the property prices, especially in the early to middle part of the 00's. Analysts and commentators agree the massive increase is in land prices.

Figure 3 (bottom panel) shows the estimated land proportion of the property value. The first few years are expectedly hard to estimate as the number of transactions is very small (top panel). However, the components estimator seems to gain traction relatively quickly and provide a very reasonable trend from period ten (1980) onwards. The bottom panel plot is based on all transactions and the estimates would indicate the proportion of the value of the properties in this suburb in the decade 2001-2010 has been at a median of 55% to 70%.

Table 2 shows the Valuer's estimated land value for the year 2009 as a ratio to the property sale price for those properties in our dataset sold in 2009. The model estimates that the land

[4] 2010 data only covers the first half of the year

value was 67% (median) of the property sale value for that year. The valuer's median for those properties is 72%. The land valuation provided to the property owner by the state valuer's is based on a three year moving average. This is viewed as a way of decreasing volatility. Our method is also weighted average of past values; however, the past information is not equally weighted as in a simple moving average. One shortcoming of an equally weighting method is that when the market takes a turn in either direction (e.g. the global financial crisis, the Brisbane floods), the land valuations' adjustment to these changes will be lagged. This was observed by rate payers after the 2011 Brisbane flood episode.

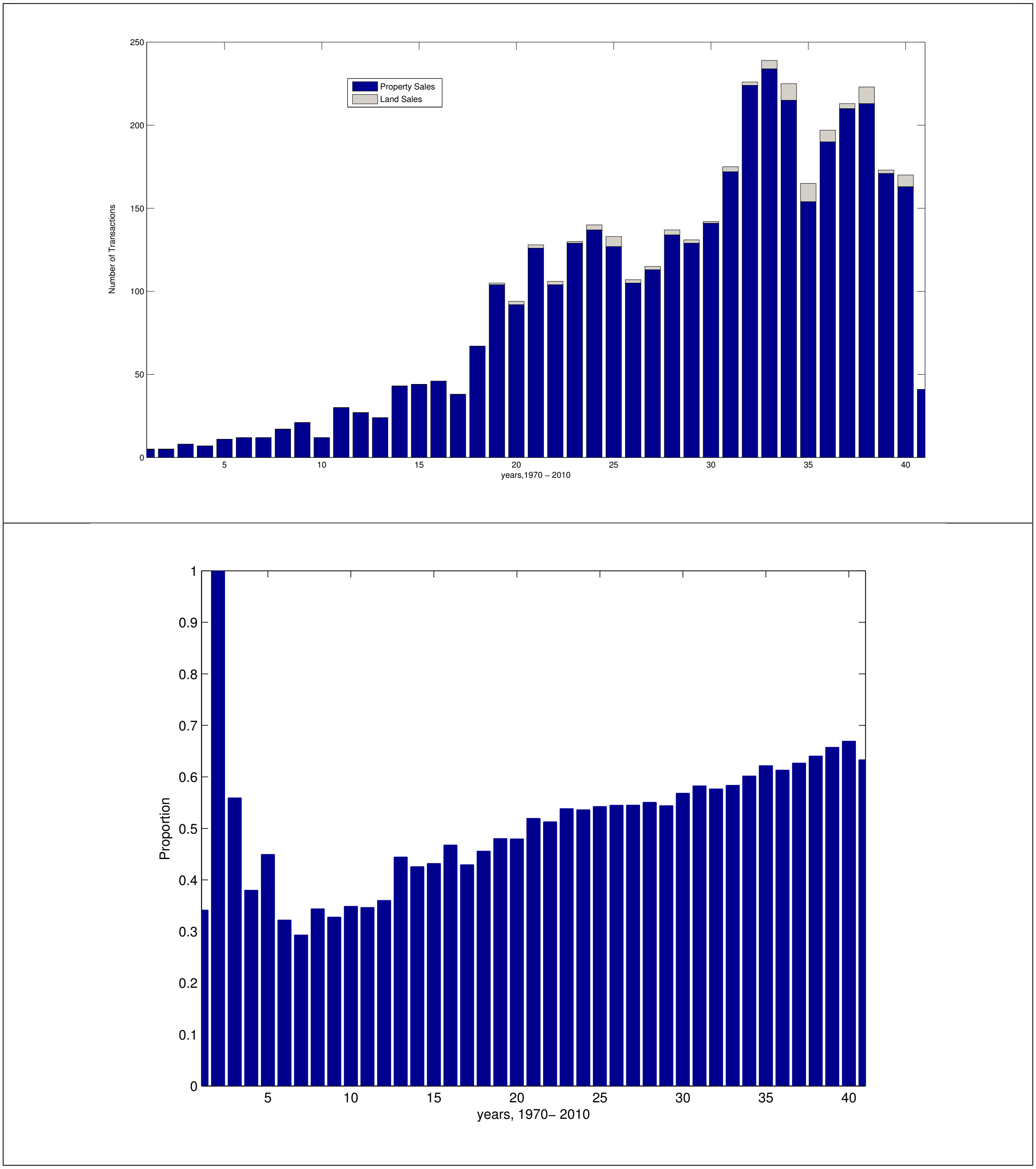


Figure 3: Brisbane Inner City - Number of (annual) Transactions (top); Median of Estimated Proportion of Land Component (bottom)

Table 2: Comparison of Land Price Valuation. State Valuer vs. Model

MonthSold	Valuer's Estimated Land Proportion of the actual sale price	# Properties
Jan-09	0.721	13
Feb-09	0.704	11
Mar-09	0.762	16
Apr-09	0.741	17
May-09	0.746	16
Jun-09	0.675	9
Jul-09	0.738	11
Aug-09	0.673	13
Sep-09	0.734	14
Oct-09	0.617	19
Nov-09	0.683	12
Dec-09	0.716	15
Valuer's Median 2009	0.716	166
Model (Median for 2009)	0.669	166

4.2 Price Indices for Land and Structure

In this section we construct price indices for land and structure using the "Town of A" dataset kindly provided by W.E. Diewert (used in Diewert et al. (2015)). A shorter and earlier version of this dataset was used by Färe et al. (2015). We clean the data following the detailed code by Diewert. The original dataset had 3543 transactions, and 3487 were kept after the cleaning process. Table 5 in Appendix Aprovides the descriptive statistics of the working sample.

