Counting multidimensional deprivations in the presence of differences in needs

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Goal of the paper

- Individuals from different demographic population subgroups and households of different size and composition exhibit different needs.

- Multidimensional deprivation comparisons in the presence of these differences in needs have yet to be analyzed.

- The paper extends the approach of Alkire and Foster by proposing a family of multidimensional deprivation indices that explicitly takes into account observed differences in needs across demographically heterogeneous units

- This is a very interesting paper but it is very long, even in its shortened version and I will not have time to discuss all the topics covered in the paper (like the section on axiomatic derivation which is new in this version)

Throughout my presentation I will use the following simple illustration:

Assume that there are three indicators (j = 1 to 3) and three households (i =1 to 3):

- Standard of living: income per adult in the household
- Education: literacy level of the adults (knows to read or not)
- Health: MUAC (mid-upper arm circumference for age)

The poverty thresholds (z_j) are:

- Standard of living: say, 1000 for the income per adult in the household
- Education: are there illiterate adults?
- Health: are there children aged 6 to 60 months with a MUAC less than 115 mm.?

Here is a simple illustration with 3 individuals. Each number x_{ij} in the following table gives the value of the indicator j for individual i. We assume that a household has to be poor in at least two dimensions to be considered as poor (k = 2)

Data for illustration: the traditional Alkire and Foster approach

Household	Income per adult	Education (illiterate adults?)	MUAC< 115mm
A	800	0	1
B	700	0	0
C	900	1	1

Taking into account the "incidence" of poverty for the various dimensions (who is poor in each dimension)

Household	Poor in Income Dimension?	Poor in Education Dimension?	Poor in Health Dimension?	Number of dimensions in which household is poor	Poor (overall), assuming the cutoff k is equal to 2?
A	1	0	1	2	1
B	1	0	0	1	0
C	1	1	1	3	1

It is then easy to see that the headcount ratio (percentage of poor) is equal to 2/3.

Let us build the censored matrix (ignoring the non poor)

Household	Income	Education	Health	Number of dimensions in which household is poor
A	1	0	1	2
B	0	0	0	0
C	1	1	1	3

- The “average deprivation share” A among the poor is hence equal to $\frac{(2+3)}{6} = \frac{5}{6} = 0.833$
- The “dimension adjusted headcount ratio” M_0 is then equal to $M_0 = HA = \left(\frac{2}{3}\right)\left(\frac{5}{6}\right) = \frac{5}{9}$=0.556
- In other words M_0 is equal to the ratio of the total number of deprivations experienced by the poor (5) divided by the maximal number of deprivation in the total population (9).

Taking needs into account

Calling as before *i* the *household* , *j* the deprivation dimension and *S* the data matrix with as typical element s_{ij}, we now define for each dimension the *applicable population*. The presence or absence of deprivation will be measured only for the applicable population.

We now define the *individual* dimensional deprivation indicator g_{ij} evaluated on its *applicable population* as

$$g_{ij}(s_j) = 1 \text{ if } s_{ij} < z_j \text{ and } s_{ij} > 0$$

$$= 0 \text{ otherwise}$$

where s_{ij} is the indicator for dimension *j* for an individual *i* belonging to the applicable population.

Each individual belongs to a specific household *h* and each household has q_h members.

Defining the $\boldsymbol{d_{ij}^{\beta}}$ −dimensional deprivation indicator for household *h* and dimension *j*:

$\boldsymbol{d_{ij}^{\beta}}$=$\left(\sum_{i\in q_h} g_{ij}(s_j)\right)^{\beta}$ if $\sum_{i\in q_h} g_{ij}(s_j) > 0$

= 0 otherwise

where $\beta \geq 0$ is the parameter of aversion to deprivation.

Note that when $\beta = 0$, the household dimensional deprivation will be equal to 0 if no household member is deprived and to 1 otherwise.

When $\beta = 1$, the dimensional deprivation is equal to the count of deprived household members in the *j*-dimension.

This β-parameter is somehow like the α-parameter defined for the FGT index. But it modulates the breadth of household deprivation in terms of the number of deprived household members.

An empirical illustration

Going back to the previous illustration assume that

- **Household A:** includes two adults, one with an income of 1600, the other without any income . The two adults know to read. There are 3 children, one less than 6 months old; the others older than 6 months but younger than 60 months and both children have a MUAC smaller than 115mm.
- **Household B**: Includes only one adult (with an income of 700) who is literate.
- **Household C**: includes two adults and a child. One adult has an income of 1200, the other of 600. One adult is illiterate. The child is two years old and has a MUAC smaller than 115mm.

