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## 34rd General Conference of the IARIW

Session 5 (plenary session): New Approaches to Studying the Causes and Consequences of Poverty, Inequality, Polarization, and Social Conflict: Multidimensionality and Growth Pro-poorness

Discussed by M. Grazia Pittau - Sapienza University of Rome



Dresden, August 26, 2016.



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#### Background, motivation and goal I

- The paper deals with the debate of economic club convergence that is when economies tend to converge in terms of per capita GDP.
- Since poor countries grow faster than rich countries the key question economists have been tried to answer is the following: Are poorer countries gradually catching up with their richer peers?
- The more recent economic literature shifted from the concept of absolute convergence to the concept of club convergence related to the presence of multiple equilibria in the economy.
- Although the so called "Twin Peaks" convergence has been first introduce by Quah in 1996, there is still no a formal definition for club convergence.

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Background, motivation and goal

# Bimodality and kernel estimation



# Bimodality: the visual impression! I

- If we observe a bi-modal or-more generally- a multimodal distribution of countries' per capita GDP, to what extend the two (or more) modes indicate two (or more) groups of homogeneous countries? How a group of countries similar in terms of per capita income can be defined as a club?
- And how these groups evolve over time? Can the modes really indicate a club convergence process?
- In both years the distribution clearly shows a high mode of poorer countries and a smaller one of rich countries.
- This bimodality in the income shape per se does not necessarily mean that club convergence has taken place between 1995 and 2010.





# Bimodality: the visual impression! II

- If these two modes have become more pronounced over time, we could have concluded that poorer and richer countries have converged towards separate points.
- The overall increase in mean income and in the distributional variance also complicates the direct comparison.
- Therefore, visual inspection of intradistributional changes can be tricky and potentially misleading.
- Starting from these questions, the paper aims to find an unambiguous formal definition for club convergence, and a distribution-based test for it.
- Particularly, the author develops a new approach for tracking polarization, that is the evolution of multiple modes over time.





The Millennium Peak in Club Convergence - A New Look at Distributional Changes in the Wealth of Nations Background. motivation and goal

# Bimodality: the visual impression! III

- The econometric framework refers to the role of the critical bandwidth in kernel density estimation and looks at the evolution of the critical bandwidth at which a distribution appears unimodal.
- Particularly the author proposes a club convergence indicator that captures underlying intradistributional changes of the distribution in only one number.
- The usefulness of the method is demonstrated with an application to the distribution of GDP per capita across countries.





The kernel density estimator and the critical bandwidth

#### Definition of kernel estimator and role of the bandwidth I

Kernel Density Estimation is a nonparametric and data-driven method to estimate a density f(x) based on n observations x<sub>i</sub>:

$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x-x_i}{h}\right) \tag{1}$$

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with kernel function  $K(\cdot)$  and bandwidth h as smoothing factor.

■ For large samples, it is well known that the nonparametric estimation is not sensitive to the different choices of kernel functions (Silverman 1986), while the selection of the bandwidth *h* is instead of crucial importance.

A very large value of h may give an oversmoothed density and, consequently, the shape of the density could be distorted.

The size of the bandwidth indicates the amount of smoothing in the ensity and determines the shape of the density!

- The kernel density estimator and the critical bandwidth

Kernel density estimation: 
$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x-x_i}{h}\right)$$



Figure: kernel density estimation and Gaussian kernel  $K\left(\frac{x-x_i}{h}\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-x_i}{h})^2}$ 





- The kernel density estimator and the critical bandwidth

#### Kernel estimation with different values of the bandwidth: true distribution has 3 modes.



The kernel density estimator and the critical bandwidth

#### The critical bandwidth in a static framework I

- The critical bandwidth  $CB^m$  for *m*-modality is defined as the smallest bandwidth still producing an *m*-modal rather than (m + 1)-modal density.
- For all bandwidths  $h < CB^m$  the estimated density will have at least m+1 modes.
- $CB^m$  can be determined by a binary search procedure.
- Based on this concept and within a **static framework** Silverman (1981) introduce a formal statistical test available for investigating the number of modes in the estimated density  $\hat{f}_h(x)$ .
- The **multimodality test**: "Does the density have m modes or at least m + 1?" uses the critical bandwidth as a statistic to test the null hypothesis that  $\hat{f}_h(x)$  has m modes versus the alternative that  $\hat{f}_h(x)$  has m + 1 modes.