We label the indices FP and FD for Fisher plutocratic and Fisher democratic, respectively (see Section 3.3). Throughout this section, we use a postfix "_L" to denote the index relating to land (e.g. FP_L), and a postfix "_S" to denote structure (e.g FP_S). In this dataset land area is the only characteristic describing land values; the structure is captured by dwelling size, age, number of floors, number of floors^2, number of rooms and number of rooms^2. The estimated discount factors for this dataset are $\hat{\delta}_L = 0.8$ and $\hat{\delta}_S = 0.9$. A plot of the estimated land proportion is presented in Appendix B.

We compare our indices to those computed by Diewert et al. (2015) and Färe et al. (2015). Diewert et al. (2015) compute Fisher indices imputed from hedonic models that are non-linear in the parameters (the economic modelling framework is labelled the Builder's Model). To assist with the identification, exogenous information in the form of a construction price index, NDOPI (New Dwelling Output Price Index, published by the Central Bureau of Statistics of Netherlands), is used as an instrument to estimate the land component. A model that uses linear splines on lotsize, but no other hedonic information, is used to compute the indices labelled DdH_P3. By adding additional characteristics to the model, such as rooms, the version labelled DdH_P4 is produced. These are computed for a quarterly frequency.

Färe et al. (2015) adopt the same approach to the modelling. They refer to this as the DS approach, derived from the work in Diewert and Shimizu (2013). In their paper the index DS2 uses piecewise linear functions of the characteristics together with the NDOPI as exogenous information. This index should be close to DdH_P4, although they were using a shorter and earlier version of the 'Town of A' dataset. Färe et al. (2015)'s contribution is to estimate the models using a distance function approach. Their models are estimated by Corrected OLS and using a Stochastic Frontier approach (SFA). Färe et al. (2015) compute Fisher indices from their SFA estimates (we label these FGSS_SFA). The index computed from their SFA approach is then used by Färe et al. (2015) as the exogenous land price index to identify the structure component in what they label DS2. We label their "Diewert and Shimizu (2013); Diewert et al. (2015)" derived indices as FGSS_DS2. Their indices are also computed for a quarterly frequency.

