Bracket Creep Revisited – With and Without r>g: Evidence from Germany

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Non-technical summary

Research question

Growth in nominal earnings to protect against inflation does not fully maintain an individual's real purchasing power under a progressive tax system as it also pushes them into a higher tax bracket. This phenomenon is known as "bracket creep" (fiscal drag/cold progression). The essential function of the tax system is to finance the government expenditure and serve the redistribution function. Since bracket creep redistributes income from households to the government, it is important to quantitatively measure how this redistribution changes the revenue and redistribution effects of the tax-benefit system. Furthermore, gross income distribution and the tax schedule evolve over time. Many developed countries witnessed, in particular, the growing inequality before the crisis and capital income expanding at a faster pace than non-capital income ($r > g$). A dual tax system to levy a flat rate on capital income and progressive rates on non-capital income has been introduced in several countries. We may expect the effect from bracket creep not to be constant. Frequent empirical assessment contributes to the understanding of the roles played by the evolution of both income distribution and the tax-benefit system.

Contribution

As a case study, we perform a tax micro-simulation on the newly available PHF data covering the entire range of German income distribution in 2009. The micro-simulation study can separate the effect of bracket creep through the tax schedule from the effect via other tax components and enables relative inflation compensation to differ between capital and non-capital incomes, and between households.

Results

Our micro-simulation illustrates an inverted U-shaped redistributive effect of bracket creep as the inflation rate increases. This contrasts with Immervoll (2005) who, in a micro-simulation study using 1998 income data on Germany, the Netherlands and the UK, documents that bracket creep can always enhance the overall redistributive effect of the tax system. Under both bracket creep and $r > g$, a dual tax system with a flat capital income tax can reduce the redistribution further. Pre-tax policies attempting to curb this degeneration can even exacerbate the situation under our assumptions.

Nichttechnische Zusammenfassung

Forschungsfrage

Nominales Einkommenswachstum zum Schutz gegen Inflation führt bei einem progressiven Steuersystem nicht zu einem vollständigen Erhalt der realen Kaufkraft der Bürger, da sie dadurch auch auf eine höhere steuerliche Progressionsstufe gezogen werden. Dieses Phänomen ist unter dem Begriff „kalte Progression“ bekannt. Die grundlegende Funktion des Steuersystems ist es, die Staatsausgaben zu finanzieren und der Umverteilung zu dienen. Da die kalte Progression Einkommen von den privaten Haushalten zum Staat umverteilt, ist es wichtig, quantitativ zu messen, wie diese Umverteilung das Einkommen und die Umverteilungswirkung des Steuer- und Sozialleistungssystems verändert. Ferner unterliegen die Bruttoeinkommensverteilung und die Steuertarife dem Wandel der Zeit. So war insbesondere in vielen entwickelten Ländern vor der Krise eine wachsende Ungleichheit zu beobachten, und die Kapitaleinkommen wuchsen schneller als die Nichtkapitaleinkommen ($r > g$). In verschiedenen Ländern wurde ein duales Steuersystem mit einer „Flatrate“ für Kapitaleinkünfte und progressiven Steuersätzen auf Nichtkapitaleinkommen eingeführt. Wir können davon ausgehen, dass die Wirkung der kalten Progression nicht konstant ist. Regelmäßige empirische Einschätzungen tragen zum Verständnis der Bedeutung bei, die der Entwicklung der Einkommensverteilung wie auch des Steuer- und Sozialleistungssystems zukommt.

Forschungsbeitrag

In einer Fallstudie führen wir eine steuerliche Mikrosimulation anhand der neu verfügbaren Daten aus der PHF-Umfrage zur gesamten Bandbreite der Einkommensverteilung in Deutschland im Jahr 2009 durch. Mithilfe der Mikrosimulation lässt sich der Effekt der kalten Progression über den Steuertarif von der Wirkung anderer Einflussfaktoren des Steuersystems trennen, sodass der relative Inflationsausgleich nach Kapital- und Nichtkapitaleinkommen sowie nach Haushalten unterschieden werden kann.

Forschungsergebnisse

Unsere Mikrosimulation fördert einen invertierten U-förmigen Umverteilungseffekt der kalten Progression mit zunehmender Inflationsrate zutage. Dies steht im Gegensatz zu Immervoll (2005), der in einer Mikrosimulationsstudie anhand von Daten zu den

Einkommen in Deutschland, den Niederlanden und dem Vereinigten Königreich aus dem Jahr 1998 nachweist, dass die kalte Progression die Umverteilungswirkung des Steuersystems insgesamt immer verstärken kann. Bei einer steuerlichen Progression und $r > g$ kann ein duales Steuersystem mit einer Flatrate-Steuer auf Kapitaleinkommen die Verteilungsfunktion weiter reduzieren. Auf die Ebene vor Steuern gerichtete Maßnahmen, die diese Verschlechterung einzudämmen versuchen, können die unseren Annahmen zugrunde liegende Situation noch verschärfen.

BUNDESBANK DISCUSSION PAPER NO 37/2015

Bracket Creep Revisited – With and Without r>g: Evidence from Germany[1]

Junyi Zhu[†]

Abstract

Using German income distribution in 2009, this paper studies the redistributive and revenue effects of bracket creep under various inflation scenarios. We develop a tax micro-simulation model for the newly available Panel on Household Finance (PHF) data. The simulation yields an inverted U-shaped overall redistributive effect of the income tax and social insurance contribution system with respect to the inflation rate, which contrasts with Immervoll (2005) who finds that fiscal drag always enhances the equalising effect. The nominal income growth as well as the deterioration of tax progression at the middle and top of the income distribution between 1998 and 2009 can be the impetus for this change. This result implies that delaying adjustment might reduce redistribution. We also suggest that these results might not be restricted solely to Germany. Additionally, when we introduce the empirical evidence that capital income grows faster than non-capital income $r > g$, the dual tax system with a flat capital income tax implemented in 2009 further disequalises the after-tax income substantially. Allowing inflation compensation to lean towards the poor by boosting their share of capital income may not be favourable to redistribution.

Keywords: Inflation, Fiscal Drag, Progressivity of Income Tax, Income Distribution, Micro-simulation, Capital income taxation

JEL-Classification: C81, H24, D31, H23

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1 Introduction

Rising earning levels result in higher tax burdens in a progressive tax system that is nominally defined, a mechanism often referred to as bracket creep or fiscal drag (see, eg Musgrave, Musgrave, and Kullmer (1994), Saez (2003), Immervoll (2005), Gutierrez, Immervoll, and Sutherland (2005), or Heer and Süssmuth (2013)).[2] Most taxpayers will be pushed up the tax schedule with higher marginal and average tax rates. The result is underproportional growth of real income or even a loss in real income, particularly when income growth is predominantly or only nominal.[3] Since bracket creep results in a real loss by taxpayers, the government naturally enjoys a real gain.

The discussion on bracket creep, similar to other topics on the personal income tax (PIT) policy, centres on revenue and redistributive effects.[4] On one hand, the public asks to what extent bracket creep raises the average tax level and government revenue without explicitly targeted legislative policies (OECD (2008a)). On the other hand, inflation can alter the distributional property of the nominally defined tax system such that the relative increases in the tax burden are uneven between the poor and the rich due to tax progressivity. Understanding the redistributive effect of bracket creep in depth can enhance the public debate and help policy makers to determine by how much each country is willing to design the countermeasure towards bracket creep (eg inflation indexing by adjusting various parameters in the tax function) to restore the redistribution effect of the tax system.

We investigate both the revenue and redistributive effects of bracket creep by conducting a case study of German income distribution, with a focus on the latter effect. Immervoll (2005) quantitatively assesses this effect using EUROMOD, a Europe-wide tax-benefit micro-simulation model, for Germany, the Netherlands and the UK. The data for Germany refer to 1998. The author finds that fiscal drag enhances the overall equalising effect of the income

[2] The German Institute for Economic Research (DIW) differentiates between the broad and narrow sense. In the broad sense, this concept refers to the disproportionate increase in income tax revenue along with any nominal income growth (DIW (2014)), while in the narrow sense, it refers to the inflation-induced increase in income tax revenue alone. We will only concentrate on the definition in the narrow sense since it draws more attention in the literature and public debate. However, the mechanisms are the same if we want to extend our discussion to the definition in the broad sense.

[3] The German literature describes this effect as "*Kalte Progression*" (see, eg Broer (2011)), which literally means "cold progression". The term "cold progression" explicitly covers all distortions of the tax function induced by inflation.

[4] As for the significant effects of the PIT policy, for many countries, PIT is an important source of revenue. For instance, in Germany, about one third of all taxes come from income taxation (Federal Ministry of Finance (2011)); on the other hand, as income taxes affect a large number of taxpayers, income taxation is deemed the main tool of redistribution. There has been a wide range of literature focusing on the effectiveness and progressivity of income taxation serving to reduce inequality. See eg Kakwani (1980) or Atkinson (1970).

tax (IT) and social insurance contribution (SIC) system. Although inflation diminishes the tax progressivity, the average tax burdens are increased substantially. The latter effect dominates in his evaluation.

Following this study, we also adopt the approach of micro-simulation using a model developed for the newly available Panel on Household Finance (PHF) data. The reference year is 2009. Similar to Immervoll (2005), in the baseline scenario, we assume full inflation compensation so that all income components grow at the same pace with inflation. As an extension, we also introduce the variation for the nominal growths between different income components, which are not bound to be the inflation rate. Our simulation covers both IT and SIC by applying the relevant fiscal rules but taking all the transfer income as given in the data.

There have been many studies which estimate the distributional feature of bracket creep by using either macro (e.g. Altig and Carlstrom (1991, 1993) and Heer and Süssmuth (2013)) or micro (e.g. Auerbach and Feenberg (2000), Steiner and Haan (2004) and Dolls (2012)) data or models. As Auerbach and Feenberg (2000) indicate, the aggregate relationship between total taxes and total income does not give any insight into the changes between income components and/or taxpayers within one tax unit, and cannot separately analyse the impacts from different parts of the tax system.[5] In particular, we contribute to the literature, taking advantage of the micro approach, by singling out the effect of bracket creep through the tax schedule from other tax components and simulating different nominal growth rates for capital and non-capital incomes.

Our result implies that the effect of the average tax burdens no longer plays a pivotal role and the impact from progressivity becomes dominant when inflation climbs. Both of these are the building blocks of the redistributive effect of the IT and SIC system. Consequently, we observe an inverted U-shaped overall redistributive effect when inflation grows. Under bracket creep, the Gini coefficient of after-IT-SIC income distribution reaches a minimum when cumulated inflation is about 20%. Assuming constant annual inflation of 2%, cumulated inflation can reach 21.9% after 10 years, which is not implausible if frequent indexation is not institutionalised.[6] On the other hand, bracket creep reduces tax progressivity and enlarges the average tax burdens. These impacts are still consistent with Immervoll (2005).

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[5] Dolls (2012) further points out that the micro approach can discriminate between each fiscal effect and also the behavioural response of agents. A macro approach based on ex-post analysis cannot do this and, moreover, suffers from endogeneity and identification problems.

[6] In the following text, we calculate cumulated inflation throughout German history. 20% is far from the upper bound among the plausible scenarios. Allowing all the social benefit income lags in inflation compensation, we show that the minimum point can be reached at a cumulated inflation of 10%. On the other hand, cumulated

Bracket creep purely through the tax schedule is remarkably overall regressive. As permitted under the dual tax system (by means of a yield test), the upper middle part of distribution would be more willing to opt the capital taxable income out of the (non-capital) income tax base than the lower middle part when nominal income grows. And tax savings through this channel are also positively correlated with the income level. With moderately low inflation cumulated to a certain degree, bracket creep can even refund the middle part due to these savings, and the effect is higher for the upper middle part.

Between 1998 and 2009, nominal income increased at a relatively faster pace with growth at both ends of income distribution, and the tax rates and progressivity for the middle and top parts of the distribution declined substantially. Our analysis suggests that they are the potential drivers in reshaping the overall redistributive effect of bracket creep documented as the difference between the results obtained by Immervoll (2005) and us. Our empirical evidence implies that delaying indexation for too long could have significant distributional implications which merit attention.

Although concerns about low inflation or deflation have emerged recently, the motivations for this topic are still valid and are shared by many countries. Market income inequality has been growing in many parts of the world. Taxpayers in many countries have witnessed a period of continuous tax reforms. The most homogenous evolution is the drop of the top PIT rates between 1998 and 2009 (Verbist and Figari (2014)). These changes naturally demand a reassessment of bracket creep. The German case serves well for such an exercise since it has followed all these developments in income distribution as well as in the IT and SIC system. As our study will demonstrate by using the recent empirical results on the relationships between income distribution, preferences for redistribution and policy alternatives, the rising role from the declining progressivity observed from our German case might not be isolated.[7]

One major tax reform in 2009 was the introduction of a dual tax system with a final withholding tax (*Abgeltungssteuer*) at the flat rate of 25% on capital income and separating it from the income tax base. Capital income was included in the income tax base in 1998.[8]

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inflation at this minimum point can vary if we adopt the generalised Gini coefficient and assign different weights for the population over the income distribution (the standard Gini measure, which we use, implies an equal weight for everyone; see Immervoll (2005) for a discussion and application of the generalised Gini coefficient).

[7] As for the German specifics, favourable economic developments, increasing scarcity on factor market incomes, loose monetary policy as well as the need to accommodate the higher subsistence standard might reinforce the impact of bracket creep (Bach (2012)).

[8] To be clear, throughout the paper, the income tax base is the one that is subject to the progressive tax schedule, which is equivalent to the non-capital income tax base since 2009 when the dual tax system was introduced.

Compared with 1998, the top of the income distribution would benefit more from this reform since their marginal income tax rate can be higher than 25% and capital income concentrates at the top. Moreover, this advantage grows with income due to tax progressivity and the increasing level of concentration. We can expect this reform to reduce the progressivity of the tax system (Jenderny (2013) and Bartels and Jenderny (2014)).[9] In terms of bracket creep, the higher the share of income subject to a flat rate tax for a tax unit, the lower the increase in real tax burden it will suffer. Therefore, by the same reasoning as above, we can expect bracket creep under the same inflation rate to induce lower progressivity through the IT and SIC system in 2009 than in 1998. Potentially, as this reduction is expected to pick up when inflation grows, this can be another driver behind the inverted U-shaped development of the redistributive effect due to bracket creep.

The other notable phenomenon in recent decades has been the rise of wealth-income ratios in several developed countries as well as a higher rate of return on capital relative to labour (ie $r > g$) as documented by Piketty and Zucman (2014) and Piketty (2014). These authors also identify $r > g$ as a driver for the increasing wealth-income ratios because faster growth in capital income would raise the capital share in total income and wealth is even more concentrated than labour income.

As another contribution to the literature, we further simulate a few scenarios which introduce $r > g$ such that the inflation-induced nominal growth on capital income is faster than that on non-capital income. For the first group of scenarios in this context, we maintain the constancy of r/g in the population. Compared with a counterfactual comprehensive tax system incorporating capital income under the same progressive tax schedule with non-capital income, the current dual tax system significantly worsens the redistribution under all bracket creep scenarios. The aggravation is even stronger as capital income grows faster. With the nominal growth rates of 2% for capital income and 0.5% for non-capital income, the redistributive effect of the comprehensive tax system can be 6.2% higher than that of the dual tax system. For the second group of scenarios, we allow the extent of inflation compensation to vary between capital and non-capital incomes, with the former always larger, across the distribution. The relative growth of the two incomes can hinge on the ratio of capital and non-capital incomes for each household. Building on a few reasonable assumptions regarding the

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Before 2009, there was no separation of the capital and non-capital income tax bases. Thus, reference is made to only one income tax base.

[9] Both document the evolution of German capital income taxation in detail.

inflation compensation process, our simulations show that the redistributive effect of the IT and SIC system under bracket creep with $r > g$ is always weakened relative to the effect without $r > g$. In the relatively plausible scenario, the peak of the inverted U-shaped development of the redistributive effect occurs at an even lower inflation rate of 10%. Further raising the relative weight of capital income for the poor, supposed a progressive measure to combat bracket creep under flat rate capital income tax, turns out to be least favourable to the poor under $r > g$.

The remainder of this paper is structured as follows: the next section describes the consequences and countermeasures of bracket creep for Germany. Section 3 presents the different approaches to measuring the progressivity of income tax schedules. Section 4 discusses the data, the tax micro-simulation model and the simulation scenarios. Section 5 provides the quantitative assessment on how bracket creep reshapes the equalising effect of the IT and SIC system. Section 6 discusses the evolution of empirical progressivity, the timing of inflation indexation and how simulation in the German case can be relevant to other countries. In section 7, we perform the simulations under $r > g$. We conclude in section 8.

2 Bracket Creep

A macroeconomic approach to assess the impact of bracket creep for a given economy is to observe the development of the income tax revenue in relation to GDP (Broer (2011)). As the statistics show, this relation has been rather stable in Germany since the mid-1970s, oscillating between 8% and 10%. This means that by means of regular and major tax reforms, progression-related revenues have been returned to income taxpayers and hence, the revenue effect of bracket creep has been indirectly phased out. The effect of bracket creep in Germany has been analysed in various studies, which we summarise below. We further describe the measures undertaken by the government to compensate for this effect.

2.1 Micro-evidence of bracket creep in Germany

Even though the relation between income tax revenue and GDP has been constant since the mid-1970s, when regarding shorter periods of time, especially those intervals when no major tax reforms occurred, significant bracket creep effects could be observed – "*(...) infrequent inflation adjustments can indeed cause additional tax burdens - even at low rates of inflation*" (Immervoll (2005), p 38). The studies below have quantified the fiscal consequences and redistributive implication of bracket creep in Germany using household micro-data.

Bach (2012) analyses the effects of bracket creep according to the draft of the German parliament (*Deutscher Bundestag)*, which proposes a tax reform to explicitly fight the cumulated effect of bracket creep for 2013 and 2014 (Bundestag (2012)). According to this evaluation, in a scenario with 4.4% cumulated inflation for 2013 and 2014, as forecast by the German government, and full inflation compensation of all incomes, the inflation-induced increase in tax revenue will amount to €6.9 billion for both years. This is in the same scale of our estimate of €5.73 billion at 4% inflation for 2009.

Similar to the findings by Bach (2012) and Immervoll (2005), Schaefer (2013) recognises that the tax burden generated through bracket creep alone is lower in absolute terms for low incomes than for high incomes, yet the ratio between this new tax burden and the remaining income taxes is higher for low incomes than for high incomes. We show the same pattern in absolute terms and, as inflation grows, in relative terms by means of an empirical analysis although our denominator in the ratio is the gross income. Beyond the current literature, our extended simulations analyse the additional distortion of the redistribution effect introduced by the heterogeneous nominal growth between income components.