Taking into account the “applicable population” we can build the two following tables for the cases where β=0 and β=1.

Measuring d_{ij}^{β} in our empirical illustration

The case where β=0:

Household	Standard of Living	Education	Health
A	1	0	1
B	1	0	0
C	1	1	1

The case where β=1:

Household	Standard of Living	Education	Health
A	1	0	2
B	1	0	0
C	1	1	1

Defining the size $\boldsymbol{n_{ij}^{\beta}}$ of household *h*’s needs on the *j* dimension:

$n_{ij}^{\beta} = \left(\sum_{i \in q_h} s_{ij}\right)^{\beta}$ if $\sum_{i \in q_h} s_{ij} > 0$

= 0 otherwise.

If $\beta = 0$, n_{ij}^{0} indicates whether household *h* had needs in dimension *j*.

If $\beta = 1$, n_{ij}^{1} gives the number of household members who have needs in dimension j.

The size of the household needs N_h^{β} is then expressed as

$N_h^{\beta} = \sum_{j \in J} n_{hj}^{\beta}$

N_h^{0} refers then to the number of dimensions in which household *h* has needs while N_h^{1} corresponds to the number of achievements for which household *h* has needs.

We therefore derive the two following matrices:

Remember the data:

- **Household A:** includes two adults, one with an income of 1600, the other without any income . The two adults know to read. There are 3 children, one less than 6 months old; the others older than 6 months but younger than 60 months and both children have a MUAC smaller than 115mm.

- **Household B**: Includes only one adult (with an income of 700) who is literate.

- **Household C**: includes two adults and a child. One adult has an income of 1200, the other of 600. One adult is illiterate. The child is two years old and has a MUAC smaller than 115mm.

Measuring n_{ij}^{β} in our empirical illustration

The case where β=0:

Household	Standard of Living	Education	Health
A	1	1	1
B	1	1	0
C	1	1	1

The case where β=1:

Household	Standard of Living	Education	Health
A	2	2	2
B	1	1	0
C	2	2	1

Measuring the burden $m_h^{\beta,\theta}$ that multidimensional deprivation places on the household

This burden is defined as

$$m_h^{\beta,\theta} = \frac{\sum_{j=1}^{J} d_{hj}^{\beta}}{\left(\sum_{j=1}^{J} n_{hj}^{\beta}\right)^{\theta}} \text{ if } \sum_{j=1}^{J} n_{hj}^{\beta} > 0.$$

= 0 otherwise.

- Note that de facto the famous Alkire and Foster approach is equivalent to using a $m^{0,0}$ metric which amounts to counting the number of dimensions in deprivation and ignoring household needs. The AF approach thus ignores the difference between non deprived and non-applicable dimensions and assigns implicitly a lower deprivation burden to small households.
- Note also that the $m^{\beta,\theta}$ measure defined previously has been defined by

- aggregating first individuals' deprivation at the household level for each dimension
- aggregating afterwards deprivations across dimensions

It is hence a *first individuals, then dimensions aggregating order*.

One could have however adopted the reverse order but then the measure would have been different.

In short:

$\beta = \mathbf{0}, \theta = \mathbf{0}$: This is the *dimensions-count-based approach* where we simply count the dimensions with at least one household member under deprivation.

$\beta = \mathbf{0}, \theta = \mathbf{1}$: This is the *dimensions-share-based approach* where we compute the share of possibly deprived dimensions.

$\beta = \mathbf{1}, \theta = \mathbf{0}$*:* This is *the deprivations-count-based approach* where we count the number of household deprivations.

$\beta = \mathbf{1}, \theta = \mathbf{1}$: *This is the deprivations-share-based approach* where we compute the share of household possible deprivations.