Figure 4 presents plots (for land and structure) showing our indices (computed monthly) as well. Tables 6 and 7 in Appendix C presents the data behind these plots. The appendix tables also show computed Fisher price indices for the property (labelled FP_p and FD_p). These are constructed by adding the land and structure predicted prices to obtain the property's price and then using it as the imputed price in the Fisher's indices formulae. Finally a median index constructed by computing the monthly median price of observed transactions and setting Feb05 =1 is also presented for reference. The DdH and FGSS indices are based on 2005:Q1=1. For the purpose of presentation the quarterly estimates have been aligned with the central monthly indices (e.g. Q1 is aligned with February, Q2 is aligned with May, Q3 is aligned with August and Q4 is aligned with November).

The DdH and FGSS_DS indices are estimated period by period which explains their more volatile nature. The FGSS_SFA index is much less volatile as the underlying econometric model is a fixed parameter model estimated with the data for the complete sample. The problem with this approach to estimation is that in practice the transactions that occurred in 2005, for example, had no information of the 2007 market; however, the model uses the 2007 information when estimating the parameters that will be then used to compute the index. Our approach has time-varying parameters that update as new information becomes available. The estimator is conditional on past and current information on the market and therefore improves on DhH by using weighted past and current information, and on FGSS as it does not use future information to estimate the parameters of the hedonic function. As a result, our estimates are less volatile than DdH, but are able to pick up the turns in market conditions due to the inherited learning nature of the Kalman filter estimator.

DdH did not compute a separate structure price index from the data, and thus DdH_P3 and DdH_P4 are identical to each other, and equal to NDOPI (their instrument). FGSS_DS2_S is computed by estimating the Builder's model with FGSS_DS2_L as the exogenous land price index (that is, as the instrument to identify the structure). Our indices, FD_S and FP_S, are based on the estimated structure component, $\hat{S}_t$, and thus no other exogenous information has been used in their computation. Further evidence of the effectiveness of our proposed approach is provided by the fact that our structure indices are very close to the NDOPI (bottom panel Figure 4). Our index is not computed from construction costs of new dwellings, instead it is using the sample information on the characteristics of the individual dwellings and controlling for depreciation due to their age.