2.2 Neutralising bracket creep

In order to offset bracket creep, many countries have institutionalised frequent indexation to modify the tax deductions, brackets and even marginal rates by referencing to the movement of cost-of-living. For instance, the USA implemented such a practice as part of the Economic Recovery Tax Act in the early 1980s, as documented in Auerbach and Feenberg (2000). Heer and Süssmuth (2013) find a more significant and contemporaneous relationship between the Gini coefficient and inflation rate dynamics if indexation is more frequent as households adjust their labour supply and saving decisions more instantly and less precipitately. The practices which Germany has introduced to adjust its tax and benefit systems for bracket creep are briefly summarised below.

In Germany there is no automatic indexation. Nevertheless, there are some legal criteria from which indexation *implications* are actually derived. For example, there is a "pension formula" for calculating adjustments of the contributions as well as the payout level (Gutierrez et al (2005)).

Concerning the income tax schedule itself with the tax rates, credits, deductions and bracket limits, discretionary amendments are driving the evolution. For instance, the adjustment of the basic allowance is not primarily motivated by changes in the CPI, but rather according to the forecast development of the minimum income needed. This level can be derived from the "margin of subsistence report" which is presented to the German parliament by the Federal government every two years.[10] The justification for the last two increases is contained in the draft law to reduce the effect of bracket creep (Bundestag (2012)). Moreover, within these adjustments, further political objectives were mixed, as is the case for changes in the minimum tax rate. These among other concerns have led to a total of 13 changes in the income tax schedule since 1990 (Hechtner, Massarrat-Mashhadi, and Sielaff (2012)). Recently, the government decided to evaluate and analyse the development of progressivity every two years via a progression report (*Progressionsbericht*). In this context, studies assessing the redistributive effect of bracket creep can be contemporaneously valuable.

2.3 The impact of distortions induced by inflation on personal tax and the income concepts adopted

Besides PIT, another source of inflation-induced real impact on personal tax can be excise tax. Since the tax bases are the volume of transactions, the real value of tax decreases as inflation takes effect. However, accounting for it in our calculations is beyond the scope of this paper since we do not have the relevant data at our disposal and, for comparability, we follow the same practice as in Immervoll (2005) and exclude such an impact. German households are also subject to a non-negligible amount of consumption tax (ie value-added tax). But this tax is not relevant in real term because it is a proportional tax on the value added.

The tax base on capital income is also nominally defined, and the part to compensate for the inflation-induced loss of capital is thus not tax-exempt (Immervoll (2000)). Some countries acknowledge this distortion and adjust for it although Germany, among others, does not take any action. However, we do not consider such a loss as belonging to the concept of bracket creep as it is not caused by tax progressivity; in particular, a flat rate on capital income has been in place in Germany for private households since 2009.

In our paper, social insurance contributions are considered when calculating the pre-tax income because, fundamentally, the net disposable income reflecting the power to consume (ie, the Haig-Simons (H-S) income concept) should be the income measurement for welfare analysis. In other words, any redistribution changing the distribution of such an income concept will have welfare implications and should be counted. On the other hand, our income concept covers all the private and public transfer income (see section 4.2 below). By

[10] The tenth "margin of subsistence report" (*Existenzminimumbericht*) was published in 2015 and can be accessed at: http://www.bundesfinanzministerium.de/Content/DE/Pressemitteilungen/Finanzpolitik/2015/01/2015-01-28-PM05-anlage.pdf?__blob=publicationFile&v=2

including the statutory social insurance contributions, we will fully derive the net disposable income. This practice is also common in the public finance literature, such as Immervoll (2005; see section 2.2), as well as recommended by the international institutes.[11]

3 Measuring Progressivity of Income Tax Schedules

Although the income tax schedule can be rather progressive in most areas, the whole tax system may not be progressive if all of these areas are underpopulated. In the following, we will present the methods to measure the (*local*) progression of an income tax schedule alone, and also its redistributive effects in terms of (*global*) progressivity when income distribution is also jointly considered.

3.1 Inequality measures related to progression and progressivity

Yet, in order to make clearer the building blocks of progression and progressivity, we will first briefly introduce the relevant inequality measures.

Lorenz curve

A Lorenz curve can be defined for a distribution of y in a population, which can be gross/net income, tax, wealth etc. Every point (p, q) on the Lorenz curve captures the share of total y, $100 \cdot q$, received/paid by the lowest $100 \cdot p$ percent population ranked by y, with p and $q \in [0,1]$. Let the density and cumulative density functions for income distribution be $f(\mathrm{y})$ and $F(\mathrm{y})$. The Lorenz curve is then defined as

$$LC_{\mathrm{y}}(p) = \int_0^{F^{-1}(p)} \frac{\mathrm{y}f(\mathrm{y})}{\mu} d\mathrm{y}, 0 < p < 1,$$

where μ represents the average income for the whole population (Lambert (2001)).

Lorenz domination

The concept of Lorenz-domination was first proposed by Atkinson (1970). Its goal is to show if one income distribution is more equal than another. To guarantee that one income distribution is more equal or less equal than another, one of the Lorenz curves must be completely above the other one (Jakobsson (1976)). Consider, for example, two distribution functions $F(\mathrm{y})$ and $G(\mathrm{y})$ and their respective Lorenz curves $L_F(p)$ and $L_G(p)$, then F is Lorenz-dominating if

[11] See, for example, the OECD's definition of disposable income: http://stats.oecd.org/glossary/detail.asp?ID=3020.

$$L_F(p) \geq L_G(p) \text{ for all } p \in [0,1].$$

A problem arises when two Lorenz curves intersect at some point. Then we cannot claim according to Lorenz-dominance which distribution is more equal (Kakwani (1986)). To still allow for a distribution ranking, the Gini coefficient is computed for every Lorenz curve.

Gini coefficient

The Gini coefficient G is an area measure of the extent to which a given Lorenz curve departs from the 45° line (Lambert (2001), p 27). It is defined as follows:

$$G = 2\int_0^1 \left(p - LC_{\mathrm{y}}(p)\right) dp.$$

As a share measure of the area below the line of perfect equality (45° line), it ranges between 0 and 1. For an income distribution, a G-value of zero means that the distribution is perfectly equal, while a G-value of one would mean that in that population all the income belongs to one single income unit, irrespective of the population size.

Concentration curve

For the measure of progressivity discussed below, it will be necessary to compute not only curves for gross taxable income shares with respect to the share of the ranked gross taxable income units, but also the relation of the tax burden distribution with respect to the gross taxable income distribution. The concentration curve $CC_{A|B}(\cdot)$ allows for such a relation as it depicts the shares of a first distribution A against the shares of a second distribution B (Lambert (2001)).

Let us assume that the gross tax function takes the form $t(\mathrm{y})$, where y represents the gross taxable income level. Further, $f(\mathrm{y})$ stands for the distribution of gross taxable income. It follows that the concentration curve of taxes with respect to gross taxable income $CC_{t(y)|y}(p)$ is defined as:

$$\int_0^{F^{-1}(p)} \frac{t(\mathrm{y})f(\mathrm{y})}{\mu_{TAX}} d\mathrm{y},$$

where μ_{TAX} is the average tax liability. Every point (p, q) on the concentration curve $CC_{t(y)|y}(p)$ captures the fraction of total tax liabilities, $100 \cdot q$ percent, paid by the poorest $100 \cdot p$ percent in terms of gross taxable income, with p and $q \in [0,1]$ (Lambert (2001)). If we replace tax by net income, we can also produce a concentration curve $CC_{x|y}(\cdot)$ for the distribution of net income x with respect to gross taxable income y.

Concentration coefficient

Likewise, the concentration coefficient for a concentration curve is defined in the same fashion as the Gini coefficient is for a Lorenz curve. As a consequence, the concentration coefficient C can be expressed as twice the area between the line of perfect equality and a given concentration curve $CC(p)$ (Lambert (2001)). Following our example from above, the concentration coefficient for the concentration curve $CC_{t(y)|y}(p)$ is defined as

$$C_{t(y)|y}(p) = 2\int_0^1 \left(p - CC_{t(y)|y}(p)\right) dp.$$

3.2 Measures of progression

First of all, it is vital to differentiate between (*local*) progression and (*global*) progressivity.[12] Progression measures identify the degree of progression of a tax schedule itself. Progressivity measures though intend to capture the change in equality of income distribution after applying a certain redistribution policy. To determine progressivity, it is indispensable to know both the pre and post-tax income distribution. However, the characteristic of the tax schedule itself is sufficient for calculating the progression.

In their seminal paper, Musgrave and Thin (1948) present four methods of measuring tax progression: average rate progression, marginal rate progression, liability progression and residual progression. All these measures describe the progression at the local point of income distribution. Therefore, they are called *local* progression. In terms of average rate progression, a tax schedule is said to be progressive (regressive) if the average tax rate increases (decreases) with income. Respectively, in terms of marginal rate progression, a tax schedule is said to be progressive (regressive) if the marginal tax rate increases (decreases) with income. Both average and marginal rate progression can help to infer that a tax schedule is progressive or regressive. But they cannot help to rank two or more tax schedules. For this purpose, Musgrave and Thin (1948) introduce liability and residual progression.

Liability progression

[12] They are also often referred to as structural progression (*local* progression) and effective progression (*global* progressivity).

Liability progression can be defined as liability elasticity – the ratio of the percentage change in tax liability to the concurrent percentage change in taxable income. It can be shown that liability progression equals

$$\frac{M(y)}{A(y)}$$

where $M(y)$ is the marginal tax rate and $A(y)$ is the average tax rate. Whenever this elasticity is larger than 1, meaning that the marginal tax rate $M(y)$ is larger than the average tax rate $A(y)$, the tax schedule is defined as "liability elastic", and accordingly, the income tax schedule is progressive. It will become clear that the redistributive property of a tax schedule is closely linked to local tax elasticity.

Residual progression

Residual progression is another measure of elasticity, which refers to net income. It is designed to analyse the residual elasticity, the percentage change in income after tax in response to a 1% increase in income before tax. It can be shown that residual progression equals

$$\frac{1-M(y)}{1-A(y)}.$$

Again, the tax schedule is progressive if the marginal tax rate is higher than the average tax rate. Contrary to liability progression, in this case the residual elasticity will be less than 1. The tax schedule is then "residual inelastic".

3.3 Measures of progressivity

We discuss the redistributive effect of tax schedules on income distribution, which is determined by both the empirical income distributions and the tax schedule. We only cover those progressivity measures that will be used in our empirical analysis.

Redistributive effect

The measure Musgrave and Thin (1984) introduce in their seminal paper captures the redistributive effect (RE) of a tax schedule by subtracting the net income Gini coefficient (G_x) from the gross taxable income Gini coefficient (G_y)

$$RE = G_y - G_x.$$

According to the definition of the Gini coefficient described above, we can also express RE as twice the area between the Lorenz curve of gross taxable income $LC_{\mathrm{y}}(p)$ and the Lorenz curve of net income $LC_x(p)$:

$$RE = 2\int_0^1 [LC_{\mathrm{x}}(p) - LC_{\mathrm{y}}(p)]dp.$$

A positive value of RE implies a progressive redistribution through taxation since it reduces the inequality of income distribution by the income tax levied. A negative value of RE implies the opposite. And a value of zero means that we are dealing with a proportional income tax system.

Kakwani (1977a and 1977b) demonstrates that the redistributive effect, as presented by Musgrave and Thin, only depicts a change in inequality, without capturing tax progressivity on its own. Plotnick (1981) recognise a further complication when comparing the Gini coefficient of the distribution of pre-tax income with that of post-tax income, namely that their respective Lorenz curves do not guarantee that, say, the 40% poorest income units in terms of pre-tax income are the same 40% poorest income units in terms of post-tax income. This problem, which is caused by income taxation due to differences in the tax treatment of income units, was categorised by Atkinson (1980) and Plotnick (1981) in the process of reranking.

The widely accepted Kakwani decomposition of the redistributive effect captures, on the one hand, the progressivity effect through the vertical effect measuring how the inequality of income distribution among income units is reduced or amplified without changing their *ex-ante* relative positions, and, on the other, the horizontal effect in terms of reranking, which only captures the change in the relative positions of income units in the income distribution after the application of the income tax schedule. Formally, the Kakwani decomposition states that

$$RE = VE - R = Kakwani\frac{ATR}{1-ATR} - R,$$

where VE is the vertical effect and R is the reranking effect. Furthermore, VE is a function of the Kakwani index ($Kakwani$) and the average tax rate (ATR). ATR is simply the ratio of aggregate tax revenue and total household pre-policy income. We next elaborate on the other factors - VE, $Kakwani$ and R.

Vertical effect

In order to be able to measure Kakwani's vertical effect (Urban (2009)) or in other words, to calculate the redistributive effect alone without allowing for reranking, Reynolds and

Smolensky (1977) propose an index RS which equals the Gini coefficient of gross taxable income G_y minus the concentration coefficient of net income with respect to gross taxable income $C_{\mathrm{x}|y}$. It can also be expressed as twice the area between the Lorenz curve of gross taxable income $LC_{\mathrm{y}}(p)$ and the concentration curve of net income with respect to gross taxable income $CC_{\mathrm{x}|y}(p)$ (Lambert (2001)):

$$RS = 2\int_0^1 [LC_{\mathrm{y}}(p) - CC_{\mathrm{x}|y}(p)]dp.$$

The VE in the Kakwani decomposition is actually equivalent to RS.

Kakwani index

As mentioned previously, Kakwani (1977a) not only shows that the redistributive effect captures a change in inequality alone, but he further introduces a progressivity measure known in the literature as the $Kakwani$ index. He argues that the progressivity of income taxation should be measured as the departure from proportionality of a certain tax system.[13] Following this logic, Kakwani (1977a) introduces an index to measure the progressivity of income taxation as the difference between the concentration coefficient of taxes with respect to gross taxable income $C_{t(y)|y}$ and the Gini coefficient of gross taxable income G_y. This index is thus twice the area between the Lorenz curve of taxable $LC_{\mathrm{y}}(p)$ income and the concentration curve of taxes with respect to gross taxable income $CC_{t(y)|y}(p)$ (Lambert (2001)):

$$Kakwani = 2\int_0^1 [LC_y(p) - CC_{t(y)|y}(p)]dp.$$

This index and ATR jointly determine VE and their impacts offset each other.

Reranking effect

Reranking occurs when income earners change their rankings in the population measured by the income distribution before and after tax. Before giving a suitable mathematical expression for the reranking effect, it is important not to mistake the reranking effect as being synonymous with the horizontal effect. The latter is caused by an unequal treatment of equals through the tax system. Yet, horizontal inequality does not necessarily need to *rerank* income units after taxation (Aronson, Johnson and Lambert (1994)). Hence, we can state that reranking always implies horizontal inequity, while the opposite statement is a misleading one.

[13] A proportional tax system will result in the same Gini coefficient for pre-policy and post-policy income.

As previously stated, it was Atkinson (1980) and Plotnick (1981) who introduced an index to measure the reranking effect (R) of income taxation as the difference between the Gini coefficient of net income G_{NI}, and the concentration coefficient of net income with respect to gross taxable income $C_{x|y}$. It can also be expressed as twice the area between the Lorenz curve of net income $LC_x(p)$ and the concentration curve of net income with respect to gross taxable income $CC_{x|y}(p)$ (Lambert (2001)):

$$R = 2\int_0^1 [CC_{x|y}(p) - LC_x(p)]dp.$$

If reranking exists, then, for instance, the 40% poorest income units in terms of pre-tax income will not be the same 40% poorest income units in terms of post-tax income. To know exactly what happened to the 40% of poorest income units after taxation, we compute the concentration curve of net income with respect to gross taxable income, which, in turn, helps us to separate the reranking effect from the overall redistributive effect.

In summary, the vertical effect or the pure progressivity effect, calculated by the Reynolds-Smolensky index, measures the total increase in equality caused through income taxation, while the reranking effect, calculated through the index introduced by Atkinson (1980) and Plotnick (1981), "(…) measures how much of this equalising effect is 'undone' by reranking" (Verbist and Figari (2014), p 6).

3.4 Channels of inflation-induced distortion on the redistributive effect

To mathematically conceptualise the distortions of the tax function through inflation, let us use a general formula for income taxes:

$$t(y) = s(y - a(y)) - c(y),$$

where *t(y)* represents taxes with respect to the pre-tax income level y, *s(.)* stands for the tax rate schedule, *a(.)* includes all the deductions, and finally *c(.)* stands for tax credits. In terms of *s(.)*, liability progressivity will drop with the rise of *y* if liability elasticity is monotonically decreasing (Keen, Papapanagos and Shorrocks (2000)).[14] Inflation-induced erosions of tax credits will always reduce liability progressivity. However, the effect is ambiguous as far as the erosion of deductions and tax bracket limits are concerned. Additionally, inflation erosion may not always increase the taxable income *y* (eg when considering the tax deductibility of social contributions).

[14] Liability elasticity for the German tax schedule in recent years is monotonically decreasing almost everywhere except the small range between €50,000 and €60,000 and at about €250,000 (see Figure 4).

Besides the liability progressivity, the size of the tax burden and income sharing within households due to splitting also contribute to the redistributive effect of a tax system.[15] The social insurance contributions constitute the other force in reshaping the distributions of disposable household income. Immervoll (2005) argues that it is necessary to apply the micro-simulation approach in order to quantitatively measure the impact when confronted with these indefinite distortions.

4 Data and Methodology

In this section, we illustrate the data, the micro-simulation model adopted and how the simulation scenarios are set up.

4.1 Data and micro-simulation model

In our model, all calculations are performed on income data from the Panel on Household Finances (PHF), a national panel survey about German household finances and wealth. The PHF is part of the Household Finance and Consumption Survey (HFCS), a joint effort which collects *ex ante* harmonised micro data in the euro-area countries (Von Kalckreuth, Eisele, LeBlanc, Schmidt and Zhu (2012)).

Between September 2010 and July 2011, a net sample of 3,565 households were surveyed on their balance sheets, pension claims, savings, incomes and other issues related to their finances. The reference year for these data was 2009.

[15] See Appendix A for a description of the tax treatment for partners with different pre-tax income forming the same household as well as of the structure of the tax function in Germany.