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- The kernel density estimator and the critical bandwidth

#### The critical bandwidth in a static framework II

- A "large" value of *CB<sup>m</sup>* indicates that the true underlying density has more than *m* modes, since a considerable amount of smoothing is required to obtain an estimated density with *m* modes from a (m + 1) modal density, thus rejecting the null.
- A bootstrap procedure determines if *CB<sup>m</sup>* is 'too high' for *m*-modal densities.





The kernel density estimator and the critical bandwidth

## Critical bandwidth and number of modes



Figure the stimated by kernel density estimation are the LARIW values of h where jumps in the step function occur. The number of modes in  $\frac{\text{DRESDEN}}{2016}$  the estimated densities are a decreasing function of the window width.

The critical bandwidth as an indicator of club convergence

#### The critical bandwidth in a dynamic framework I

- The basic idea of the paper: If the two modes become more pronounced, more smoothing is necessary to obtain a unimodal density.
- Therefore: increases in the smoothing parameter constitute evidence of a trend toward a bipolar distribution or club convergence, decreases indicate a trend toward uni-modality.
- In order to detect club convergence, this papers looks at the distribution over time rather than at a given point in time and- in particular- observe whether the modes become more pronounced.
- Particularly, this is done by looking at changes in the critical bandwidth: changes in the critical bandwidth measure how the shape of the distribution has changed.

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The critical bandwidth as an indicator of club convergence

#### The critical bandwidth in a dynamic framework II

If the two modes of a bimodal distribution become more pronounced, the critical bandwidth for unimodality goes up as more smoothing must be applied to obtain a unimodal density.

## Dynamic setting:

- The critical bandwidth based on raw data is sensitive to changes affecting the whole distribution, typically the increase of variance in the worldwide distribution;
- an indicator of club convergence should only reflect how pronounced the modes and should be invariant to changes in the overall distributional variance.
- Solution: standardization of the densities to eliminate the influence of time-varying variance.
- No matter how well-pronounced the two modes are in the beginning, the paper looks at the changes over time that are crucial for the population of the second sec

Only intradistributional changes can show the dynamics at work 2016

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The critical bandwidth as an indicator of club convergence

#### The critical bandwidth in a dynamic framework III

Changes in the critical bandwidth for unimodality are therefore related to club convergence:

the Critical Bandwidth as an Indicator of Club Convergence:

- Let f(x) be a standardized income per capita density with at most two clusters.
- The density is observed at two points in time, t=1,2 and the critical bandwidths for unimodality at t = 1 and t = 2 are calculated as CB<sub>t1</sub> and CB<sub>t2</sub>. In this setting:
- we experience **club convergence** if and only if  $CB_{t2} > CB_{t1}$ .
- we experience instead **de-clubbing** if and only if  $CB_{t2} < CB_{t1}$ .
- Intuitively when the two modes become more (less) pronounced, the critical bandwidth for unimodality, increases (decreases) because more (less) smoothing needs to be applied to make the bimodal share turn into a unimodal one.

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The critical bandwidth as an indicator of club convergence

#### Implementation and Properties I

- Within this framework, *CB* (critical bandwidth for **unimodality**) provides a club convergence indicator in just **one number**: in fact, club convergence can result from an increase in between-cluster separation, an increase of within-cluster concentration or a combination of both. All of these developments will be reflected in an increase in *CB*.
- Changes in *CB* can easily be calculated: Track the test statistic of Silverman's (static) multimodality test over time.
- Asymptotic properties: Consistent estimation of the change in CB as  $n \to \infty.$
- Suggestion of a bootstrap procedure involving longitudinal correlation (based on Biewen, 2002) to determine significance of the change in CB.