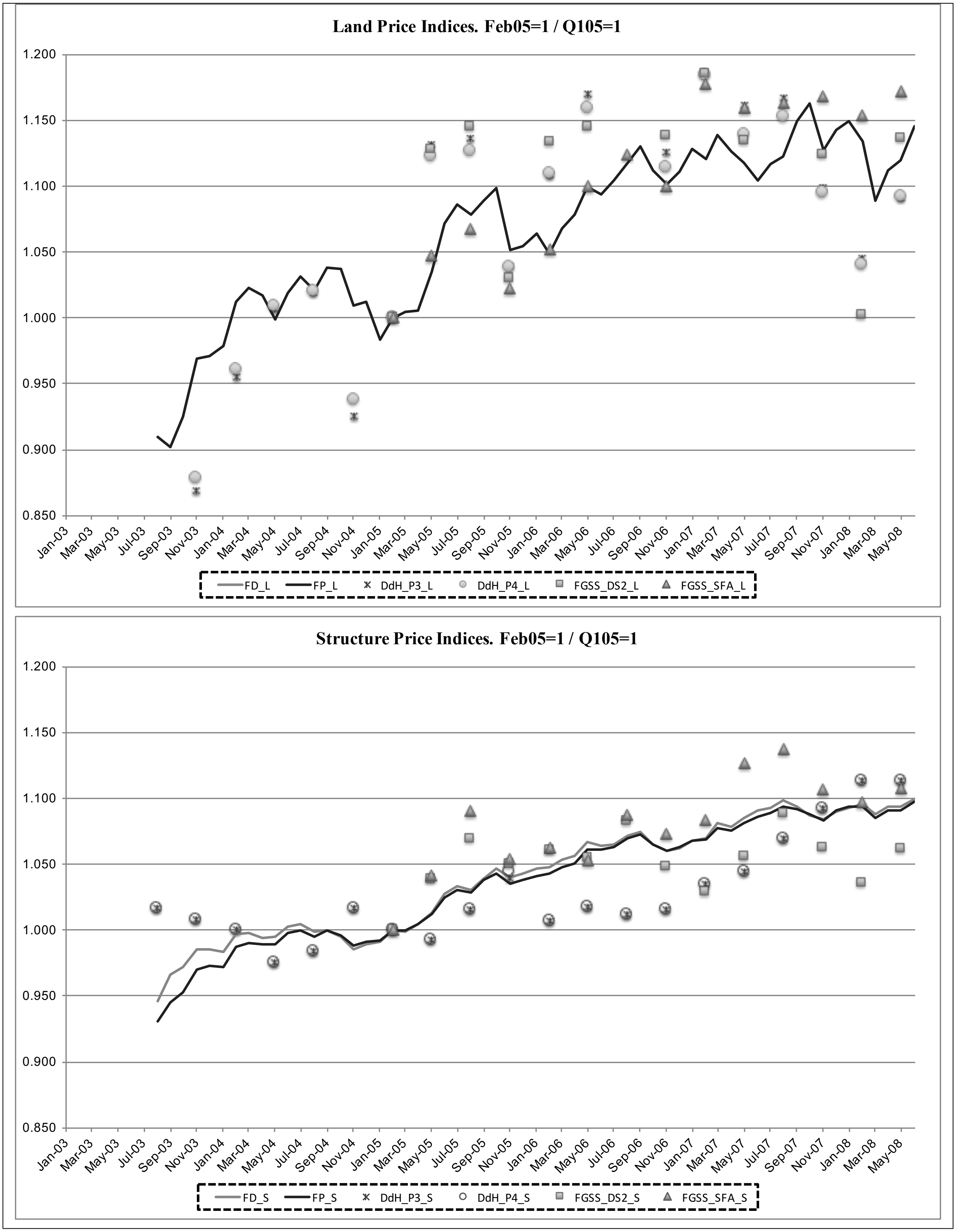


Figure 4: Dutch "Town of A" - Comparison of Computed Monthly Indices to DdH and FGSS Quarterly Alternatives (Data in Tables 6 and 7 (Appendix C))

5 Conclusion

We have proposed a new approach to decoupling property prices into land and structure and have applied it to three widely different datasets. The approach is generalisable, applicable to most datasets and simple to implement. We show our approach uses only prior sales information, and has the flexibility to deal with varying data frequency. We have demonstrated that it produces predictions of the land component of property values that are comparable to those provided by the state valuer's office. The imputed price indices for the land component we generate are smoother than those produced by alternative methods. Importantly, our computed indices do not rely on exogenous non-market price information on construction costs in order to identify the land component of the price.

Developing better approaches for decoupling property prices into land and structure is both important, and increasingly feasible. Both fair land taxation and economically sensible urban development projects need to be informed by an understanding of what proportion of the asset base is an appreciating asset (i.e. land), and what proportion is a depreciating asset (i.e. house structure). Issues like climate change are raising the profile and importance of making sound adaptation decisions. At the same time, electronic data on housing characteristics are becoming more readily available, making approaches such as ours more accessible to regular valuation.

Our paper starts by providing a simple underlying economic model, labelled the valuer's problem, whose task is to provide a valuation of either the property or the land. We then propose an econometric representation of the problem. The identification of land and structure as two components is achieved through the use of dynamic discounting factors in a modified form of the Kalman filter. Using well established behavioural patterns in the prices of urban land and construction costs, we can impose econometrically a different dynamic behaviour in each of these components. We adopt a classical approach to the estimation, which entails running the Kalman filter to evaluate the log-likelihood of the unknown covariance parameters. The outcome of our paper is a generalisable approach that we hope responds to emerging needs and data availability.

Appendices

A Descriptive Statistics

Table 3: Descriptive Statistics Bay side

	Min	Max	Mean	Median	St.Dev
Sale Price (in 1000)	15.50	1250.00	191.77	161.50	129.44
Land Characteristics					
Flood Plain Dummy	0.00	1.00	0.06	0.00	0.23
Large Plot Dummy	0.00	1.00	0.09	0.00	0.28
Land area (hectarea)	0.03	1.06	0.10	0.06	0.11
dist_coast (Km)	0.02	5.78	1.39	1.38	0.93
dist_waterway (Km)	0.01	0.86	0.27	0.25	0.16
dist_OffenIndus (Km)	0.18	8.38	2.87	2.27	1.83
dist_parks (Km)	0.01	0.98	0.13	0.11	0.11
dist_busStop (Km)	0.02	4.35	0.47	0.22	0.80
dist_Schools (Km)	0.01	6.55	0.65	0.32	1.09
dist_Shops (Km)	0.01	4.80	0.53	0.40	0.49
dist_BoatRamp (Km)	0.06	6.29	1.97	1.60	1.46
dist_PubsClubs (Km)	0.01	5.34	1.44	1.24	0.93
dist_Hospitals (Km)	0.01	13.26	3.20	2.26	2.71
Structure Characteristics					
Structure=1	0.00	1.00	0.75	1.00	0.43
Age (years)	0.00	86.00	11.94	10.00	11.44
Structure Footprint (hectare)	0.00	0.09	0.02	0.02	0.01
Number of Bathrooms	0.00	4.00	1.06	1.00	0.78
Number of Bedrooms	0.00	8.00	2.52	3.00	1.58
Number of Parking Spaces	0.00	5.00	1.39	1.00	1.12
Total number of Transactions	13088				
Number of Months	233	(1991:5	2010:9)		