Table 1 Income variables relevant to tax conversion and microsimulation

Income variable	Household (h) or personal (p) level	Label	Accounting period variable[1)]	Variable to determine whether income flow is received throughout the year[2)]	Months received if the income flow is not received throughout the year[3)]	Gross-net option[4)]
dhg0200	h	INCOME FROM REGULAR SOCIAL TRANSFERS	dhg0100	dhg0110	dhg0120	
dhg0400	h	INCOME FROM REGULAR PRIVATE TRANSFERS	dhg0300	dhg0310	dhg0320	
dhg0600	h	RENTAL INCOME FROM REAL ESTATE PROPERTY	dhg0500	dhg0530	dhg0510	
dhg1000	h	INCOME FROM PRIVATE BUSINESSES OR COMPANIES	dhg0900	dhg0910	dhg0920	
dhg0800	h	INCOME FROM FINANCIAL INVESTMENT				available
dhg1150	h	INCOME FROM OTHER REGULAR SOURCES				
dhg1200	h	INCOME FROM OTHER IRREGULAR SOURCES				
dpg0200	p	EMPLOYEE INCOME	dpg0100	dpg0110	dpg0150	available
dpg0400	p	SELF-EMPLOYMENT INCOME	dpg0300	dpg0310	dpg0320	available
dpg0600	p	INCOME FROM STATUTORY PENSION	dpg0500	dpg0530	pg0510	available
dpg0800	p	INCOME FROM PRIVATE PENSIONS	dpg0700	dpg0730	dpg0750	available
dpg1000	p	INCOME FROM UNEMPLOYMENT BENEFITS	dpg0900	dpg0910	dpg0920	
dpg0210	p	INCOME FROM BONUS PAYMENT				available[5)]

Notes: 1) Income value can be provided on either an annual or monthly basis. This variable does not exist when we only ask for annual income. 2-3) When we ask for annual income only, this variable does not exist. 4) Respondents can provide either gross or net figures. We assume that the gross income is always reported if this option is not available. Usually, they are not liable for tax pursuant to the tax code or the respondents most probably provide the gross value given the special institutional setting in Germany (eg unemployment income (dpg1000) is partially levied since it enters the tax base to boost the marginal tax rate for the other income components due to progressivity, but the tax on it is waived). 5) In German upfront taxation, bonus income (dpg0210) is added to annual employment income (dpg0200) to determine the total annual tax liability. Then the tax liability for the annual employment income is calculated separately. The difference becomes the tax liability for the bonus income which we perceive in forming the net bonus income reported.

The tax micro-simulation model we use is developed from a module of the net-gross conversion of income which is embedded in the imputation process for the PHF. The PHF questionnaire allows the respondent to select from a flexible dimension of formats regarding income information: components (eg labour, capital, pension, social benefits,…), individual and household levels, time (yearly, monthly, quarterly, other specified duration or months whenever the flows are incomplete throughout the year), gross or net, quantity in brackets and different currencies. Table 1 summarises these specifications for all the income variables involved with tax conversion and micro-simulation.

Therefore, we have to convert all the income variables to a common unit – the annual gross concept – in order to ensure a legitimate basis for implementing imputation algorithms and miscellaneous estimates. Appendix B describes the main assumptions and procedures involved with the gross-net conversion.

The tax micro-simulation can be deemed a gross-to-net conversion where the input is all the gross income components liable to tax aggregated from all members of each tax unit. The tax unit is formed by either single persons or married heterosexual couples. Compared to the tax conversion, the procedures involved in tax class choice are omitted. We assume that spouses always file a joint tax return and that capital income is treated separately by the flat rate withholding tax (*Abgeltungssteuer*).

This model is similar in many aspects to the Siena Micro-simulation Model (SM2) and EUROMOD, particularly regarding the rules on tax treatment and social insurance contributions.[16] The main difference between the SM2 and the model we used is that we do not perform the iterative process between imputation and net-gross income conversion when some information required for conversion is missing.[17] In the case of social benefits, we only use the self-reported benefit incomes. We use all the lump-sum tax allowances for each tax unit. Instead, EUROMOD imputes the social benefits income by assuming full take-up and matches with official tax statistics to simulate the

[16] The SM2 is a flexible tool for net-gross income conversion and imputation used in some countries. EU-SILC (EU Statistics on Income and Living Conditions) data (Betti, Donatiello and Verma (2011)).EUROMOD, a tax-benefit micro-simulation model for the European Union (EU), assesses the effects of taxes and benefits on household incomes in a comparative manner. For the case of Germany in 2009, it uses EU-SILC data (Gallego Granados and Ochman (2012)).

[17] The iterative process consists of applying imputation and modelling routines iteratively and in combination (Betti et al (2011)). The iterative process seems to be ideal but is rather resource-demanding.

individual-specific tax expenses.

Table 2: Income distribution of tax payers: Wage and Income tax statistics vs PHF

Brackets of total income (Summe der Einkünfte)			2010 wage and income tax statistics		PHF	
			Persons liable for tax	%	Persons liable for tax	%
0[1)]	-	5,000	7,725,718	16.9	12,115,932	18.8
5,000	-	10,000	5,486,101	12.0	6,636,510	10.3
10,000	-	15,000	5,255,273	11.5	7,732,530	12.0
15,000	-	20,000	4,669,959	10.2	8,030,584	12.5
20,000	-	25,000	4,195,657	9.2	6,533,975	10.2
25,000	-	30,000	3,901,934	8.5	5,095,524	7.9
30,000	-	35,000	3,366,891	7.4	4,171,173	6.5
35,000	-	40,000	2,637,706	5.8	2,812,074	4.4
40,000	-	45,000	1,956,018	4.3	2,411,172	3.7
45,000	-	50,000	1,434,029	3.1	1,709,532	2.7
50,000	-	60,000	1,775,482	3.9	2,289,549	3.6
60,000	-	70,000	1,017,166	2.2	1,463,212	2.3
70,000	-	80,000	626,003	1.4	887,994	1.4
80,000	-	90,000	411,558	0.9	610,824	0.9
90,000	-	100,000	274,976	0.6	402,759	0.6
100,000	-	125,000	389,225	0.9	596,308	0.9
125,000	-	250,000	489,366	1.1	701,465	1.1
250,000	-	500,000	119,367	0.3	97,763	0.2
500,000	-	1,000,000	30,567	0.1	26,601	0.0
1,000,000	oder mehr		13,909	0.0	1,000	0.0
	Sum		45,776,905	100	64,326,480[2)]	100

Notes: 1) The bracket with negative income is collapsed into the first one since negative figures are not allowed in the PHF. 2) We assume the PHF covers all the residents liable to income tax, which is the entire population of 20 years and older, less young adults eligible for child benefit. This size estimate is close to the estimates of the population of 20 years and older between 1992 and 2005 in Table A2 of Bach et al (2013a), which is stable at around 65 million. The same estimate from the PHF is around 65.4 million.
Sources: Wage and income tax statistics (Table 3, Federal Statistical Office, 2014, p.14) and own calculation using PHF data and the same concept of total income (*Summe der Einkünft*, § 2, German income tax law (*EStG*)).

Next, we assess the representativeness of our income distribution. The weighted proportion of the total income (*Summe der Einkünfte*) earners estimated from our original data according to the fine brackets in the wage and income tax statistics (*Lohn- und Einkommensteuerstatistik 2010*, Federal Statistical Office (2014)) agrees with the official statistics satisfactorily (see Table 2).[18] One potential source of deviation lies in the fact that the wage and income tax statistics collect only information on the full population of tax filers but the population of non-tax filers is possibly incomplete.[19] Such a difference is reflected by the evidence that the count is almost always higher in the PHF data, which are supposed to cover all potential taxpayers. This table shows that there is a slightly higher coverage of low income earners, which can be accounted for by the concentration of potential non-tax filers among those marginally employed.[20] Although wealthy households are oversampled in the PHF data (Schmidt and Eisele (2013)), the top rich households are still underrepresented in the PHF income distribution.

To further assess our coverage, we calculate or collect the summary statistics for the total population, the number of tax filing units, gross market income, and relevant income components calculated from income tax statistics, PHF data and, for comparison, the national accounts. This is simply to reproduce Table A2 in Bach et al (2013a) for the case of 2009 using PHF data. That table presents a comparison of the structure of the ITR-SOEP data base with the national accounts. All the figures in our 2009 PHF estimates are very close to those in the ITR-SOEP results, particularly in 2005.[21] The tax filing units represent 57.5% of potential tax units in the 2009 PHF data.[22] The corresponding figure for the 2005 ITR-SOEP data is 57.2%. Given the fact that the total numbers of tax filing units and potential tax units should be stable within such a short time interval, this similarity implies that the PHF data represent the potential tax unit population well. Compared to the national account aggregates, our coverage of income components in the 2009 PHF data resembles that in the 2005 ITR-SOEP data. In the

[18] The 2009 wage and income tax statistics are unavailable. Since there were few changes to the income tax code between 2009 and 2010 and income distribution should be stable within such short duration in Germany, using the benchmark from 2010 should be justified. We could consider updating our distribution to 2010 in future.

[19] German legislation on filing tax returns is outlined in Appendix A. Among the non-tax filers for whom filing tax is not compulsory, the following populations are not represented or are underrepresented in the wage and income tax statistics: marginally employed, retirees/pensioners and those capital income earners with few other sources of income.

[20] Appendix A explains the cause of such a concentration from the legislative perspective.

[21] Bach et al (2013a) match the Income Tax Return data (ITR) from income tax statistics and the German Socio-Economic Panel (SOEP) in such a way as to cover the whole German income distribution (1992-2005) from the bottom to the top. This integrated data is denoted ITR-SOEP. Given the similarity of the results, we decide not to show them in order to save space. However, they are available on request.

[22] Total potential tax units consist of the entire population of 20 years and older, less young adults eligible for child benefit; married couples count as one tax unit. We estimate the total of potential tax units from PHF data instead of census statistics as in Bach et al (2013a).

2009 PHF data, the gross market income totalled 81.2% of primary income of private households, as documented by the national accounts statistics. This is in the range of the ratios in other years. The wage income coverage is 98.1% in the 2009 PHF data, which is close to and better than the counterparts in other years. Business and capital income from the 2009 PHF data as a percentage of entrepreneurial and property income from national accounts is 58.7%, again close to 57.7% of 2005 ITR-SOEP.

Table 3: Income deciles from the PHF and decile means from survey data in 2009: equivalised disposable household income (€ per year)

Decile	Thresholds from PHF	PHF	EU-SILC	SOEP
1st	8,899	5,701	6,985	4,014
2nd	12,000	10,644	10,969	9,656
3rd	14,376	13,230	13,439	12,330
4th	16,587	15,520	15,570	14,460
5th	18,750	17,648	17,664	16,573
6th	21,398	19,918	19,849	18,751
7th	24,309	22,872	22,361	21,373
8th	28,310	26,326	25,680	24,720
9th	37,241	32,018	30,707	30,284
10th	-	57,591	50,362	55,764
Overall:				
Median		18,758	18,678	18,586
Mean		22,148	21,264	21,223
Gini		32.81	29.26	29.10

Notes: The "modified OECD" scale is used for equalising incomes of households of different structure and size. The respective weights are 1 (first adult), 0.5 (subsequent adults) and 0.3 (children aged below 14).
Source: Own results using PHF data, EU-SILC micro data for 2009 and SOEP micro data from the 2010 wave referring 2009. Disposable income from SOEP has been adjusted to match the EUROMOD/EU-SILC concept where losses and costs from renting and leasing as well as capital investment have been excluded (Gallego, Granados and Ochmann (2012)).

Table 3 provides a comparison of equivalised disposable household income from different sources. The PHF results are by and large close to the other three sources. In particular, the PHF decile means are almost always within the interval between EU-SILC and SOEP for the lower half of the distribution. The PHF data overrepresent deciles six to nine compared with EU-SILC and SEOP, which potentially indicates a higher degree of oversampling. This is also reflected in the higher median and mean from the PHF data. The Gini index is also higher for the PHF. Together with the evidence in Table 2, the PHF income distribution seems to exhibit a longer and thinner tail among the top income earners.

Given these comparison results, we assert that the PHF should be capable of representing the whole distribution of income except the very top. To assess the tax system empirically, it is essential that both the overall income tax revenue and the tax burden shares as simulated by our model are consistent with the benchmark statistics. To meet the first condition, it is necessary to "reweight" the PHF income distribution to amend the missing top distribution. This exercise uses the benchmark statistics from the Income Tax Statistics (*Einkommensteuerstatistik*, Federal Statistical Office (2013)) in 2009 which releases only the tabulated distribution of tax filing units in Table 3. These statistics are preferred over the 2010 wage and income tax statistics discussed above because the former refers to 2009 and the tax unit is the direct basis in deriving income tax instead of the individual tax payer. The fluctuation of the top distribution between neighbouring years might not be negligible and almost all of them should have filed a tax return given the relatively higher incentive for them to take the rich set of allowances and deductions available only when filing a tax return. Since the PHF gathers no tax units in the brackets of € 2,500,000 or more, we multiply the weights of the richest subsample available in the PHF (those tax units in the bracket between €1,000,000 and €2,499,999.99) by a factor high enough so that the weighted aggregate gross taxable income from the tax units in the bracket between €1,000,000 and €2,499,999.99 is equal to the figure summing the corresponding figures belonging to the top three brackets starting with €1,000,000 in the income tax statistics.

4.2 Simulation scenarios

In this section, we describe the set-up we use to produce the empirical findings. For each inflation scenario, we proceed in the same manner as Immervoll (2005). We prorate all the incomes according to the inflation rate specified.[23] Micro-simulation is then performed under different inflation scenarios based on the income with and without nominal growth. The net income data under each regime are all saved toanalyse the redistributive and revenue effects.

[23] Our study also partly relies on the assumption about zero real growth in income. Brenke and Grabka (2011) argue that real gross hourly wages in Germany have stagnated on average during the last decade.

For our simulations, we used a range of inflation rates between 1 and 90 percent. Taking a historical perspective, over a few periods when there is no adjustment in combating bracket creep in Germany, the loss of real purchasing power as inflation accumulates can climb by a non-negligible degree. As mentioned above, the German government only takes discretionary measures against bracket creep by mixing such a purpose into various tax reforms aimed at relieving the general tax burden. In a narrow sense, only the year with tax relief for the whole population to lower the average rates, progressivity and total tax revenue can be deemed a moment to offset the bracket creep effect.[24] As documented by Corneo (2005) and Bach et al (2013a), there have been two periods, since 1958, in which there was no such tax relief in Germany.[25] One is from 1958 to 1974, and the other is from 1991 to 2000. Cumulated inflation is 71.4% for the former, and 26.9% for the latter.[26] We observe that there is almost a continuous increase in average tax rates and progressivity from 1958 to 1990. This period can also be considered, in a broad sense, as an inflation cumulating duration in the context of bracket creep, with (cumulated) inflation accruing to 185.4%. Recently, there was no major tax relief between 2006 and 2014. This period produces cumulated inflation of up to 15.3%. By taking various standards, our simulated inflation scenarios seem to be reasonable.

Throughout our paper, the economic income concept is adopted such that all sources of incomes are counted regardless of their taxability. Our gross income includes wage income, business income, capital income, private and public transfer income (see Bach et al (2013a) for the definitions which we also apply). The net, or disposable, income is derived as gross income minus income taxes, solidarity surcharge, church tax, capital income tax and social insurance contributions.

In this exercise, we profit from the strength of the micro-simulation approach, which "*(…) lies precisely in its ability to analyze one type of change at a time while holding 'everything else' constant*" (Immervoll (2005), p 44). On the other hand, we recognise

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[24] One major approach to reduce the general tax burden is to drop the average tax rate for all. In terms of bracket creep, depressing the progressivity can also work, sometimes independent of the change in average tax rate. Besides these options, other policy changes can also deflate the effective tax load, eg by shrinking the tax base, which can be indirectly measured by the change in tax revenue.

[25] The author chooses 1958 as the starting year, when income splitting for spouses was introduced into the current system of family taxation.

[26] We calculate these from the annual inflation available in the "Consumer Prices (MEI)" dataset of OECD statistics (http://stats.oecd.org/index.aspx?DatasetCode=MEI_PRICES, accessed on 7 May 2015).

that the accuracy of our empirical findings might be enhanced by taking into account potential behavioural reactions to a real income drop owing to bracket creep. Furthermore, there are some differences between our simulation environment and reality. Similar to the drawbacks in the EUROMOD summarised by Immervoll (2005), and in spite of the fulfilling recovery of the distribution reported by income tax statistics and other sources, there are still uncertainties in defining what is counted in a given tax category, tax evasion, less than perfect representation of tax rules in model algorithms and, importantly, shortcomings in the underlying micro-data, such as underrepresentation of high income groups or missing information about tax expenditure. Furthermore, while our reweighting on the top rich should have pulled the revenue effect in to line with income tax statistics, we do not recover the distribution perfectly. The real income distribution should be more concentrated because we retrieve the missing income from the top rich by inflating the population of the less rich subgroup instead of imputing this small group at the very end of the income distribution. We can then infer that our result is simply a conservative estimate.

5 Empirical Findings

This section provides quantitative answers as to how bracket creep changes the equalising effect of the IT and SIC system.

5.1 Inverted U-shaped relationship of RE and inflation rate

Before proceeding with the empirical analysis, we should review the concepts on RE defined in section 3.2. Figure 1 illustrates the measurement of RE and the further decomposition, which can reinforce our intuitive understanding of the following results. RE is simply the area between the Lorenz curves (LC) for taxable and net income. If we assume the population rankings were not disturbed by the redistribution, our measure of RE is exactly the VE, which is the area between the LC for taxable income and the concentration curve (CC) for net income. In most cases, the rankings using taxable and net income are not constant. Therefore, the difference between RE and VE constitutes the reranking effect (R) which is, as observed from Figure 1, the area between CC for net income and LC for net income.