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Empirical analysis

#### The distribution of per capita GDP I

- The Data Set:
  - GDP per capita measured at PPP, taken from Penn World Tables 8.0
  - Yearly data 1970-2011
  - 123 countries: no oil producers nor tiny states (population below 300,000)
- Steady increase of mean and standard deviation:





#### Empirical analysis



Figure: Descriptive Statistics for per Capita GDP in the 123-Country Data set





Empirical analysis



Empirical analysis



Empirical analysis

Main results

### Main Results: The Evolution of CB and the millennium peak

- Silverman's Static Test: After 1983, always reject unimodality (95% level).
- Bimodality  $\neq$  Club Convergence, look at evolution of CB:
  - CB varies around a constant level from 1970 to the middle of the 1980s.
  - CB exhibits a notable increase afterwards and reaches its highest value of 0.6251 in 2002.
  - After that CB falls again until reaching levels of the 1970s and early 1980s.
- Club Convergence into two modes of rich and poor countries in the 1980s/1990s,
- Statistically significant decrease after the Millennium Peak in 2002: tendency of de-clubbing with modes becoming again less clearly.
   Separated.

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#### Empirical analysis

#### Main results



Figur Strotztion of the CB for Unimodality Over Time and p-Values of ARW Silverman's (1981) Bootstrap Test (5000 Replications) 2016

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— Empirical analysis

Main results

#### Significance of the Evolution of CB

- The p-values associated to the null hypothesis of equality between CB in 1970 and later years form a U-shape around the Millennium Peak: in the 1980s/1990s the alternative is strongly rejected while in the late 1990s and early 2000s CB is not significantly different from the 1970s anymore.
- Further insights from the second test: **null hypothesis of equality between CB in 2002 and later years** starting from 2005.



Empirical analysis

Main results

#### Countries' club membership

- The **antimode** between the two modes in the kernel density plot is taken a cut-off to allocate the countries between groups.
- Working with standardized data implies to focus on the relative rather than absolute per capita income: the cut-off in 1970 lies at 1.00 standard deviation above the mean (USD 9,9) while the cut-off in 2011 at 0.58 (USD 21,4).
- This division of countries into the poor and rich club confirms very low mobility: 109 out of the 123 countries stay in the same club for each of the 42 years from 1970 to 2011.
- The 28 countries in the rich club are essentially OECD members.
- 14 "mobile" countries changed clubs at least once in the 1970-2011:
  - Asian tigers: (Korea and Taiwan) and some EU countries (Ireland, Spain, Cyprus) from the poor to the rich;

**27. Bahamas**: from the poor to the rich (70s) and back to the poor **3 Israel** from the rich to the poor and back to the rich again.

- Empirical analysis
  - Main results



Figure: Trajectories of Selected Countries' Standardized Income per Capita Over Time





# Caveat, specific comments, suggestions |

- Interesting elegant and simple contribution to convergence measurement literature which provides another instrument for practitioners.
- I am skeptical about the use of the CB as a measure for detecting multimodality. The bandwidth is very sensitive to changes in the density especially in presence of "outliers".
- As a general principle in applied work it is not a good idea to "leave out" outliers since it can be argued that they contain a lot of information to be explained.
- Actually population weighting the sample would solve the problem in the present case. How would the CB behave with weighed data?

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# Caveat, specific comments, suggestions ||

- New evidence on economic convergence suggesting that the distribution of per-capita income of countries may display more than two convergence club: in fact some distributions (1970, 1985 and 2000) look tri-modal. How would CB work in a regime switch environment where the number of clubs is changing either up or down?
- The method is strictly related to small samples. If the sample size increases the CB becomes very difficult to manage, even worst with the bootstrap.
- There is no possibility to interpret in an economic way the groups of countries. What if the number of groups remain the same but their characteristics (means, variances, weights) change over time?
- The definition of antimode could be arbitrary.





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- thanks to the organizers for the opportunity to read these interesting papers
- thanks to Melania for help
- congrs to Melania since the paper is a very good paper but... move on to mixtures!