Table 4: Descriptive Statistics BrisSub Data

	Min	Max	Mean	Median	St.Dev
Sale Price (in 1000)	2.60	4710.00	305.22	215.00	269.48
Land Characteristics					
Flood Dummy	0.00	1.00	0.04	0.00	0.20
Land area (hectareas)	0.02	0.22	0.06	0.06	0.02
dist_waterway (Km)	0.01	1.62	0.57	0.53	0.38
dist_river (Km)	0.95	4.77	2.97	3.04	0.87
dist_industry (Km)	0.00	2.62	1.00	0.91	0.66
dist_park (Km)	0.01	0.56	0.18	0.16	0.12
dist_bikeway (Km)	0.01	1.51	0.57	0.56	0.35
dist_busstop (Km)	0.01	0.50	0.20	0.18	0.11
dist_TrainStn (Km)	0.01	3.17	1.38	1.40	0.82
dist_school (Km)	0.04	1.23	0.47	0.45	0.24
dist_shops (Km)	0.00	1.09	0.36	0.33	0.19
dist_CBD (Km)	2.46	5.77	3.97	3.97	0.82
Structure Characteristics					
Pre-War	0.00	1.00	0.49	0.00	0.50
War/Post War	0.00	1.00	0.37	0.00	0.48
Late 20th C	0.00	1.00	0.07	0.00	0.26
Contemporaneous	0.00	1.00	0.04	0.00	0.20
Structure=1	0.00	1.00	0.98	1.00	0.15
Structure footprint	0.00	0.10	0.02	0.02	0.01
Number of Levels	0.00	4.00	1.10	1.00	0.36
Number of Bathrooms	0.00	6.00	1.37	1.00	0.67
Number of Bedrooms	0.00	8.00	3.04	3.00	0.91
Number of Parking Spaces	0.00	8.00	1.66	2.00	0.78
Total number of Transactions	3944				
Number of Years	41	(1970	2010)		

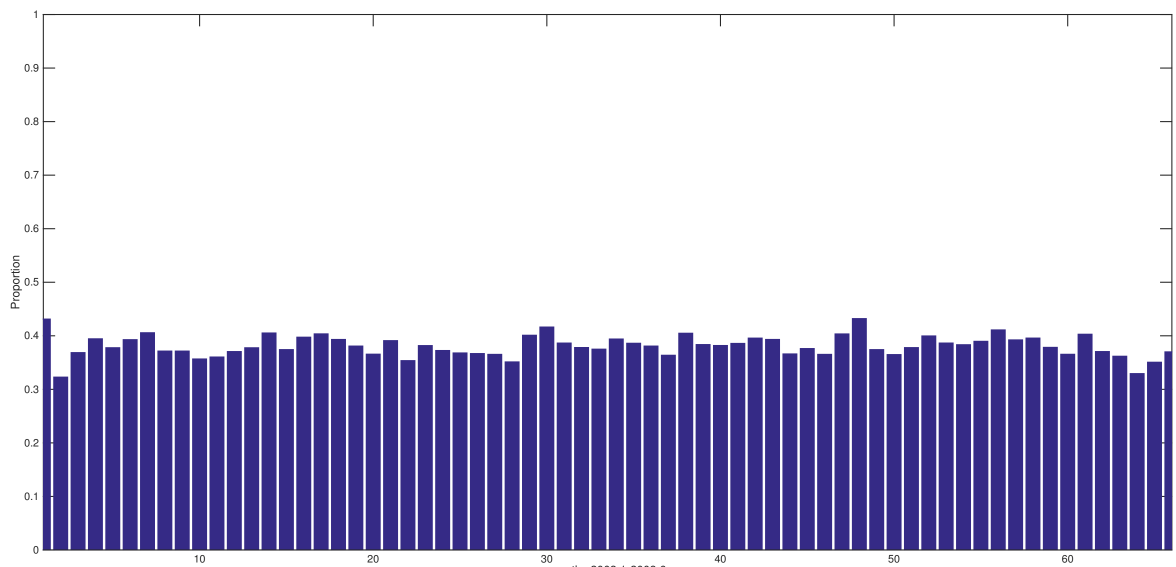
Table 5: Town of A Descriptive Statistics