Figure 1 A graphical exposition of the redistributive effect, vertical effect and reranking

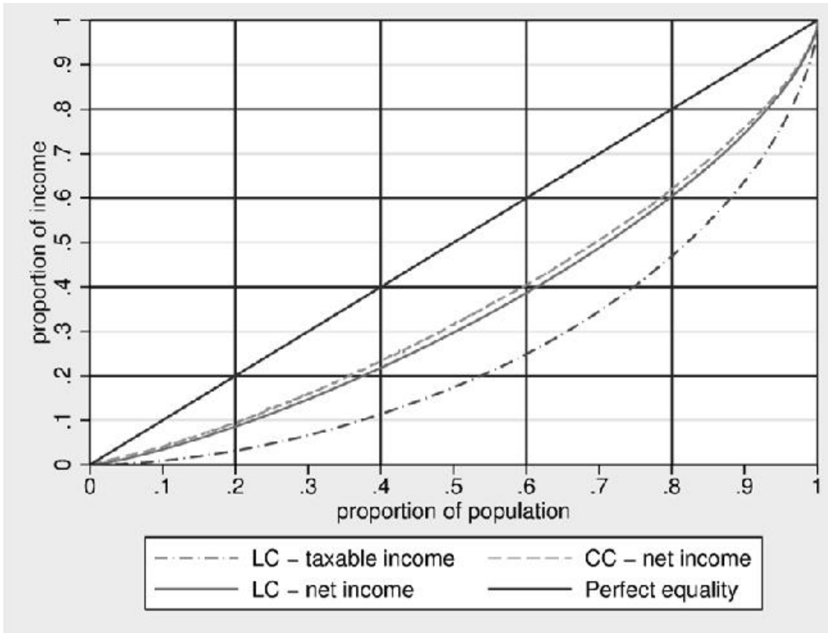


Table 4: Loss of household net incomes as a percentage of gross income induced by bracket creep under different inflation scenarios

Inflation rate %	Deciles									
	1	2	3	4	5	6	7	8	9	10
1	-0.031	-0.045	-0.076	***-0.186***	-0.162	-0.157	-0.161	-0.137	-0.119	-0.101
2	-0.039	-0.074	-0.147	-0.306	-0.300	-0.309	***-0.311***	-0.268	-0.227	-0.197
4	-0.062	-0.146	-0.293	-0.549	-0.571	***-0.608***	-0.607	-0.524	-0.440	-0.386
5	-0.070	-0.182	-0.371	-0.669	-0.706	***-0.756***	-0.753	-0.650	-0.545	-0.478
10	-0.143	-0.392	-0.783	-1.281	-1.365	***-1.468***	-1.449	-1.233	-1.051	-0.918
15	-0.315	-0.640	-1.213	-1.885	-2.006	***-2.157***	-2.088	-1.772	-1.530	-1.329
20	-0.371	-0.891	-1.646	-2.473	-2.638	***-2.807***	-2.677	-2.283	-1.991	-1.716
25	-0.475	-1.182	-2.082	-3.048	-3.249	***-3.416***	-3.222	-2.770	-2.434	-2.079
30	-0.529	-1.558	-2.526	-3.604	-3.830	***-3.996***	-3.708	-3.221	-2.857	-2.421

Notes: Decile groupings are determined by the distribution of the equivalised household disposable income (EHDI) in the regime without indexation. The "modified OECD" scale is used for equivalising incomes of households of different structure and size. The respective weights are 1 (first adult), 0.5 (subsequent adults) and 0.3 (children aged below 14). Relative loss is calculated for each household as the difference of net income before and after bracket creep divided by the gross income and multiplied by 100. Each cell contains the decile mean of the relative loss from all the households within deciles. In each row, darker background shading emphasises lower values. The smallest value per row is in bold and italics.
Source: Tax micro-simulation model using PHF data.

We first examine how differently German households along the net income distribution would incur losses from bracket creep under different inflation scenarios. Table 4 presents this evaluation. Every column depicts the mean percentage loss of German household net income with respect to gross income in every decile of the distribution.[27] For instance, with the nominal gross income growing at the same pace with 1% of inflation, the household in the 7th decile can experience a loss of real net income averagely equivalent to 0.161% of their gross income. Irrespective of the decile observed, the higher the inflation rate involved in bracket creep, the higher the loss of all household incomes. Nevertheless, regarding the losses in household income distribution, an uneven development becomes clear. The poorest as well as the richest households will not lose as much in relative terms from bracket creep as those households at the middle of the income distribution. This pattern is strongly related to the varying progression along the income tax schedule. A large number of household incomes will remain below the basic tax allowance while, at the opposite end of the income distribution, the top two richest deciles will face the flat marginal income tax rates. At the same time, all the other household incomes that are in the more progressive part of the income tax function will thus experience a higher relative income tax load for every additional euro they earn. Bach et al (2013b) gather similar results when analysing the relief effect for households after applying an indexation proposal which recovers the average tax rates for almost all parts of the income distribution.[28]

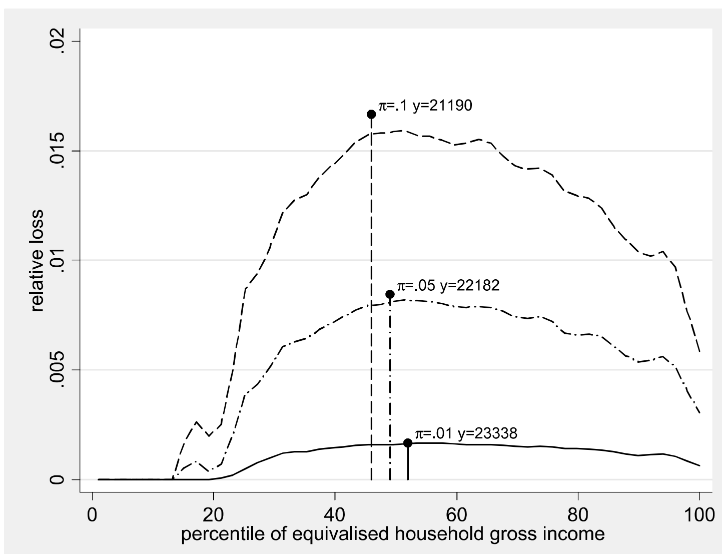
In addition, the rank of relative loss is not constant under different inflation scenarios. As inflation grows above 1%, the poorer receive a greater relative loss than the richer as the shading becomes increasingly darker in the lower part of the distribution represented by the heat map. The largest loser shifts from the richer middle class (7th decile) to the poorer middle class (6th decile; see the column highlighted in bold font). The exception at the case of 1% seems to be driven by the outliers. As a robust illustration, we present the distribution of relative loss in an alternative Figure 2 when the inflation rate is 1%, 5% and 10%. The relative loss is defined by the same approach as before. However, this display differs from the previous tabulation in three respects: first, the income

[27] Table 13 in Appendix C contains the decile mean value of the absolute loss. For instance, in terms of equivalised household disposable income, the households in the 6th decile lose an average of €153 for a one-shot nominal income growth at 4% cumulated inflation. Due to tax progressivity, the absolute loss of bracket creep grows monotonically with the income level.

[28] This indexation is thus a mirror image of bracket creep on the tax schedule.

distribution is defined over the equivalised gross household income (EGHI); second, the percentile is used to decompose the income distribution; third, the median of relative loss is calculated for each percentile. This figure demonstrates the local polynomial smooth plot of the bivariate relationship between relative loss and income distribution and ignores the underlying scatterplot for clarity.[29] We also mark the peak and the associated EGHI where the relative loss achieves the maximum for each scenario. Besides the bell curve development of the relative loss and ever increasing relative loss with inflation as observed from previous tabulation, we can identify exactly the EGHI at which the relative loss starts to fall. For example, when nominal income growth is 5% for the whole distribution, the relative loss due to bracket creep increases with EGHI, as long as it is smaller than € 22,182, and falls thereafter. The monotonic downward trend of the maxima as inflation grows starting from 1% supports our outlier judgment for the case of 1% in the previous tabulation.

Figure 2 Relative loss (change of net income divided by gross income) over gross income level owing to bracket creep



Notes: The deciles for the equivalised household gross income are 1st – 9,533, 2nd – 13,000, 3rd – 16,000, 4th – 19,000, 5th – 23,000, 6th – 27,070, 7th – 32,361, 8th – 40,000 and 9th – 53,733. Three inflation scenarios (1%, 5% and 10%) are illustrated and indicated on the top of the fitting cureves. Besides, the peak and the associated equivalised gross household income (*y*) where the relative loss achieves the maximum for each scenario are represented by the droplines.
Source: Tax micro-simulation from PHF data.

To investigate the underlying mechanism, we turn to the Kakwani decomposition of RE

[29] The 95% confidence interval almost collapses along the curves so that we choose not to display it.

introduced in section 3. Table 5 contains this result by presenting the redistributive effect of the IT and SIC system hit by bracket creep under different inflation scenarios.

Table 5: Redistributive effect (RE) of the IT and SIC system with bracket creep under different inflation scenarios

Inflation rate %	G_pre	G_post	RE	ATR	Kakwani	VE	R
0	0.4000	0.3287	0.0713	0.2559	***0.2221***	0.0764	0.0051
1	0.4000	0.3287	0.0713	0.2571	0.2209	0.0764	0.0051
4	0.4000	0.3285	0.0716	0.2602	0.2181	0.0767	0.0052
10	0.4000	0.3282	0.0718	0.2662	0.2125	0.0771	0.0053
20	0.4000	0.3280	***0.0720***	0.2754	0.2035	***0.0773***	0.0054
30	0.4000	0.3281	0.0719	0.2837	0.1952	0.0773	0.0054
60	0.4000	0.3292	0.0708	0.3042	0.1746	0.0763	***0.0055***
90	0.4000	***0.3305***	0.0695	***0.3198***	0.1593	0.0749	0.0054

Notes: G_pre represents the Gini of equivalised gross income and G_post is the Gini of EHDI after inflation. RE represents the redistributive effect, ATR denotes the average tax rate, Kakwani means the Kakwani index, VE stands for the vertical effect and R is the reranking effect.
In each column, darker background shading emphasises higher values. The highest value per column is in bold and italics.
Source: Tax micro-simulation model using PHF data.

It becomes evident that there is an inverted U-shaped RE evolution when inflation mounts up. This simply echoes the evidence found above on the relative loss. The highest RE is associated with about 20% inflation. In the other exercise, in which all the social benefit incomes lag in nominal compensation, the peak arrives at about 10% of inflation.[30] This fact is in contrast with Immervoll (2005), who shows that fiscal drag always intensifies the RE and this effect is monotonic with inflation. Similar to his study, bracket creep worsens the progressivity of the IT and SIC system as the Kakwani index drops when inflation grows but the ever increasing average tax counteracts with the progressivity effect. We show that in 2009 the effect of average tax no longer dominates as claimed by Immervoll (2005). Since RE is the sum of VE and R, and R is much smaller in relative terms, VE also bears the pattern of inverted U-shaped development, which is the function of the Kakwani index and ATR.

To explore the causes of the inverted U-shaped development, we further decompose the RE introduced purely by bracket creep into two parts of the contributions: income tax schedule (ITS) and deductions – taxable income – social insurance contributions (DTS) as explained in section 3.4. To do so, we isolate the RE due to bracket creep (on ITS

[30] See Table 14 in Appendix D. In this scenario, there is zero nominal growth of social benefit incomes but full inflation compensation for all other sources of income.

and DTS) from the part inherent in the IT and SIC system itself.[31] We perform an indexing exercise to separate the effects on ITS and DTS. This indexation is to adjust all the slopes and corners on the bracket such that the nominally inflated taxable income should be subject to the same average tax rate as the one when income is not inflated. As mentioned above, this indexation is a mirror image of the bracket creep on the tax schedule only. Table 6 presents this result.

Table 6: Separate redistributive effects (RE) owing to the bracket creep effect on the income tax schedule (ITS) and deductions - taxable income - social insurance contributions (DTS)

Inflation rate %	ITS							DTS
	G_pre	G_post	RE	ATR	Kakwani	VE	R	RE
1	0.3210	0.3281	-0.0071	***0.0152***	-0.4943	-0.0076	***-0.0005***	***0.0071***
4	0.3212	0.3279	-0.0067	0.0120	-0.5993	-0.0073	-0.0006	0.0067
10	0.3216	0.3276	-0.0061	0.0056	-1.1828	-0.0066	-0.0006	0.0061
20	0.3222	0.3275	-0.0053	-0.0046	***1.3217***	-0.0060	-0.0006	0.0053
30	0.3228	0.3277	***-0.0049***	-0.0142	0.3992	***-0.0056***	-0.0007	0.0049
60	0.3238	0.3288	-0.0050	-0.0399	0.1544	-0.0059	-0.0010	0.0050
90	0.3239	***0.3302***	-0.0063	-0.0607	0.1325	-0.0076	-0.0012	0.0063

Notes: For the ITS, G_pre represents the Gini of EHDI after indexation and G_post is the Gini of EHDI after inflation but without indexation. Here the sequence of pre- and post- policy (bracket creep) EHDIs is the inverse of the one in our simulation steps; the latter has the indexation as the mirror policy of the former. For the DTS, G_pre represents the Gini of EHDI when inflation is nil and G_post is the Gini of EHDI after indexation. RE represents the redistributive effect, ATR denotes the average tax rate, Kakwani means the Kakwani index, VE stands for the vertical effect and R is the reranking effect. To save space, we only demonstrate the RE for the DTS.
In each column, darker background shading emphasises higher values. The highest value per column is in bold and italics.
Source: Tax micro-simulation model using PHF data.

Surprisingly, RE is negative for the bracket creep on ITS. If there is no change between gross income and taxable income as defined in the tax code, bracket creep can be overall regressive in terms of the redistribution through ITS only. A dual tax system between income tax and capital income tax induces such a remarkable change. Through a yield test, taxpayers can file capital income tax together with non-capital income as long as the total tax liable drops compared with the one, as a default, from the sum of two separate taxes – a capital income tax withheld at source and an income tax on non-capital income. This potential saving consists of two parts. The first part derives from capital income which depends on the difference between the marginal tax rate of the

[31] See the notes below Table 6 for the approach to reach such decomposition.

total taxable income (aggregating capital taxable income and non-capital taxable income) and the flat capital income tax rate of 25%. The second part comes from the non-capital income which is determined by the difference between the marginal tax rates of two tax bases – non-capital taxable income added with and without capital taxable income. These two parts of savings, for most at both ends of the distribution, are meaningless because their marginal tax rates are far away from 25% and they are situated in the flat tax rate (either zero or the top rates, 42% or 45%). Therefore, only the middle part of distribution is relevant. Due to the progressivity and the positive relationship of the concentration of capital income and total income, both parts of saving are more likely to fall below zero for the relatively richer middle class as nominal income grows. Consequently, the extensive margin is higher for some of them to switch from filing both incomes together to the default state. The intensive margin of such a switch is also higher for the richer at the middle of distribution as trivially inferred from the structure of these two parts of savings. On the whole, the part of bracket creep induced by the tax schedule renders a regressive redistribution. It reflects that the effect from the yield test of a dual tax system dominates the effect from tax progressivity.

The overall equalising effect measured by RE from income tax again bears an inverted U-shaped relationship with the inflation rate. This should be explained again by the interaction of progressivity (Kakwani index) and the average tax rate. However, we also observe the kink points on both evolutions of the average tax rate and the Kakwani index. As inflation moves above 20%, the average tax load starts to become negative and Kakwani switches from negative to positive. This means some populations even gain considerably from the bracket creep on tax schedule only. The intensive margin of the switch discussed above contributes to such a gain.

The overall RE development from DTS, as inflation rises, illustrates the monotonically decreasing progressivity. Therefore, we can infer that the offsetting redistribution effects developed with the growth of the inflation rate between ITS and DTS introduce the other channel for the inverted U-shaped relationship in the overall IT and SIC system, besides the interaction between progressivity of the tax schedule and the ever increasing tax burden.

5.2 Revenue effect of bracket creep

We next evaluate the revenue effect of bracket creep. Table 7 demonstrates the tax revenue gain from bracket creep under four different inflation scenarios. For instance, if inflation as well as nominal incomes had increased by 4%, the government would gain around €5.73 billion more in real income tax revenue, even though real incomes experienced no change at all. This €5.73 billion represents a redistribution of income from households to the government which is to be fully attributed to the effect of bracket creep.

Table 7: Revenue gain after bracket creep under different inflation scenarios

Inflation rate %	Real revenue gain (in € billion)
1	1.44
4	5.73
10	14.10
15	20.81

Source: Tax micro-simulation model using PHF data.

We observe that the ratio of real revenue gains in either of the two scenarios is very close to the ratio of their inflation rates. Under a moderately low inflation rate, this can be considered as a rule of thumb as we can show that the relative real revenue gain is more or less the inflation rate – which generally holds under both the flat rate and piecewise-linear taxes with lump-sum allowances.[32] In a country where there is no automatic indexation, it is not unrealistic to build up relatively high (cumulative) inflation, implying that the revenue effect can grow to a considerable degree from

[32] Let the tax schedule function be $t(y) = ay + b$, where y is the taxable income, for a bracket in the range of p to q. The total tax revenue under no inflation is thus $r_0 = \int_p^q t(y)dy = \frac{1}{2}a(p^2 - q^2) + b(p - q)$ if there is a uniform income distribution on this bracket (this can be considered as an approximate to reality or we can imagine dividing one bracket into however many brackets where each distribution is close to uniform). It is then trivial to derive the total real tax revenue under inflation $r_\pi = \int_{p(1+\pi)}^{q(1+\pi)} t(y)dy/(1+\pi)$ and show $(r_\pi - r_0)/r_0 \cong \pi$ as long as b is negligibly small compared with $a(p+q)$, which is the case for Germany. Summing up all the brackets, we should achieve the same result. We ignore the jump up to the higher bracket for the top end as we assume a moderately low inflation rate and some degree of offsetting from the other top end in the lower bracket.
The quadratic term in the German tax schedule function is almost zero. Thus, the German tax schedule function approximates to a piecewise-linear tax with lump-sum allowances.

bracket creep, unless there are some other compensation mechanisms.[33] Then the size of redistribution can grow proportionally with cumulated inflation as we show in Table 7 even when inflation does not pick up dramatically.

6 Discussion

Our finding of the inverted U-shaped development of the redistributive effect would contribute more economic insight if we also studied the relevant root in income distribution and tax schedule between 1998 (reference year in Immervoll (2005)) and 2009. On the other hand, our evaluation can shed some light on the discretionary inflation indexation which is typically practiced in Germany.

6.1 The underlying changes in income distribution and tax schedule between 1998 and 2009[34]

To understand how much income distribution and tax schedule changed between 1998 and 2009, we plot the taxable income distribution from PHF data as well as the marginal tax rate for 2009 and align them with the same plot for 1998 (Figure 1, Immervoll (2005)). Such a comparison is shown in Figure 3.[35] We observe that the whole income distribution shifts upwards. In particular, the highest peak at the very low end grows at the fastest pace.[36] Using official income tax statistics, Table 4 in Hechtner et al (2012) documents the same pattern with the poorest two decile means of taxable income increasing the most between 1998 and 2009. As for the tax schedule, the marginal tax rate drops everywhere. Corneo (2005) indicates that such a sharp downturn started in 1999 together with progressivity and tax revenue.[37] Figure F.1 in Online Appendix depicts this drop for local progression (ie, an increase in residual progression almost everywhere). The most significant evolution occurs at the highest peak in the

[33] Germany employs these discretionary alternatives in offsetting the revenue effect, which usually does not fully account for the redistribution effect.

[34] A rigorous examination should be built on an analytical tool linking local progression and global progressivity. Jakobsson (1976) and Kakwani (1977b) present the theories. Nevertheless, their conditions are too strong and of little practical value for the German tax system. Our exploration borrows the intuition from them but can be speculative.

[35] The rich tax (*Reichensteuer*) area is not included. With the domain extended to cover it, none of the conclusions change.

[36] This is a result of the behavioural response predicted from the standard labour supply model (see the analysis and empirical evidence presented by Saez (2010)).

[37] Figure 1 in Bach et al (2013a) documents the decrease in average and marginal tax rates everywhere between 1998 and 2005 (the tax schedule of 2009 deviates little from that of 2005).

density plot of taxable income. It moves from an area much below the basic allowance in 1998 to somewhere in the second tax bracket. The outcome of this evidence is the dramatic increase between 1998 and 2009 in local empirical progression and the tax burden for the lower part of taxable income distribution, specifically the second decile (corresponding to the highest peak), which is documented in Figure F.2 and Figure F.3 in Online Appendix. The effect of income growth dominates the overall downgrading of local progression for this part of distribution. On the other hand, the tax rates and progression drop more remarkably at the middle and top of the distribution (see also Corneo (2005) and Bach et al (2013a)).[38]

There can be two parallel versions of analysis on progressivity of the tax schedule by adopting either liability or residual progressions as introduced in section 3.2. We first explore by using the latter approach since it can better accommodate the joint development of the income distribution and tax schedule. The other approach is then offered as a comparison of the same analysis in Immervoll (2005).

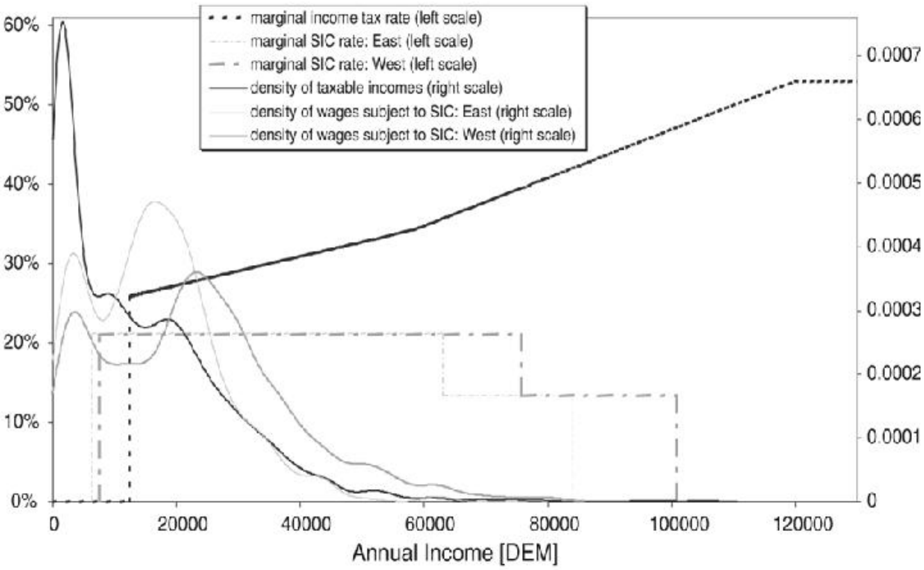
We can now consider the bracket creep effect as the shift of income distribution to the right with the tax curve fixed. In 1998, as the local maxima of taxpayer density moved to start paying tax and towards the more progressive tax zone, global progressivity was gradually enhanced. But in 2009, this mass of population was already situated in the very progressive area and heavily levied (see Figure F.2 and Figure F.3 in Online Appendix).[39] Moving to the right means that they are pushed to a less progressive area. Regarding the right tail of the distribution, it is still far from the flat tax zone contributing little to progressivity in 1998. But it is not so far in 2009. Additionally, the middle and high income groups are confronted with much lower progression as stated above. Their evolution due to bracket creep becomes the other channel to lower the redistributive effect of the IT and SIC system in 2009 compared with 1998.

[38] Besides the decline in tax rates, structural change between 1998 and 2009 in the tax system fuels the drop in progression markedly: the threshold to enter the top brackets with marginally flat tax is lower so that more of the middle income population is no longer subject to directly progressive tax (the marginal tax rates increase with taxable income); additionally, capital income was excluded from the progressive tax schedule and replaced with a flat rate much lower than the top marginal tax rates (25% compared with 42% and 45%) which dampens the tax load and progression of the middle and top distribution more than the lower part.

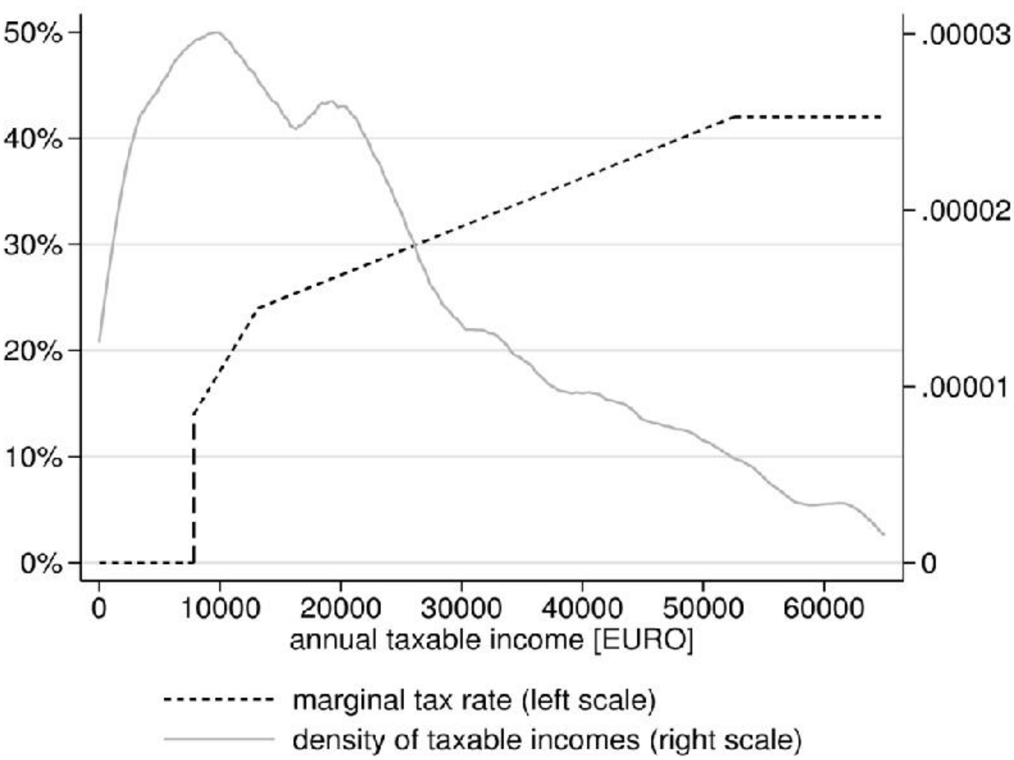
[39] These areas for both years correspond to the first valley at the basic allowance in the residual progression curves as displayed in Figure F.1 in Online Appendix.

Figure 3: Taxable income distribution and marginal tax rates in Germany (1998 vs 2009)

A. 1998

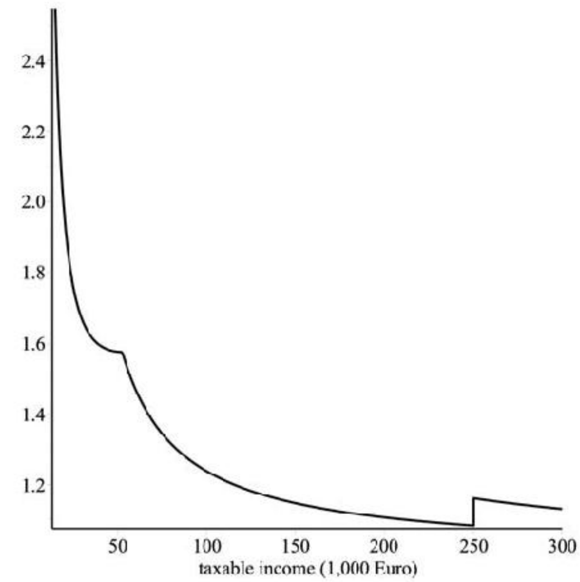


B. 2009



Source: A. Figure 1, Immervoll (2005); B. own derivation using PHF data

Figure 4 Liability progression of the German income tax schedule (2009) over taxable income



Source: Own derivation

The above analysis takes the perspective of residual progression. We can also perform the other version of analysis by adopting liability progression and linking it to global progressivity via the Kakwani decomposition. Figure 4 plots the liability progression of the 2009 German income tax schedule.[40] In 1998, the evidence that the local maxima of distribution from the bottom would join the "tax-paying club" was the main driver to lower liability progressivity. However, this movement from the bottom part of the distribution is slightly offset by a movement in the opposite direction as the middle and top of the distribution are simultaneously pushed to a more progressive tax zone. On the other hand, many households have their taxable income inflated and spread onto the linear progressive part of the tax schedule (the most progressive zone) from the zero tax

[40] The basic shape of the counterpart for the 1998 tax schedule resembles this one.

zone. Consequently, there is a significant increase in the overall tax burden, which Immervoll (2005) empirically shows, dominating the decreasing liability progressivity. In 2009, the offsetting force against lowering liability progressivity was weaker or probably even reversed, as we can draw from the discussion above. Furthermore, the increase in the average tax rate as induced by nominal income growth was also weaker since the tax rates were cut strongly at the middle and the top of distribution and less of the population switched from paying nothing to a positive tax. The role of the rising average tax burden does not dominate much, especially when inflation is cumulated up to a certain point.

6.2 Delaying indexation

Given our results, the debates on the cost – benefit analysis of specific inflation targets might benefit from including the redistributive effect through bracket creep, which is echoed in Altig and Carlstrom (1991, 1993). Besides the buildup of sizeable income redistribution from households to the government as shown in the preceding section, bracket creep may raise concerns about inequality. We argue, based on our result, which differs from Immervoll (2005), that delaying indexation aimed at fixing the distortion of tax redistribution, and allowing inflation to cumulate, does not always enlarge the equalising effect of the IT and SIC system. Delaying indexation for long enough can yield different redistribution implications.

6.3 Is the German evidence relevant for other countries?

Immervoll (2005) documents that the monotonically enhancing equalising effect introduced by higher inflation-induced erosion is present not only in Germany but also in the Netherlands and the UK. The assessment in Levy, Nogueira, Siqueira, Immervoll and O'Donoghue (2010) yields the same pattern for Brazil's income data in 2003. As nominal income grows over the whole distribution, the average tax rate is higher and the liability progressivity as indicated by the Kakwani index is lower in all these countries. These dynamics still persist in our simulation. But the dominant influence from the increasing average tax rate is not preserved. In our result, the increasing tax burden for the rich cannot compensate the reduced progressivity. The question that follows is whether this is a development specific only to Germany and whether the role of falling progressivity also expands to turn around the positive growth path of the redistributive effect in other countries' tax-benefit systems when inflation cumulates. This investigation can be very meaningful for all the countries without automatic inflation indexing or with only infrequent adjustment.[41]

Using EUROMOD on the income distribution of EU countries in both 1998 and 2008, Verbist (2004) and Verbist and Figari (2014) find two lines of relationships between inequality and the redistribution preference to be always significant: a negative association between the pre-tax income inequality and the average tax level as well as a negative one between the average tax level and local progressivity, as measured by the Kakwani index. The former relationship seems to confirm the perception that more unequal societies demand less redistribution by lowering the average tax level.[42] The latter can be interpreted as a result of the political bargaining process. It is easier to convince the higher part of the distribution to take on a greater tax burden when the tax weight is rather mild. But when the aggregate tax volume reaches a certain level, a much broader part of the distribution would be required by the others to share the burden so that their relative difference is less prominent.

Over the last decade many studies have demonstrated the remarkable rise of pre-tax income inequality (OECD (2008b and 2011) and Salverda, Nolan, Checchi, Marx, McKnight, Tóth and van de Werfhorst (2014)). According to the relationships discussed above, we should observe a trend of a lower average tax level as well as higher progressivity in those countries with growing inequality. These combinations represent a new range of both the average tax level and the Kakwani index which have not been documented in the previous tax-benefit systems before and after bracket creep.[43]

[41] OECD (2008a) shows that there are several countries which do not implement automatic inflation indexing in Annex Table S.A4. Box 1 in the same paper reports that the scope of adjustments is not complete, time lags can be notable and the targeted compensation may be insufficient if the inflation threshold is not reached.

[42] In the political economy literature, the "Meltzer-Richard model" argues that growing inequality tends to impoverish the median voters and hence lends support to more redistribution. The "Moene-Wallerstein model" disagrees but points out that common factors exist for both higher inequality and less redistribution, eg capital market imperfection, strong belief in individual responsibility and the market. See section 5 of Verbist and Figari (2014) for this debate. Also note that the scope and measurement adopted by the studies in Verbist (2004) and Verbist and Figari (2014) are close to ours although the empirical evidence on this debate is not unanimous due to different data sources and years of studies.

[43] Following our argument, the starting point of the average tax rate in our 2009 German case is lower than that in 1998 reported by Immervoll (2005) and the starting point of the Kakwani index in the 2009 case is higher than the counterpart in 1998. As discussed above, the empirical results always present an increasing average tax rate and a declining Kakwani index as inflation grows. Therefore, as inflation climbs, the simulated range of the average tax rate may always be lower in our study than that in

Therefore, it leads us to postulate a possible switch of regime such that reduced progressivity plays a more pivotal role in changing the redistributive effect when inflation accumulates. Potentially, the lower average tax level and higher progressivity can be the driving forces such that the marginal contribution from the former is weaker and the marginal contribution from the latter is stronger as inflation increases.

7 Capital income grows faster under flat rate tax

In the next stage of simulation, we start by introducing the environment of $r > g$, which sets the inflation compensation to differ between capital and non-capital incomes, with the former always larger. There are two groups of scenarios: r and g are homogeneous or heterogeneous. First, the assumptions are proposed. Outcomes and investigation then follow.

7.1 Assumptions and scenarios

A more interesting question arises. How would the redistributive effect of the IT and SIC system under bracket creep change if there were a dual tax system as well as $r > g$? We provide a simulation exercise below using our German case. However, this is not an issue specific only to Germany since such a dual income taxation has been introduced in Nordic countries, Austria and the Netherlands. Before describing the scenarios to be simulated, we first lay out and justify our assumptions.

One straightforward scenario to start with is to allocate the constant nominal growth rates (r and g) for both capital (C) and non-capital (N) incomes over the whole population and set r/g, as a measure of relative inflation compensation between capital and non-capital incomes, to be a number larger than one.[44] Specifically, we consider three combinations of r and g: π and π, π and $\pi/2$ and 2π and $\pi/2$. With this setting, it makes sense to compare the first and second combinations as well as the second and third combinations to identify the effect from larger r/g, since, at least, one component of incomes grows at the same pace for the same π. To demonstrate the contribution of

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Immervoll (2005) and the simulated range of the Kakwani index in our study may always be higher than that in Immervoll (2005).

[44] We associate g with the total non-capital income which mainly includes the labour income for most of the population. Also, for clarity, we actually treat r and g as the growth multipliers instead of growth rates in the following text. For example, the C after nominal growth is rC.

the dual tax system to redistributive effect, a parallel simulation is always executed, replacing the dual tax system by a comprehensive tax system for each combination. We restore the comprehensive tax system by adding all the capital incomes to rental income and leaving capital incomes at zero. Under German tax law, rental income is included in the non-capital income tax base. Throughout the paper, the scope of capital incomes in simulation is, in line with German tax law, the capital return from private businesses or companies, financial investment and private pensions.[45]

It is trivial to speculate that imposing the comprehensive tax system to subject capital income to progressive tax should enhance the redistributive effect of the IT and SIC system. As capital income is concentrated, incorporating capital income into the same income tax base should raise the average and marginal tax rates of the top distribution. Additionally, by dropping the dual tax system, we eliminate one significant regressive redistribution channel through bracket creep in the German dual tax system revealed in section 5.1 – there is no longer any option available for the rich middle class to switch between filing capital income together with non-capital income as in the comprehensive tax system for tax return and treating them separately as in the dual tax system. However, when $r > g$ is introduced, the total effect of bracket creep might not be clear. On the one hand, $r > g$ suggests that the total gross income would increase faster for the rich than for the poor because of the concentration of capital income. As a result, the tax rates for the rich would be boosted even more than the others, which helps to equalise the distribution. But the faster growth of nominal income at the top could also mean the opposite because more rich households would be pushed into the flat rate zones in the German tax schedule, as illustrated in section 6.1, which reduces the redistributive effect through bracket creep.

As inflation hits, everyone seeks to secure inflation compensation from the various income sources owned. Under this reality, the previous group of scenarios seem to be

[45] As shown in Table 1, they correspond to dhg1000, dhg0800 and dpg0800. dhg1000 should include profit distributions from a participating interest in a non- publicly traded business or a private partnership not wholly owned by a household. dhg0800 is supposed to be mainly dividend and interest income. dpg0800 covers the distributions from private pensions. There is not enough information to derive the share of earnings from a private pension which is taxable (*Ertragsanteil*). We treat 100% of dpg0800 as capital income. Alternatively, we also experiment with a share of 18% which is considered, according to the German Tax Code (Abgabenordnung), as the capital-earning part of most other private pensions which are not occupational or Riester or Rürup pensions. The results change little.

rather unrealistic since r and g, in particular, r/g, could be as heterogeneous as the income composition can be. A more reasonable assumption is that r/g depends on the relative share k of both incomes (ie C/N) in each household.[46] The most plausible postulation is that r/g is positively related with k, which argues that each household makes more of an effort and/or has more bargaining power to demand inflation compensation from the income source which contributes the higher share. Alternatively, we also entertain with a contrasting idea that r/g is negatively dependent on k.[47] Additionally, we always assume that households with maximum k can obtain the largest r/g among their choice set (*maximum capital income bargaining restriction*). This is plausible since these households should have virtually no non-capital income as capital income concentration is quite extreme.[48]

As before, we still assume that total nominal growth only compensates the aggregate real loss due to inflation but not beyond. This is represented by an identity $rkN + gN = (N + kN)(1 + \pi)$, which is transformed as the *inflation compensation restriction* $F(r, g; k) = rk + g - (1 + k)(1 + \pi) = 0$, where π is the inflation rate. This again reflects that there is only inflation-driven nominal growth. Furthermore, in doing so, we do not have to confront the difficulty, inherent in the previous homogeneous r and g group of scenarios, of separating the redistributive effect introduced by the bracket creep effect on the IT and SIC system from the effect induced by the shift of pre-tax income.[49]

Finally, we impose $g \geq 1$ to reflect a *wage rigidity restriction*, and the core, $r > g$ *restriction*.

[46] We assume that households with the same k share an identical r/g.
[47] This scenario might occur as a result of government interference in wealth concentration, for example, by rationing the capital return conditional on the income composition. A portfolio reshuffling can also be a driving force, for instance, due to asymmetric information on return-risk generated by a varying degree of involvement in the capital market proxied by k.
[48] The maximum k observable can be around 150.
[49] To compare two income -tax systems which differ in both dimensions, Pogorelskiy, Seidl and Traub (2010) as well as Dardanoni and Lambert (2002) independently proposes two approaches. It is beyond the scope of this paper to proceed in that direction.