	MIN	MAX	MEAN	MEDIAN	STDEV
PRICE (000 EUROS)	70	550	182.260	160	71.316
Land Characteristics					
LAND (SQ MTS)	70	1344	258.060	217	152.310
Structure Characteristics					
HOUSE (SQ MTS)	65	352	126.560	120	29.841
AGE (YEARS)	0	4	1.895	2	1.231
FLOORS	1	6	2.878	3	0.478
ROOMS	2	10	4.730	5	0.874
Number of Transactions	3487				
Number of months	66	(2003:1	2008:6)		

The data were cleaned following Diewert et al (2015). See footnotes 11,12,13.

We are indebted to W.E. Diewert for providing the data and cleaning code

B Land Component Proportion of the Price (monthly) - Dutch "Town of A"



C Price Indices for the Town of A

Table 6: "Town of A" Land Price Indices

Month	FD_p	FP_p	Median_I	FP	FD	Quarter	DdH P3	DdH P4	FGSS DS2	FGSS SFA
Aug-03	0.932	0.923	0.927	0.910	0.910	03:Q3	0.826	0.837		
Sep-03	0.942	0.930	1.096	0.902	0.902					
Oct-03	0.954	0.943	0.937	0.925	0.925					
Nov-03	0.979	0.970	0.904	0.969	0.969	03:Q4	0.869	0.879		
Dec-03	0.979	0.972	0.955	0.971	0.971					
Jan-04	0.981	0.975	1.013	0.979	0.979					
Feb-04	1.002	0.997	0.997	1.012	1.012	04:Q1	0.956	0.961		
Mar-04	1.007	1.003	0.915	1.023	1.023					
Apr-04	1.002	0.999	1.125	1.017	1.017					
May-04	0.996	0.993	1.046	0.998	0.998	04:Q2	1.007	1.009		
Jun-04	1.008	1.006	0.977	1.019	1.019					
Jul-04	1.014	1.012	0.952	1.031	1.031					
Aug-04	1.006	1.005	0.924	1.021	1.021	04:Q3	1.019	1.020		
Sep-04	1.014	1.014	1.043	1.038	1.038					
Oct-04	1.011	1.012	0.963	1.038	1.038					
Nov-04	0.995	0.997	0.963	1.010	1.010	04:Q4	0.925	0.938		
Dec-04	0.998	1.000	0.993	1.013	1.013					
Jan-05	0.988	0.988	0.940	0.983	0.983					
Feb-05	1.000	1.000	1.000	1.000	1.000	05:Q1	1.000	1.000	1.000	1.000
Mar-05	1.001	1.002	1.080	1.004	1.004					
Apr-05	1.005	1.004	1.030	1.005	1.005					
May-05	1.021	1.021	1.110	1.035	1.035	05:Q2	1.131	1.123	1.128	1.048
Jun-05	1.043	1.042	1.120	1.072	1.072					