Figure 5 Simulation assumptions and scenarios for compensating inflation when r>g

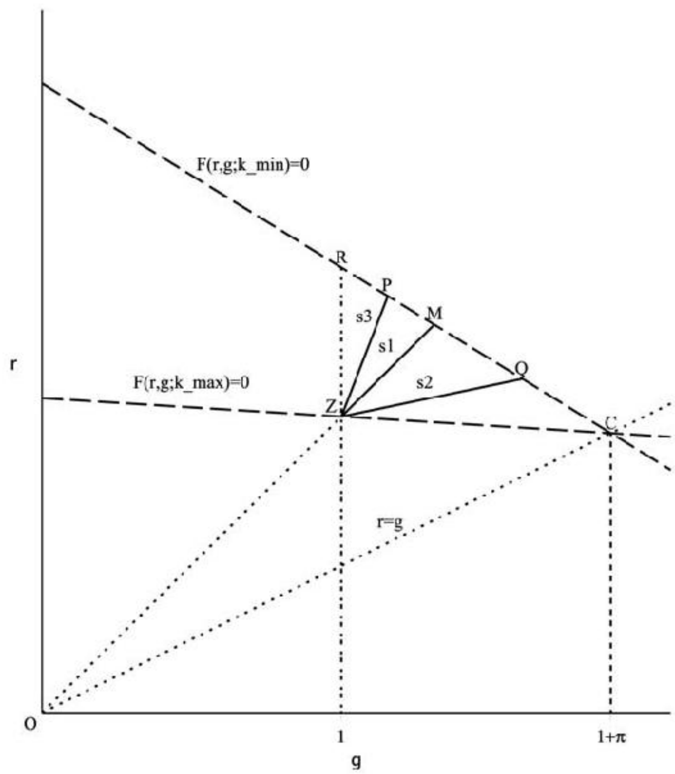

Figure 5 illustrates the permissible combination of inflation compensation through capital and non-capital incomes, r and g, and the simulation scenarios, given the ratio of capital and non-capital incomes k for each household. The area RZC defines the choice set for all the k. $F(r, g; k\_max) = 0$ and $F(r, g; k\_min) = 0$ are the borders of *inflation compensation restrictions*, where k_max and k_min are the maximum and minimum k observed in our sample. All the households are situated at one point on their own restriction $F(r, g; k) = 0$, which is a straight line within the borders and passes through point C. $r > g$ *restriction* rules out the area below the line CO. The vertical line RZ represents the *wage rigidity restriction*. On the other hand, by our assumptions the maximum non-capital income growth is $1 + \pi$, under which the capital income growth is the same. Point C is in such a situation. This set should always include point Z as required by *maximum capital income bargaining restriction*. In the end, each scenario should be defined by a set of points in the area RZC, which intersects with each line $F(r, g; k) = 0$, and neither of these two points stay on the same line $F(r, g; k) = 0$.

We conduct simulations on three scenarios as indicated by s1 to s3 in Figure 5.[50] The following text describes the main motivations behind and we present the detailed construction of, the scenarios in Appendix E. s1 can be considered a *steady state scenario* – r/g is constant for all, which simply becomes a straight line ZM as an extension of the line OZ. The remaining two scenarios allow full heterogeneity in r, g, and r/g. As already emphasised, the most important mechanism to relieve bracket creep under a dual tax system is to raise the share of capital income, which is the equivalent of lifting r/g. With respect to the distribution effect, we postulate that the higher the r/g assigned to the poor relative to the rich, the more equalising effect of bracket creep on the IT and SIC system.[51] The income level is positively associated with capital income concentration, ie k, and the line $F(r, g; k) = 0$ becomes steeper as k drops. Therefore, s2 is a *pro-rich biased scenario* and s3 is a *pro-poor biased scenario*.[52] s1 might be ranked between the other two scenarios. Alternatively, we should expect that s3 can improve (or worsen) the equalising effect of the IT and SIC system under bracket creep more than (or less than) the other two can. The same postulation applies to the relationship between s1 and s2.

Both gross capital and non-capital incomes are inflated according to r and g, located by finding the intersection point of each scenario path and $F(r, g; k) = 0$ given k for each household in the data. We consider the households with only capital (non-capital) income by assigning full nominal growth equal to the inflation rate to capital (non-capital) income. The same simulation procedure as described in the previous section is then executed for each scenario.

[50] To make our exercise simpler, we only allow the combination of r and g to move on a continuous straight line as k decreases.
[51] The following analysis proves that this speculation is not sophisticated enough.
[52] Among the pro-rich and pro-poor groups of candidates, these two scenarios are both the median ones because point P is the middle point in the line RM, and point Q is also the middle point in the line MC.

7.2 Results and analysis

We then present the simulation outcomes for both homogeneous and heterogeneous r and g groups of scenarios.

Table 8 Redistributive effect (RE) of the IT and SIC system under dual and comprehensive tax systems between r>g and r=g when r and g are homogeneous

Inflation rate % - π	r	g	G_pre	Dual tax system		Comprehensive tax system		RE relative change btw two systems %	RE/G_pre Relative change from the comparision scenario %[1)]	
				RE	RE/G_pre	RE	RE/G_pre		Dual tax system	Comprehensive tax system
1			0.4000	0.0719	0.1798	0.0763	0.1907	6.1	-	-
4			0.4000	0.0721	0.1803	0.0766	0.1914	6.2	-	-
10			0.4000	0.0724	0.1810	0.0770	0.1924	6.3	-	-
20	π	π	0.4000	***0.0725***	***0.1812***	0.0773	0.1932	6.6	-	-
30			0.4000	0.0723	0.1809	***0.0773***	***0.1932***	6.8	-	-
60			0.4000	0.0712	0.1781	0.0765	0.1913	7.4	-	-
90			0.4000	0.0698	0.1745	0.0753	0.1883	***7.9***	-	-
1			0.4000	0.0718	0.1796	0.0762	0.1906	6.1	-0.1	-0.1
4			0.4002	0.0719	***0.1796***	0.0764	0.1909	6.3	-0.4	-0.3
10			0.4004	***0.0719***	0.1796	0.0766	0.1914	6.5	-0.7	-0.5
20	π	π/2	0.4008	0.0719	0.1794	0.0769	0.1919	7.0	-1.0	-0.6
30			0.4012	0.0717	0.1787	0.0770	***0.1920***	7.4	-1.2	-0.6
60			0.4021	0.0709	0.1764	***0.0771***	0.1918	8.7	-0.9	0.3
90			***0.4029***	0.0699	0.1734	0.0768	0.1906	***10.0***	-0.7	1.2
1			0.4001	***0.0718***	***0.1794***	0.0762	0.1905	6.2	-0.1	0.0
4			0.4005	0.0717	0.1790	0.0764	0.1907	6.5	-0.4	-0.1
10			0.4013	0.0714	0.1780	0.0766	0.1908	7.2	-0.9	-0.3
20	2π	π/2	0.4026	0.0709	0.1761	0.0768	***0.1908***	8.4	-1.8	-0.6
30			0.4038	0.0702	0.1739	0.0769	0.1905	9.6	-2.7	-0.8
60			0.4072	0.0681	0.1674	***0.0770***	0.1892	13.0	-5.1	-1.4
90			***0.4100***	0.0659	0.1608	0.0767	0.1871	***16.4***	-7.3	-1.9

Notes: 1) The comparision scenario is the one sharing either r or g under the same inflation rate. Thus, the comparision scenario for anyone in the second block is the one under the same inflation rate in the first block. And the comparision scenario for anyone in the third block is the one under the same inflation rate in the second block. Refer to Table 5 for an explanation of the heat map.
Source: Tax micro-simulation model using PHF.

The results for the homogeneous r and g group of scenarios are summarised in Table 8. For each combination of r and g, Table 8 displays the G_pre, Gini of total gross income distribution, redistributive effect (RE) and a normalised measure RE/G_pre of the IT and SIC system under both the dual and the comprehensive tax systems, and a comparison – the relative percentage change of RE between the two tax systems. With $r > g$ and homogeneous r and g, those with relatively more capital income would enjoy faster nominal growth of total gross income. Hence, G_pre is no longer constant in these two scenarios. Furthermore, we observe a growing G_pre as inflation rises. This widening of inequality is even stronger as the gap between r and g rises. For example, the G_pre when capital income grows by 20% and non-capital income grows by 5% is 0.4013. It is larger than the counterpart when capital income grows by 10% and non-capital income grows by 5%, which is 0.4004. This development of inequality serves as evidence for the uneven distribution of capital income. Higher inflation triggers more intense concentration of capital income. Besides, due to this complication, we have to normalise the RE measure by dividing it with G_pre when we compare the scenarios when $r > g$ and those when $r = g$.

The reinstatement of the comprehensive tax system is always conducive to the redistribution. The relative change in RE between the comprehensive and the dual tax systems is always well above zero. This change is persistently larger as r/g moves up from 1 to 2 and from 2 to 4. When the gap between r and g expands, the RE-suppressing effect of $r > g$ due to pushing the rich faster towards the flat tax zones under the comprehensive tax system is increasingly dominated by the RE enhancing effects – higher total average and marginal tax rates at the top of distribution as well as the disappearance of the tax arbitrage opportunity for the rich middle class. The intensification on redistribution by a switch to the comprehensive tax system is quite sizable, and is further enlarged by $r > g$. For instance, even with 2% nominal growth on capital income and 0.5% nominal growth on non-capital income, the relative change of RE between the two systems amounts to 6.2%, which is larger than 6.1%, a counterparting measure when capital income grows at a slower rate of 1% and non-capital income grows at the same rate of 0.5%. For a highly skewed distribution, this magnitude is rather significant. As a benchmark, when $r = g$, the largest RE relative change between various other inflation scenarios and the 1% scenario occurs when inflation hits 90%, which is just 2.9% in absolute terms.

As capital income grows faster, under the current dual tax system redistribution of the IT and SIC system always seems to be weakened. The most straightforward evidence is that relative changes of RE/G_pre from the comparison scenario are always negative for all the inflation rates under the dual tax system. Again, the size of these effects is significant. The other supporting sign is that the peak of RE/G_pre or RE developments arrives at much lower inflation rates when $r > g$. In particular, when capital income grows at twice the inflation rate and non-capital income grows at half the inflation rate, RE/G_pre or RE are always monotonically decreasing with a higher inflation rate.

In keeping with the previous evidence that the crossing marginal contribution of $r > g$ on the introduction of the dual tax system is negative to redistribution, relative changes of RE/G_pre from the comparison scenario mostly become much closer to zero as we switch from the dual tax system to the comprehensive one. This contribution can even offset the deterioration of RE by bracket creep as some relative changes of RE/G_pre are positive. The other indication about the positive effect of recovering a comprehensive tax system is that the peak of RE/G_pre or RE developments arrives at much larger inflation rates under the comprehensive tax system for all three combinations of r and g.

Table 9 Redistributive effect (RE) of the IT and SIC system under the bracket creep scenarios when r>g and r and g are heterogeneous

	RE			Percentage change of RE w.r.t. the benchmark		
	Scenarios			Scenarios		
Inflation rate %	1	2	3	1	2	3
1	0.0719	0.0719	***0.0714***	-0.04	-0.02	***-0.73***
4	0.0720	0.0721	0.0703	-0.14	-0.08	***-2.48***
10	***0.0721***	***0.0722***	0.0681	-0.35	-0.20	***-5.94***
20	0.0720	0.0722	0.0644	-0.70	-0.42	***-11.19***
30	0.0716	0.0719	0.0609	-1.06	-0.64	***-15.89***
60	0.0697	0.0703	0.0516	-2.19	-1.34	***-27.49***
90	0.0675	0.0684	0.0443	-3.32	-2.06	***-36.51***

Notes: RE represents the redistributive effect. Benchmark refers to the initial bracket creep scenario when there is the same level of inflation compensation for both capital and non-capital incomes whose RE is presented in Table 5.
In each column of the left panel, darker background shading emphasises higher values. The highest value per column is in bold and italics. In each row of the right panel, darker background shading emphasises lower values. The lowest value per row is in bold and italics.
Source: Tax micro-simulation model using PHF data.

Table 10 Percentage change of non-capital taxable income between the s3 bracket creep scenario when r>g and r and g are heterogeneous and the benchmark one

Inflation rate %	Deciles 1	2	3	4	5	6	7	8	9	10
1	***-0.28***	-0.30	-0.36	-0.47	-0.51	-0.56	-0.65	-0.73	-0.76	-0.82
2	***-0.55***	-0.58	-0.71	-0.92	-1.00	-1.11	-1.28	-1.44	-1.49	-1.63
4	***-1.06***	-1.15	-1.38	-1.78	-1.94	-2.18	-2.51	-2.82	-2.92	-3.19
5	***-1.31***	-1.42	-1.70	-2.20	-2.39	-2.69	-3.10	-3.49	-3.61	-3.94
10	***-2.49***	-2.66	-3.20	-4.13	-4.48	-5.06	-5.87	-6.61	-6.86	-7.51
15	***-3.52***	-3.78	-4.52	-5.85	-6.35	-7.19	-8.37	-9.42	-9.79	-10.74
20	***-4.42***	-4.79	-5.69	-7.38	-8.02	-9.10	-10.63	-11.97	-12.46	-13.68
25	***-5.23***	-5.69	-6.74	-8.75	-9.52	-10.82	-12.67	-14.28	-14.88	-16.38
30	***-5.99***	-6.50	-7.69	-9.99	-10.87	-12.39	-14.54	-16.39	-17.09	-18.85

Notes: Benchmark refers to the initial bracket creep scenario when there is the same level of inflation compensation for both capital and non-capital incomes. Decile groupings are determined by the distribution of the equivalised household disposable income (EHDI) in the regime without indexation. In each row, darker background shading emphasises higher values. The highest value per row is in bold and italics.
Source: Tax micro-simulation model using PHF data.

Next, we turn to the results simulated from the heterogeneous r and g group of scenarios. Table 9 presents the redistributive effects of the IT and SIC system under three scenarios when r and g can be heterogeneous. Again, we observe from the left panel the inverted U-shaped development of RE with respect to inflation for the s1 and s2 scenarios. But the maximum is reached at a lower inflation rate of 10% compared with the benchmark bracket creep scenario when r=g (see Table 5). RE starts to decline even from the very low inflation for s3. More surprisingly, all scenarios introduce a greater disequalising effects as, in the right panel, the percentage change of RE with respect to the benchmark scenario is always negative, even for the *pro-poor biased scenario* s3. Contrary to our prediction, this fall is most significant in s3, followed by s1 and then s2. The difference is fairly sizeable in s3 as, for instance, a fall in 2.48% of RE compared with the benchmark can arise at 4% of inflation.

The other influential channel which distorts the redistributive effect we ignored is the crowding-out of non-capital income by capital income. Triggered by the *inflation compensation* and $r > g$ *restrictions*, g is smaller than $1 + \pi$, so that all the households have less nominal non-capital income after inflation compensation than the benchmark scenario. Under our *maximum capital income bargaining restriction*, this reduction is larger for those with a higher k (the rich) than for those with a lower one (the poor). As demonstrated by Figure 5, every point is horizontally farther away from $1 + \pi$ as the households move towards the point Z, with k rising, over any of these three scenario paths. Consequently, the rich can shift more income from non-capital to capital components in relative terms (with respect to non-capital income), and also in absolute term, since they usually have more non-capital income than the poor. Due to the progressivity on the tax for non-capital income as we explained before, the top of the distribution should enjoy more tax relief relative to their tax base than the bottom part.[53] Taking scenario s3 as an example, Table 10 displays the relative decline of non-capital taxable income compared with the benchmark scenario at the same inflation rates. This drop prevails over the whole distribution and grows as inflation rises. As expected, as income deciles move downwards, this shrinkage of the non-capital income base depreciates monotonically.[54]

Furthermore, as the scenario becomes less pro-poor biased, this crowding-out effect declines for all. Figure 5 shows that each point on the scenario paths, defined by the *inflation compensation restriction* given the same k, moves closer to $1 + \pi$ when the scenario shifts from s3 to s1, and then s2, or the path becomes flatter. As we can see from Figure 5, when the scenario path spirals downwards, this increase in non-capital income is stronger for the rich than for the poor in both relative and absolute terms. Table 11 and Table 12 provide such evidence regarding the relative change of non-capital taxable income from s3 and s1, and from s1 to s2 respectively. These changes are all positive across the distribution, and they are almost always positively related with the income level.[55] Tax progressivity further reinforces such an uneven rise of non-capital income so that the ranking of RE drops in s3, s1 and s2 from high to low.

[53] This statement might not be accurate enough if there is a non-negligible part of the top distribution with a share of non-capital income that is much smaller than the share of capital income. However, this situation is rather unlikely in Germany.
[54] The same pattern occurs for the other two scenarios, although evidence for this is not shown in order to save space.
[55] The exception occurs in the first decile of Table 12, and is most likely caused by non-monotonicity, at the bottom of income distribution, of the procedure for deriving the taxable income from gross income in the German tax system (ie the impacts from tax deductions, social insurance contributions…).

Table 11 Percentage change of non-capital taxable income between the bracket creep scenarios s1 and s3 when r>g and r and g are heterogeneous

Inflation rate %	Deciles 1	2	3	4	5	6	7	8	9	10
1	0.25	0.29	0.35	0.45	0.49	0.55	0.62	0.71	0.73	***0.77***
2	0.50	0.57	0.69	0.90	0.97	1.08	1.24	1.41	1.45	***1.54***
4	0.97	1.12	1.35	1.76	1.90	2.13	2.45	2.78	2.87	***3.07***
5	1.21	1.39	1.66	2.18	2.35	2.65	3.05	3.46	3.58	***3.82***
10	2.32	2.63	3.18	4.17	4.49	5.10	5.94	6.77	7.02	***7.54***
15	3.30	3.77	4.53	6.01	6.48	7.39	8.68	9.93	10.32	***11.14***
20	4.17	4.83	5.77	7.70	8.32	9.54	11.27	12.96	13.50	***14.65***
25	4.96	5.78	6.90	9.26	10.02	11.55	13.73	15.85	16.56	***18.06***
30	5.71	6.65	7.93	10.69	11.60	13.43	16.06	18.63	19.50	***21.38***

Notes: Decile groupings are determined by the distribution of the equivalised household disposable income (EHDI) in the regime without indexation. In each row, darker background shading emphasises higher values. The highest value per row is in bold and italics.
Source: Tax micro-simulation model using PHF data.