Table 6: "Town of A" Land Price Indices

Month	FD_p	FP_p	Median_I	FP	FD	Quarter	DdH P3	DdH P4	FGSS DS2	FGSS SFA
Jul-05	1.051	1.050	1.080	1.086	1.086					
Aug-05	1.048	1.047	1.023	1.079	1.079	05:Q3	1.136	1.127	1.145	1.068
Sep-05	1.057	1.056	1.066	1.089	1.089					
Oct-05	1.065	1.064	1.017	1.099	1.099					
Nov-05	1.044	1.041	1.096	1.051	1.051	05:Q4	1.040	1.038	1.030	1.023
Dec-05	1.046	1.044	1.063	1.054	1.054					
Jan-06	1.052	1.049	1.068	1.064	1.064					
Feb-06	1.047	1.044	1.096	1.049	1.049	06:Q1	1.108	1.109	1.134	1.052
Mar-06	1.057	1.055	1.088	1.067	1.067					
Apr-06	1.063	1.061	1.142	1.079	1.079					
May-06	1.077	1.075	1.256	1.099	1.099	06:Q2	1.170	1.159	1.145	1.100
Jun-06	1.075	1.073	1.206	1.094	1.094					
Jul-06	1.079	1.079	1.080	1.104	1.104					
Aug-06	1.088	1.088	1.194	1.118	1.118	06:Q3	1.225	1.208	1.231	1.124
Sep-06	1.095	1.094	1.063	1.130	1.130					
Oct-06	1.083	1.083	1.133	1.112	1.112					
Nov-06	1.076	1.077	1.129	1.101	1.101	06:Q4	1.126	1.114	1.139	1.100
Dec-06	1.082	1.082	1.198	1.111	1.111					
Jan-07	1.092	1.092	1.075	1.129	1.129					
Feb-07	1.089	1.089	1.080	1.120	1.120	07:Q1	1.205	1.184	1.185	1.177
Mar-07	1.102	1.101	1.136	1.139	1.139					
Apr-07	1.097	1.096	1.228	1.126	1.126					
May-07	1.098	1.096	1.256	1.117	1.117	07:Q2	1.161	1.139	1.134	1.160
Jun-07	1.097	1.095	1.156	1.104	1.104					

Table 6: "Town of A" Land Price Indices

Month	FD_p	FP_p	Median_I	FP	FD	Quarter	DdH P3	DdH P4	FGSS DS2	FGSS SFA
Jul-07	1.103	1.101	1.027	1.117	1.117					
Aug-07	1.109	1.106	1.043	1.122	1.122	07:Q3	1.167	1.153	1.208	1.163
Sep-07	1.117	1.116	1.238	1.149	1.149					
Oct-07	1.118	1.119	1.118	1.163	1.163					
Nov-07	1.103	1.102	1.158	1.127	1.127	07:Q4	1.099	1.095	1.124	1.168
Dec-07	1.112	1.112	1.163	1.142	1.142					
Jan-08	1.117	1.117	1.151	1.150	1.150					
Feb-08	1.111	1.110	1.163	1.134	1.134	08:Q1	1.046	1.041	1.002	1.153
Mar-08	1.091	1.089	1.196	1.088	1.088					
Apr-08	1.102	1.100	1.093	1.112	1.112					
May-08	1.105	1.103	1.147	1.120	1.120	08:Q2	1.092	1.092	1.136	1.172
Jun-08	1.118	1.117	1.179	1.146	1.146					

FD_p: Fisher Democratic (property); FD_l: Fisher Democratic (land);

FP_p: Fisher Plutocratic (property); FP_l: Fisher Plutocratic (land);

DdH_P3, DdH_P4: Land Price Indices; Diewert et al. (2015). Tables 1 and 2

FGSS_DS2, FGSS_SFA: Land Price Indices; DS and SFA computed by Färe et al. (2015) Table 5

Table 7: "Town of A" Structure Price Indices

Month	FD_p	FP_p	Median_I	FP	FD	Quarter	DdH P3	DdH P4	FGSS DS2	FGSS SFA
Aug-03	0.932	0.923	0.927	0.946	0.931	03:Q3	1.016	1.016		
Sep-03	0.942	0.930	1.096	0.966	0.946					
Oct-03	0.954	0.943	0.937	0.972	0.953					
Nov-03	0.979	0.970	0.904	0.985	0.970	03:Q4	1.008	1.008		
Dec-03	0.979	0.972	0.955	0.985	0.972					
Jan-04	0.981	0.975	1.013	0.984	0.972					
Feb-04	1.002	0.997	0.997	0.996	0.988	04:Q1	1.000	1.000		
Mar-04	1.007	1.003	0.915	0.998	0.990					
Apr-04	1.002	0.999	1.125	0.994	0.989					
May-04	0.996	0.993	1.046	0.995	0.989	04:Q2	0.975	0.975		
Jun-04	1.008	1.006	0.977	1.002	0.998					
Jul-04	1.014	1.012	0.952	1.004	1.000					
Aug-04	1.006	1.005	0.924	0.998	0.995	04:Q3	0.984	0.984		
Sep-04	1.014	1.014	1.043	1.000	1.000					
Oct-04	1.011	1.012	0.963	0.995	0.996					
Nov-04	0.995	0.997	0.963	0.986	0.989	04:Q4	1.016	1.016		
Dec-04	0.998	1.000	0.993	0.989	0.991					
Jan-05	0.988	0.988	0.940	0.991	0.992					
Feb-05	1.000	1.000	1.000	1.000	1.000	05:Q1	1.000	1.000	1.000	1.000
Mar-05	1.001	1.002	1.080	0.999	0.999					
Apr-05	1.005	1.004	1.030	1.005	1.004					
May-05	1.021	1.021	1.110	1.014	1.012	05:Q2	0.993	0.993	1.039	1.041
Jun-05	1.043	1.042	1.120	1.028	1.025					
Jul-05	1.051	1.050	1.080	1.033	1.030					

Table 7: "Town of A" Structure Price Indices

Month	FD_p	FP_p	Median_I	FP	FD	Quarter	DdH P3	DdH P4	FGSS DS2	FGSS SFA
Aug-05	1.048	1.047	1.023	1.031	1.028	05:Q3	1.015	1.015	1.070	1.090
Sep-05	1.057	1.056	1.066	1.039	1.038					
Oct-05	1.065	1.064	1.017	1.047	1.043					
Nov-05	1.044	1.041	1.096	1.040	1.035	05:Q4	1.039	1.045	1.050	1.054
Dec-05	1.046	1.044	1.063	1.043	1.038					
Jan-06	1.052	1.049	1.068	1.046	1.041					
Feb-06	1.047	1.044	1.096	1.048	1.043	06:Q1	1.007	1.007	1.061	1.062
Mar-06	1.057	1.055	1.088	1.054	1.048					
Apr-06	1.063	1.061	1.142	1.057	1.051					
May-06	1.077	1.075	1.256	1.067	1.061	06:Q2	1.017	1.017	1.055	1.053
Jun-06	1.075	1.073	1.206	1.064	1.061					
Jul-06	1.079	1.079	1.080	1.065	1.063					
Aug-06	1.088	1.088	1.194	1.071	1.070	06:Q3	1.012	1.012	1.082	1.087
Sep-06	1.095	1.094	1.063	1.074	1.073					
Oct-06	1.083	1.083	1.133	1.065	1.065					
Nov-06	1.076	1.077	1.129	1.061	1.060	06:Q4	1.015	1.015	1.048	1.073
Dec-06	1.082	1.082	1.198	1.062	1.063					
Jan-07	1.092	1.092	1.075	1.068	1.068					
Feb-07	1.089	1.089	1.080	1.070	1.069	07:Q1	1.034	1.034	1.029	1.084
Mar-07	1.102	1.101	1.136	1.081	1.078					
Apr-07	1.097	1.096	1.228	1.078	1.075					
May-07	1.098	1.096	1.256	1.085	1.081	07:Q2	1.045	1.045	1.056	1.127
Jun-07	1.097	1.095	1.156	1.091	1.086					
Jul-07	1.103	1.101	1.027	1.093	1.089					

Table 7: "Town of A" Structure Price Indices

Month	FD_p	FP_p	Median_I	FP	FD	Quarter	DdH P3	DdH P4	FGSS DS2	FGSS SFA
Aug-07	1.109	1.106	1.043	1.099	1.094	07:Q3	1.069	1.069	1.088	1.138
Sep-07	1.117	1.116	1.238	1.094	1.092					
Oct-07	1.118	1.119	1.118	1.087	1.088					
Nov-07	1.103	1.102	1.158	1.084	1.083	07:Q4	1.092	1.092	1.063	1.106
Dec-07	1.112	1.112	1.163	1.090	1.090					
Jan-08	1.117	1.117	1.151	1.093	1.094					
Feb-08	1.111	1.110	1.163	1.096	1.094	08:Q1	1.113	1.113	1.036	1.097
Mar-08	1.091	1.089	1.196	1.088	1.085					
Apr-08	1.102	1.100	1.093	1.094	1.091					
May-08	1.105	1.103	1.147	1.094	1.091	08:Q2	1.113	1.113	1.061	1.107
Jun-08	1.118	1.117	1.179	1.100	1.098					

FD_p: Fisher Democratic (property); FD_S: Fisher Democratic (structure);

FP_p: Fisher Plutocratic (property); FP_S: Fisher Plutocratic (structure);

DdH_P3, DdH_P4: Structure Price Indices; Diewert et al. (2015) Tables 1 and 2. Statistics Netherlands "New Dwelling Output Price Index"

FGSS_DS2, FGSS_SFA: Structure Price Indices; DS and SFA computed by Färe et al. (2015) Table 7.

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