Table 12 Percentage change of non-capital taxable income between the bracket creep scenarios s2 and s1 when r>g and r and g are heterogeneous

Inflation rate %	Deciles 1	2	3	4	5	6	7	8	9	10
1	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	***0.02***
2	0.02	0.01	0.01	0.01	0.02	0.02	0.02	0.03	0.03	***0.04***
4	0.04	0.02	0.02	0.02	0.04	0.04	0.05	0.05	0.06	***0.09***
5	0.05	0.02	0.03	0.03	0.04	0.05	0.06	0.06	0.07	***0.11***
10	0.09	0.05	0.06	0.06	0.08	0.10	0.12	0.13	0.14	***0.21***
15	0.14	0.07	0.09	0.08	0.12	0.14	0.18	0.18	0.21	***0.31***
20	0.18	0.09	0.11	0.11	0.16	0.19	0.24	0.24	0.27	***0.41***
25	0.21	0.11	0.14	0.13	0.20	0.23	0.29	0.30	0.33	***0.50***
30	0.25	0.13	0.17	0.16	0.24	0.27	0.34	0.35	0.39	***0.59***

Notes: Decile groupings are determined by the distribution of the equivalised household disposable income (EHDI) in the regime without indexation. In each row, darker background shading emphasises higher values. The highest value per row is in bold and italics.
Source: Tax micro-simulation model using PHF data.

Our simulation results imply that the effect of biasing the r/g towards the poor is dominated by the other effect of crowding out non-capital income. In particular, under our set-up, these two effects are positively related. The net effect is always to worsen the redistributive effect of the IT and SIC system under bracket creep. We also perform the robustness check to redo the simulation as we restrict the maximum and minimum k in order to exclude the influence of outliers. The results are maintained.

8 Conclusions

Over the years before the crisis, there was a wide and noticeable rise in market income inequality, and governments in the developed countries undertook continuous tax legislation reforms, with most of them bringing a decline in the top PIT rates.[57] An empirical assessment of the effects of bracket creep is needed against this background. Germany serves well as a case study since it has experienced an income and tax development featuring many common trends. The effect of bracket creep is more pronounced among taxpayers in Germany due to the continuously increasing marginal rates. On the other hand, the government can also lose real revenue on particular taxes such as excise tax due to inflation erosion. Therefore, these aspects have triggered many debates to address this issue.

Our study provides an answer by investigating both the redistributive and revenue effect of bracket creep. In our analysis, we use a tax micro-simulation model developed for the 2009 income distribution from the newly available PHF data.[58] By simulating under a more complete list of inflation rates than Immervoll (2005), we produce an inverted U-shaped overall redistributive effect from the IT and SIC system when inflation grows. The reduction of tax progressivity and growing average tax burdens are still consistent with the findings by Immervoll (2005). Furthermore, bracket creep purely through the tax schedule is strikingly overall regressive since the possibility of switching the capital income into and out of the income tax base, as nominal income grows, benefits the upper middle part of distribution more than the lower middle part. The recent empirical investigation shows that the rise in market income inequality is associated with a lower average tax level and higher tax progressivity. Therefore, it is worth examining whether the current widening income inequality among many countries might have caused them to enter a new regime – the initial average tax level is too low (or tax progressivity as the offsetting force is high enough) so that the development of the redistribution effect is no longer dominated by the upward movement of the average tax level induced by

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[57] Förster, Llena-Nozal and Nafilyan (2014, Figure 18) illustrate that top PIT rates were about 60-70% in major OECD countries in the mid-1970s, but they fell to about 40% on average by the late 2000s. Verbist and Figari (2014, Table 1) show that, on average, top rates declined from 44% to 36.6% between 1998 and 2008 in the EU-15 countries.

[58] Most of the results should still be valid in 2014 (Hechtner et al, 2012; Figure 8 in this paper depicts the empirical residual elasticity for deciles and p99 of income distribution shifts little). Fundamentally, there have been no major changes in income tax and social insurance since 2009.

bracket creep. In this sense, our simulation with the German data can hopefully contribute to this examination.

Between 1998 and 2009, nominal gross income in Germany increased over the whole distribution at both ends at a faster pace. Meanwhile, the tax rate and progression of the tax schedule declined significantly at the middle and top of the distribution. These factors contributed to a diminishing of the redistributive effect in the IT and SIC system when inflation introduces nominal income growth, since a large mass of taxpayers were pushed to a relatively less progressive tax zone in 2009 compared with 1998. This finding implies that frequent monitoring of the redistributive effect in bracket creep is necessary because it provides the timely evaluation of the progressivity aspect of tax reform.

Assuming the current development of income distribution and tax progression persists, we claim that delaying indexation might not always enhance the equalising effect of the IT and SIC system. Our results suggest that a decision to act against bracket creep should take this development into account.

The other part of simulations incorporates the contemporaneous evidence $r > g$ such that the nominal growth needed to compensate inflation is stronger in capital income than in non-capital income. We divide the scenarios by allowing the heterogeneity in r and g or not. When r and g are homogeneous within the population, our simulation shows that a restoration of the comprehensive tax system always significantly reduces the inequality of disposable income under all kinds of simulated inflation scenarios. For instance, even with 2% nominal growth on capital income and 0.5% nominal growth on non-capital income, relative change of RE between the current dual tax system and the counterfactual comprehensive tax system amounts to 6.2%. With higher nominal growth on capital income than non-capital income, this effect is even intensified. With a shift to the comprehensive tax system and faster growth in capital income, the RE-enhancing path can be prolonged until a peak which is much further above the initial 20% in the equal growth scenario under the dual tax system. Under the current dual tax system, faster growth in capital income exasperates the redistribution dynamics as inflation picks up. The redistributive effect can, with rising inflation, start to enter the dropping path with inflation much lower than 20%.

Although the scenarios simulated with heterogeneity in r and g are not exhaustive, the redistributive effect of the IT and SIC system under bracket creep deteriorates for all the scenarios simulated under a few plausible assumptions regarding the inflation compensation process under $r > g$. Crowding-out the non-capital income to a greater extent at the top of distribution becomes the dominating mechanism to explain the increasing disequalising effect.

We would suggest pursuing this direction of research further in the future. Capital income and wealth data can be enriched by the approaches in Bartels and Jenderny (2014) as well as Saez and Zucman (2014). We can explore the extent to which our results can hold good if we revise some of our assumptions and extend beyond the context of inflation compensation. Many critical questions regarding the design of capital income taxation remain to be answered, particularly if asset price booms are fuelled by the extensively expansionary monetary policy around the world.

We also acknowledge that caveats exist in our data and simulation. The very top of the income distribution might not be precisely represented, although our reweighting partially resolves the problem in terms of tax revenue. The actual tax and contributions to which households are subject may depend on several factors (eg exemptions) and rules we do not fully account for. However, using a new data set which oversamples households at the top of income distribution, our study can provide valuable insights into the redistributive effects in different parts of the distribution.

Appendix A German system of tax and social security contributions

The German income tax schedule includes a basic allowance, the directly progressive and the marginally flat tax brackets. Nominal taxable incomes above the basic allowance are subject to a marginal tax rate increasing linearly with income until a threshold is hit. Incomes exceeding this threshold are taxed by the constant marginal tax rates.[59] Married couples can enjoy the marriage tax benefit mainly through a splitting rule for spouses (joint taxation of couples) such that the joint tax liability is twice the tax liability derived from the point of the tax function where taxable income is half of the couple's income. Due to tax progressivity, almost all couples can reduce the total tax liability by filing tax jointly.

Furthermore, since 2009, capital incomes of private households have been – in the normal case – subject to a flat tax rate of 25% for the part above a saver's allowance. They are treated as a withholding tax separate from the above-mentioned income tax. However, tax payers can instead include the capital incomes in the income tax base if filing jointly is more favourable. This can be done when they file their annual tax return. The authorities then perform a yield test to determine whether withheld tax is to be refunded. Initially designed for funding German re-unification, a surcharge of 5.5% on both income and capital income tax burdens (as well as on the corporate tax burden for corporations) is additionally levied. Members of the catholic and protestant churches (and some smaller churches) are supposed to pay a church tax, with income and capital income tax burdens as the tax base. There is a flat rate of 8% or 9% on this tax base, depending on the federal state.

Wage tax and withholding taxes on interest and dividends have to be paid upfront. Then, by filing a tax return every year, the discrepancies between up-front and final tax liability are resolved. However, the obligation to file tax is binding only for the self-employed and anyone with multiple income sources, but only in particular cases for wage earners. It is much more favourable for high income earners to file tax because they can claim itemised deductions as well as other allowances to save tax; this possibility is only available when filing a tax return. Consequently, low income earners are more likely to be underrepresented in the income tax statistics covering only the tax filer population.

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[59] There was only one marginal flat tax bracket before 2007. The rich tax (*Reichensteuer*) was introduced in 2007 with the other bracket bearing a higher flat marginal tax rate at the top of the distribution. In 2009, a flat 42% was levied in respect of taxable income between €52,552 and €250,400. For taxable income above €250,400, a flat 45% tax was levied.

Many Germans are subject to statutory social insurance contributions. Theese include pension insurance, health insurance, long-term care insurance, statutory unemployment insurance, and accident insurance. Employees and employers are obliged to pay statutory social insurance contributions. These levies are progressive for the lower part of the income distribution because there are limits below which contributions on earnings are waived. For earners with gross income above the limits, contributions are proportional to nominal gross income up to a ceiling. Gross income above this ceiling is disregarded. Income ceilings are adjusted roughly according to wage growth.

Civil servants and the self-employed are not obligated to pay compulsory social insurance contributions. Many opt out of such insurance and choose to be covered by the private system, eg private health insurance. Pensioners only have to contribute to health insurance and long-term care insurance.

Appendix B Gross-net tax conversion

The raw income data in the PHF contains either gross (*brutto*) or net (*netto*) values. While the survey asked for gross figures to be provided wherever possible, some of the income questions in the questionnaire gave the option of providing net figures. Interviewers recorded this distinction in one two-level categorical BR variable. In order to perform a consistent analysis, we converted all the reported figures so that they are all on the annual gross base.

The conversion carries a few assumptions. The main one is that the respondent reports the upfront net income figure which is the gross figure minus the source tax (usually wage tax / *Lohnsteuer*) and default social insurance contributions. This is usually what the employee respondent can gather from his/her payslip which conforms to the information-retrieving process they should follow. We also assume the same context for pension and self-employment incomes as the setting is similar for these incomes.

The second important assumption is that spouses choose the tax class (*Lohnsteuerklasse*) which ensures that the aggregate upfront net employment income for both is highest among all the alternative combinations: III/V, IV/IV or V/III. One complication in German upfront taxation is the choice of tax class by married partners. In almost all cases, couples have to decide between two arrangements: first, that one spouse receives the full marriage benefits (class III) while the other receives none (class V), and, second, that each shares the marriage benefits (IV for both). However, the choice of tax class is not asked for in the questionnaire. Therefore, we have to carry out an optimisation which is equivalent to a rationality assumption. We ignore the possibility of choosing tax class VI, which applies to income from second and subsequent employment.[60] We apply tax class I to all single persons with tax liability who are not single mothers. Tax class VI is assigned to the latter. Tax class I defines the lowest tax deductions and those with class VI can enjoy some additional tax benefits.

The gross to net calculator is specified by strictly following the protocol published by the Federal Ministry of Finance as well as the rules regarding social insurance contributions.[61] Note that capital income is always treated separately with regard to the flat rate withholding tax (*Abgeltungssteuer*).

[60] Everyone is asked to report employment income as one figure which we cannot separate among multiple jobs, if any.

[61] See PAP 2009 (*Geänderter Programmablaufplan für die maschinelle Berechnung der vom Arbeitslohn einzubehaltenden Lohnsteuer, des Solidaritätszuschlags und der Maßstabsteuer für die Kirchenlohnsteuer* in 2009:

We then discuss the structure of the tax conversion. The conversion is composed of three major steps.

First, all of the reported income variables are converted one by one to the gross figures under all the eligible tax classes the respondents can take if the BR indicators show they were the net figures. This is achieved by a non-linear solver based on the gross to net tax calculator. The tax class is one input parameter that this tax calculator uses.

Next, for all spouses who are eligible for classes III/IV/V we calculate the alternative aggregate net figures under tax class IV using the version of the gross figures under classes III and V. Using the total net figures across all the income components reported by both partners, a yield test algorithm determines the optimal tax class.

Finally, we assign the gross figure to each variable which is associated with the optimal tax class as produced in the first step.

The bonus income (dpg0210) should be entered into the tax base together with the employment income to determine the joint marginal tax rate as stipulated by the German Tax Code. Our conversion also takes this complex matter into consideration.

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http://www.bundesfinanzministerium.de/Content/DE/Downloads/Steuern/Steuerarten/Lohnsteuer/Programmablaufplan/012_PAP_2009_a.pdf?__blob=publicationFile&v=4, accessed on 7 May 2011).

Appendix C Absolute real loss of household net incomes owing to bracket creep

Table 13: Absolute real loss of household net incomes owing to bracket creep under different inflation scenarios

Inflation rate %	Deciles 1	2	3	4	5	6	7	8	9	10
1	2	5	12	33	35	39	47	47	53	75
2	3	9	24	55	65	78	92	93	101	147
4	4	19	48	101	124	153	181	183	196	288
5	5	24	61	124	153	191	225	227	243	357
10	9	52	131	243	300	375	437	433	471	687
15	22	86	206	364	447	556	635	626	689	995
20	27	123	284	484	595	731	819	811	901	1,284
25	35	166	364	604	741	897	990	991	1,107	1,557
30	40	220	448	722	882	1,058	1,143	1,159	1,306	1,814
40	66	317	632	951	1,148	1,357	1,419	1,473	1,684	2,281
50	80	427	806	1,173	1,400	1,612	1,679	1,768	2,039	2,693
60	96	543	975	1,382	1,634	1,833	1,921	2,046	2,367	3,057
70	118	660	1,139	1,578	1,839	2,040	2,149	2,309	2,672	3,385
80	140	778	1,291	1,765	2,016	2,231	2,367	2,558	2,950	3,680
90	164	897	1,433	1,940	2,180	2,410	2,574	2,794	3,204	3,947

Notes: Decile groupings are determined by the distribution of the equivalised household disposable income (EHDI) in the regime without indexation. The "modified OECD" scale is used for equivalising incomes of households of different structure and size. The respective weights are 1 (first adult), 0.5 (subsequent adults) and 0.3 (children aged below 14). Absolute real loss is calculated as the difference of EHDI before and after bracket creep deflated by inflation. Each cell contains the decile mean of the absolute real loss. The minus sign is omitted which avoids overcrowding.
Source: Tax micro-simulation model using PHF data.

Appendix D Redistributive effect (RE) of the IT and SIC system with bracket creep on all sources of income except social benefit incomes

Table 14 Redistributive effect (RE) of the IT and SIC system with bracket creep on all sources of income except the social benefit incomes under different inflation scenarios

Inflation rate %	G_pre	G_post	RE
0	0.4000	0.3281	0.0719
1	0.4003	0.3284	0.0719
4	0.4011	0.3292	0.0720
10	0.4027	0.3307	***0.0720***
20	0.4050	0.3333	0.0717
30	0.4071	0.3359	0.0712
60	0.4119	0.3430	0.0688
90	0.4153	***0.3490***	0.0663

Notes: Refer to Table 5 for an explanation of each column and the heat map.
Source: Tax micro-simulation model using PHF data.

Appendix E Mathematical formulations of scenarios when r and g are heterogeneous

We discuss the parameterisation of scenarios s1 - s3 presented in section 7.1 when r and g can be heterogeneous. In the end, r and g for each household can be denoted by the observables k, k_{min}, k_{max} as well as the hypothetical π. We use r_x and g_x to represent the solution in the scenario sx ($x = 1, 2$ or 3) and r_y and g_y to denote the coordinate for point y. The following derivation refers to the geographical representation in Figure 5. The basic process is to deduct the algebraic expression of each scenario path (ie ZM, ZQ and ZP lines), and they are solved jointly with the *inflation compensation restriction* $F(r, g; k) = rk + g - (1 + k)(1 + \pi) = 0$.

s1: $\mathrm{r/g}$ shares a constant equal to the r value in point Z, since g value in point Z is one. Letting $\mathrm{g} = 1$ in $\mathrm{F(r, g; k)} = 0$, we solve the g to have this slope. The multiplication of this slope and g is then inserted into r in $\mathrm{F(r, g; k)} = 0$ in order to solve $\mathrm{g_{s1}}$ and then $\mathrm{r_{s1}}$. The results are

$$r_1 = \frac{\left(\pi + k_{\max} + k_{\max}\pi\right)(1 + k)\left(1 + \pi\right)}{k_{\max} + k\pi + k\,k_{\max} + k\,k_{\max}\pi} \qquad g_1 = \frac{(1 + k)\left(1 + \pi\right)k_{\max}}{k_{\max} + k\pi + k\,k_{\max} + k\,k_{\max}\pi}$$

s2: r_M and g_M are first obtained by inserting the multiplication of the slop solved above and g into r in $\mathrm{F}(\mathrm{r}, \mathrm{g}; k_{min}) = 0$. r_C and g_C are both $1 + \pi$. We then solve the Q as the midpoint of line MC. With the coordinates of both Z and Q, we derive the expression of line ZQ and finally to achieve

$$\begin{aligned} r_2 = \Big(& k_{\max}^2\pi^2 k_{\min} + k_{\max}^2\pi^2 + k_{\max}^2\pi^2 k \\ & - 2k\pi^2 k_{\min}k_{\max} + k_{\max}\pi^2 + \pi^2 k\,k_{\max} \\ & - 2k\pi^2 k_{\min} - k_{\min}\pi^2 + 2k_{\min}k_{\max}^2\pi + 3 \\ & k_{\max}^2\pi + 2k\,k_{\max}^2\pi - 4k\,k_{\min}k_{\max}\pi \\ & - k_{\min}k_{\max}\pi + k_{\max}\pi - 2k\,k_{\min}\pi - k_{\min}\pi \\ & + k_{\min}k_{\max}^2 + 2k_{\max}^2 + k\,k_{\max}^2 - 2k\,k_{\min}k_{\max} \\ & - k_{\min}k_{\max} - k\,k_{\max}\Big) \Big/ \Big(-k_{\min}k_{\max} + k_{\min} \\ & k_{\max}^2 + 2k_{\max}^2 + k_{\min}k_{\max}\pi + k_{\min}k_{\max}^2\pi \\ & - k\,k_{\max} + k\,k_{\max}^2 - 2k\,k_{\min}\pi + k\,k_{\max}\pi + k \\ & k_{\max}^2\pi - 2k\,k_{\min}k_{\max}\pi - 2k\,k_{\min}k_{\max}\Big) \end{aligned}$$

$$\begin{aligned} g_2 = -\Big(& -k_{\max}^2\pi^2 k_{\min} + k\pi^2 k_{\min}k_{\max} - k_{\max}\pi^2 k_{m} \\ & + k\pi^2 k_{\min} - 2k_{\min}k_{\max}^2\pi - k\,k_{\max}^2\pi - 2 \\ & k_{\max}^2\pi + 3k\,k_{\min}k_{\max}\pi + k\,k_{\max}\pi + k\,k_{\min}\pi \\ & - k_{\min}k_{\max}^2 - 2k_{\max}^2 - k\,k_{\max}^2 + 2k\,k_{\min}k_{\max} \\ & + k_{\min}k_{\max} + k\,k_{\max}\Big) \Big/ \Big(-k_{\min}k_{\max} + k_{\min} \\ & k_{\max}^2 + 2k_{\max}^2 + k_{\min}k_{\max}\pi + k_{\min}k_{\max}^2\pi \\ & - k\,k_{\max} + k\,k_{\max}^2 - 2k\,k_{\min}\pi + k\,k_{\max}\pi + k \\ & k_{\max}^2\pi - 2k\,k_{\min}k_{\max}\pi - 2k\,k_{\min}k_{\max}\Big) \end{aligned}$$

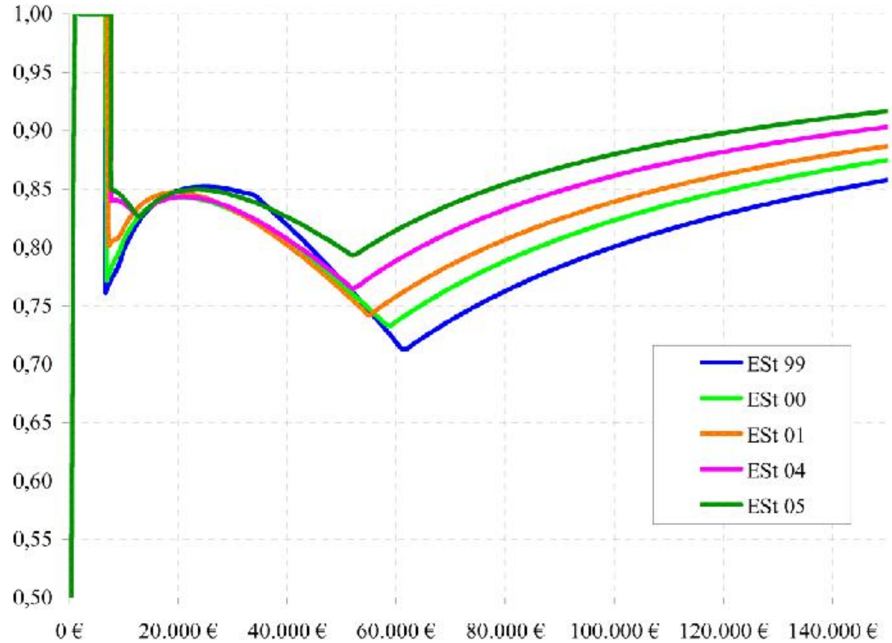
s3: With $g_R = 1$, we solve r_R by insert 1 into g in $\mathrm{F}(\mathrm{r}, \mathrm{g}; k_{min}) = 0$. P is the midpoint of line RM. The rest of derivation follows the same steps as in s2, which attains

$$r_3 = \left(k\,k_{\max} + k_{\max}\,\pi + 2\,k\,k_{\min}\,k_{\max} + k\,k_{\max}\,\pi + 4\,k\,k_{\min}\,k_{\max}\,\pi + 2\,k\,\pi^2\,k_{\min}\,k_{\max} + k_{\min}\,k_{\max} + 3\,k_{\min}\,k_{\max}\,\pi + 2\,k_{\max}\,\pi^2\,k_{\min} + 2\,k_{\min}\,\pi^2 + 2\,k\,\pi^2\,k_{\min} + 2\,k\,k_{\min}\,\pi + k_{\min}\,\pi\right) \Big/ \left(k\,k_{\max} + k_{\min}\,k_{\max} + 2\,k\,k_{\min}\,k_{\max}\,\pi + 2\,k\,k_{\min}\,k_{\max} + 2\,k\,k_{\min}\,\pi\right)$$

$$g_3 = \left(2\,k\,k_{\min}\,k_{\max}\,\pi + k_{\min}\,k_{\max}\,\pi + k\,k_{\max} + 2\,k\,k_{\min}\,k_{\max} + k_{\min}\,k_{\max} + k\,k_{\min}\,\pi\right) \Big/ \left(k\,k_{\max} + k_{\min}\,k_{\max} + 2\,k\,k_{\min}\,k_{\max}\,\pi + 2\,k\,k_{\min}\,k_{\max} + 2\,k\,k_{\min}\,\pi\right)$$

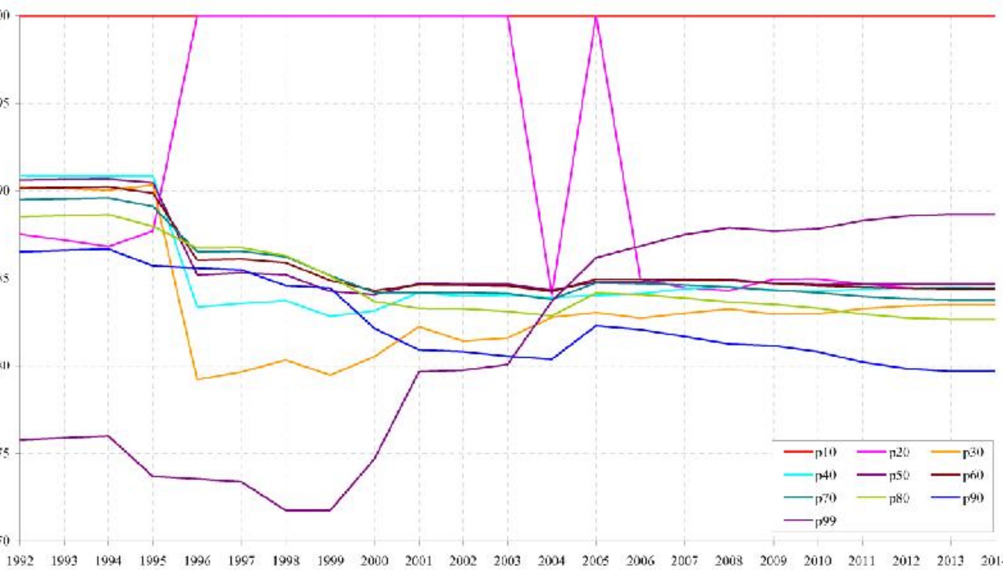
Appendix F

Figure F.1 Residual progression of German income tax schedule (ESt) over taxable income between 1999 and 2005



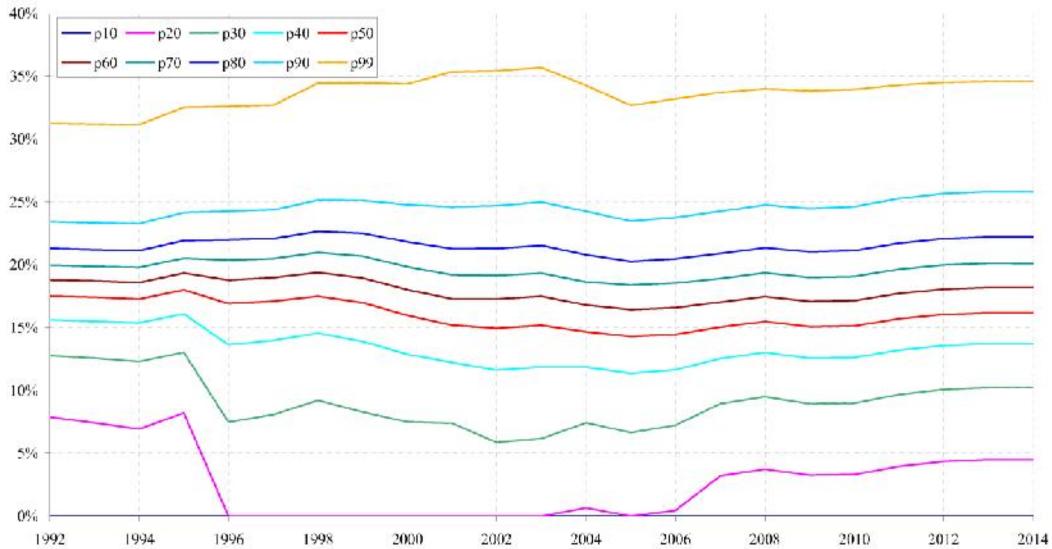
Notes: Given only few tax reforms between 1998 and 1999 as well as 2005 and 2009, we can consider ESt 99 (the lowest curve for the bottom and top of the distribution) and 05 (the highest curve for the bottom and top of the distribution) as close approximations to ESt 98 and 09.
Source: Figure 6, Hechtner et al (2012)

Figure F.2 Development of the residual progression for empirical income deciles from 1992 to 2014



Source: Figure 8, Hechtner et al (2012)

Figure F.3 Development of the average tax burden for the empirical income deciles from 1992 to 2014



Source: Figure 4, Hechtner et al (2012)

References

Altig, D., Carlstrom, C.T., 1991. Bracket creep in the age of indexing: have we solved the problem? Federal Reserve Bank of Cleveland Working Paper Series, 9108.

Altig, D., Carlstrom, C.T., 1993. Using bracket creep to raise revenue: a bad idea whose time has passed. Federal Reserve Bank of Cleveland Economic Review, 29, 2-11.

Aronson, J. R., Johnson, P. and Lambert, P. J. (1994). Redistributive Effect and Unequal Income Tax Treatment, The Economic Journal, 104(423), 262-270.

Atkinson, A. B. (1970). On the Measurement of Inequality, Journal of Economic Theory, 2(3), 244-263.

Atkinson, A. B. (1980). Horizontal Equity and the Distribution of the Tax Burden. In H. J. Aaron and M. J. Boskin (eds), The Economics of Taxation, 3-18. Brookings, Washington D.C.

Auerbach, A.J. and Feenberg, D. (2000). The significance of federal taxes as automatic stabilizers. Journal of Economic Perspectives 14, 37-56.

Bach, S. (2012). Abbau der kalten Progression: Nicht die einzige Herausforderung beim Einkommensteuertarif, Wochenbericht des DIW Berlin, No 12/2012**.**

Bach, S., Corneo, G. and Steiner, V. (2013a). Effective Taxation of Top Incomes in Germany, German Economic Review, 14(2), 115-137.

Bach, S., Haan, P. and Ochmann, R. (2013b). Reformvorschläge zur Einkommensteuer: Mehr echte und weniger kalte Progression, Wochenbericht des DIW Berlin, No. 30/2013.

Bartels, C., and Jenderny, K. (2014). *The role of capital income for top incomes shares in Germany* (No. 2014/32). Discussion Paper, School of Business & Economics: Economics, Free University, Berlin.

Betti, G., Donatiello, G. and Verma, V. (2011). The siena micro-simulation model (SM2) for net-gross conversion of EU-Silc variables, International Journal of Micro-simulation, 4(1), 35-53.

Brenke, K. and Grabka, M. (2011). Schwache Lohnentwicklung im letzten Jahrzehnt, Wochenbericht des DIW Berlin, No. 45/2011.

Broer, M. (2011). Kalte Progression in der Einkommensbesteuerung, Wirtschaftsdienst, 91(10), 694-698.

Bundestag (2012). Entwurf eines Gesetzes zum Abbau der kalten Progression. Drucksache 17/8683.

Corneo, G. (2005). The rise and likely fall of the German income tax, 1958-2005, *CESifo Economic Studies*, *51*(1), 159-186.

Dardanoni, V and Lambert PJ 2002, 'Progressivity comparisons', Journal of Public Economics, Vol 86, 99-122.

Dolls, M. (2012). *Automatic Stabilization and Redistribution in Europe and the US* (Doctoral dissertation, Universität zu Köln).

Federal Ministry of Finance (2011). Structure and distribution of tax revenue. http://www.bundesfinanzministerium.de/Content/DE/Monatsberichte/Standardartikel_Migration/2011/06/analysen-und-berichte/b04-struktur-und-verteilung-der-steuereinnahmen/struktur-und-verteilung-der-steuereinnahmen.html-doc215728bodyText2, accessed on 19 February, 2014.

Federal Statistical Office (2014). Wage and Income tax statistics 2010. https://www.destatis.de/DE/Publikationen/Thematisch/FinanzenSteuern/Steuern/LohnEinkommensteuer/LohnEinkommensteuer2140710109004.pdf?_blob=publicationFile, accessed on 3 March, 2015.

Federal Statistical Office (2013). Yearly income taxes statistics 2009. https://www.destatis.de/DE/Publikationen/Thematisch/FinanzenSteuern/Steuern/LohnEinkommensteuer/Einkommensteuerstatistik2140711097004.pdf?__blob=publicationFile, accessed on 15 March, 2014.

Förster, M., Llena-Nozal A. and Nafilyan V. (2014), "Trends in Top Incomes and their Taxation in OECD Countries", *OECD Social, Employment and Migration Working Papers*, No. 159, OECD Publishing, Paris.

Gallego Granados, P. and Ochmann, R. (2012). EUROMOD Country Report – Germany. Discussion Paper No. 12.EUROMOD.

Gutierrez, R., Immervoll, H. and Sutherland, H. (2005). How European Union Member States adjust tax and benefit systems for inflation. EUROMOD Working Paper.

Hechtner, F., Massarrat-Mashhadi, N. and Sielaff, C. (2012). Eine Analyse zur Einkommensteuerbelastung und Wirkung der kalten Progression der vergangenen 20 Jahre in Deutschland. Discussion Paper No. 137, Arbeitskreis Quantitative Steuerlehre (arqus).

Heer, B. and Süssmuth, B. (2013). Tax Bracket Creep and its Effects on Income Distribution. Journal of Macroeconomics, Elsevier, Vol 38(PB), 393-408.

Immervoll, H. (2005). Falling up the stairs: The effects of “bracket creep” on household incomes, Review of Income and Wealth, 51(1), 37-62.

Jakobsson, U. (1976). On The Measurement of The Degree of Progression, Journal of Public Economics, 5(1-2), 161-168.

Jenderny, K. (2013). Declining Tax Progression and the German Dual Income Tax. In *Annual Conference 2013 (Duesseldorf): Competition Policy and Regulation in a Global Economic Order* (No. 80039). Verein für Socialpolitik/German Economic Association.

Kakwani, N. (1977a). Measurement of Tax Progressivity: An International Comparison, The Economic Journal, 87(345), 71-80.

Kakwani, N. (1977b). Applications of Lorenz Curves in Economic Analysis, Econometrica, 45(3), 719-728.

Kakwani, N. (1980). On a Class of Poverty Measures, Econometrica, 48(2), 437-446.

Kakwani, N. (1986). Analyzing redistribution policies: a study using Australian data. Cambridge University Press, Cambridge.

Keen, M., Papapanagos, H. and Shorrocks, A. (2000), "Tax reform and progressivity", Economic Journal, 110, 50-68.

Lambert, P. J. (2001). The distribution and redistribution of income. Manchester University Press, Manchester.

Levy, H., Nogueira, J. R., Siqueira, R. B. D., Immervoll, H., and O'Donoghue, C. (2010). Simulating the impact of inflation on the progressivity of personal income tax in Brazil. *Revista Brasileira de Economia*, *64*(4), 405-422.

Musgrave, R. A. and Thin, T. (1948). Income Tax Progression, 1929-48, Journal of Political Economy, 56(6), 498-514.

Musgrave, R. A., Musgrave, P. B. and Kullmer, L. (1994). Die öffentliche Finanzen in Theorie und Praxis. Updated 6th edition. J. C. B. Mohr, Tübingen.

OECD (2008a). *Taxing Wages 2006-2007,* Special Feature: Tax Reforms and Tax Burdens 2000-2006.

OECD (2008b). *Growing Unequal? Income distribution and poverty in OECD countries*, Paris, OECD.

OECD (2011). *Divided We Stand: Why Inequality Keeps Rising*, Paris, OECD.

Piketty, T., and Zucman, G. (2014). Capital is Back: Wealth-Income Ratios in Rich Countries 1700-2010. *The Quarterly Journal of Economics*, *129*(3), 1255-1310.

Piketty, T.. *Capital in the Twenty-first Century*, Harvard University Press, 2014.

Plotnick, R. (1981). A Measure of Horizontal Equity. Review of Economics and Statistics, 63(2), 283-288.

Pogorelskiy, K., Seidl, C. and Traub, S. (2010). *Tax progression: International and intertemporal comparison using LIS data* (No. 2010, 08). Economics working paper/Christian-Albrechts-Universität Kiel, Department of Economics.

Reynolds, M., and Smolensky, E. (1977). Public expenditures, taxes, and the distribution of income. Academic, New York.

Saez, E. (2003). The effect of marginal tax rates on income: a panel study of 'bracket creep'. Journal of Public Economics, 87(5-6), 1231-1258.

Saez, E. (2010). Do taxpayers bunch at kink points? American Economic Journal: Economic Policy, 180-212.

Saez, E. and Zucman, G. (2014): "Wealth inequality in the United States since 1913: evidence from capitalised tax return data", *NBER Working Papers*, No 20625.

Salverda, W., Nolan, B., Checchi, D., Marx, I., McKnight, A., Tóth, I. G., and van de Werfhorst, H. (eds). (2014). *Changing Inequalities in Rich Countries: Analytical and Comparative Perspectives*. Oxford University Press.

Schaefer, T. (2013). Kalte Progression. Mikrosimulationsanalyse der Auswirkungen inflationsbedingter Einkommensteuererhöhungen. Study commissioned by the Initiative for a New Social Market Economy (INSM), Institut der deutschen Wirtschaft Köln.

Schmidt, T. and Eisele, M. (2013). Oversampling vermögender Haushalte im Rahmen der Studie „Private Haushalte und ihre Finanzen (PHF)". http://www.bundesbank.de/Redaktion/EN/Downloads/Bundesbank/Research_Centre/phf_oversampling.pdf?__blob=publicationFile, accessed on 10 May 2014.

Steiner, V. and Haan, P. (2004). *Distributional and Fiscal Effects of the German Tax Reform 2000: A Behavioral Microsimulation Analysis* (No. 419). DIW- Discussion Paper.

Straubhaar, T., Vöpel, H., Illing, G., Flassbeck, H., Spiecker, F., Bach, S., and Wagner, G. G. (2012). Inflation und Schuldenabbau. *Wirtschaftsdienst*, *92*(9), 583-598.

Urban, I. (2009). Kakwani decomposition of redistributive effect: Origins, critics and upgrades. Working Paper, ECINEQ (Society for the Study of Economic Inequality).

Verbist, G. (2004), “Redistributive effect and progressivity of taxes: an international comparison across the EU using EUROMOD”, EUROMOD Working Paper Series: EM5/04.

Verbist, G., and Figari, F. (2014). *The redistributive effect and progressivity of taxes revisited: An International Comparison across the European Union* (No EM6/14). EUROMOD Working Paper.

Von Kalckreuth, U., Eisele, M., Le Blanc, J., Schmidt, T. and Zhu, J. (2012). The PHF: a comprehensive panel survey on household and wealth in Germany. Discussion Paper No. 13, Deutsche Bundesbank.