Remember the data:

The value of d_{ij}^{β} in our empirical illustration when β=0:

Household	Standard of Living	Education	Health
A	1	0	1
B	1	0	0
C	1	1	1

The value of n_{ij}^{β} when β=0:

Household	Standard of Living	Education	Health
A	1	1	1
B	1	1	0
C	1	1	1

We therefore derive that when β = 0 and θ = 0:

Household	$m_h^{0,0}$
A	2
B	1
C	3

But when β = 0 and θ = 1, we get:

Household	$m_h^{0,1}$
A	(2/3)
B	(1/2)
C	(3/3)

Remember the data:

The value of d_{ij}^{β} in our empirical illustration when β=1:

Household	Standard of Living	Education	Health
A	1	0	2
B	1	0	0
C	1	1	1

The value of n_{ij}^{β} when β=1:

Household	Standard of Living	Education	Health
A	2	2	2
B	1	1	0
C	2	2	1

We hence derive that when β=1,$\theta = 0$:

Household	$m_h^{1,0}$
A	3
B	1
C	3

Whereas when β = 1, $\theta = 1$, we get:

Household	$m_h^{1,1}$
A	$(3/6) = (1/2) = 0.5$
B	$(1/2) = 0.5$
C	$(3/5) = 0.6$

Using a threshold k (which clearly will be a function of β and θ), a binary indicator p_h of the presence or absence of multidimensional deprivation will be defined as

p_h=1 if $m_h^{\beta,\theta} \geq k$

= 0 otherwise.

Then, the simplest metric to represent the overall society multidimensional deprivation incidence $MD^{\beta,\theta}$ is equal to the average over all households of the binary indicator p_h.

Note that $MD^{0,0}$ corresponds to the M_0 metric of Alkire and Foster.

The empirical illustration of the authors

For the empirical analysis in this paper, a multidimensional deprivation index is built using the 2013 Paraguayan Household Survey (PHS).

Table 1: Dimensions, indicators, weights, applicable population subgroups and deprivation criteria

Dimension	Deprivation indicator	Applicable population subgroups where the indicator is relevant to be measured	A person from the applicable population is deprived if:
Access to health services	Health insurance non-coverage	Any person	Is not covered by any health insurance
Access to health services	Non-access to health services when needed	Any person that was sick or had an accident during the 90 days previous to the interview	Did not receive institutional care
Education	Non-school attendance	5 - 17 years old population	Is not attending school
Education	Low educational achievement	Population 18 years old and over	Has less than 9 years of completed education
Dwelling conditions	Sub-standard housing	Any person	Lacks at least 2 of the following 3 dwelling conditions: flooring different from earth or sand; adequate material of ceilings and adequate material of walls

Table 2: Percentage of households with at least one deprived person from the applicable population

Persons per household

	1	2	3	4	5	6	7 or more	Total
(1) Health insurance non-coverage	70.3	75.2	79.9	81.0	84.3	91.4	93.8	81.3
(2) No access to health service	12.5	17.7	15.1	19.7	20.1	23.6	29.6	19.0
(3) Non-school attendance	0	1.9	2.9	4.4	5.8	9.7	21.4	5.5
(4) Low educational achievement	61.4	64.8	57.8	65.4	68.6	78.3	88.3	67.0
(5) Sub-standard housing	25.5	25.0	18.5	19.9	23.5	24.9	34.2	23.3
Number of households	593	836	1,135	1,108	771	466	514	5,423
% of individuals	2.8	8.0	16.3	21.2	18.4	13.4	19.9	100

Table 2 gives for each household size the proportion of households with at least one deprived household member in each of the 5 indicators (mean of the d^{0}_{hj} for the 5,423 Paraguayan households).

It appears that larger households have generally a higher proportion of dimensional deprivation.

There is a positive link between dimensional deprivation and household size because the number of persons in the applicable population rises with the household size (e.g. non-school attendance).

It is then preferable to compare the multi-dimensionally deprived population on the basis of the $m^{\beta,\theta}$ indicators.

For each couple (β,θ) the 40% most deprived (2168 households) are identified as multi-dimensionally deprived. This criterion is different from the one used in our simple illustration (determining a value of *k* which depends on the β and θ parameters) and better because the different deprived populations can be compared on an equal basis.

Figure 1 plots the H-multidimensional deprivation index by household size for m^{00}, m^{01}, m^{10} and m^{11} (vertical axis: proportion of households multi-dimensionally deprived).

Thus 80% of the 514 households with 7 or more members are deprived when m^{10} is used.

We observe that when no adjustment is made for needs (θ=0) H is higher among larger households. This is not true for θ=1 where there is rather a U-shaped relationship between H and the household size.

One can in fact consider that difference in needs are a legitimate source of difference in multidimensional deprivation incidence . But what value of θ should be used? This is not a simple. Question.

The authors state that an *unbiased* multi-dimensional deprivation incidence profile is such that it is unable to distinguish between two population subgroups that have no systematic differences in deprivation between each other but only different sets of needs.

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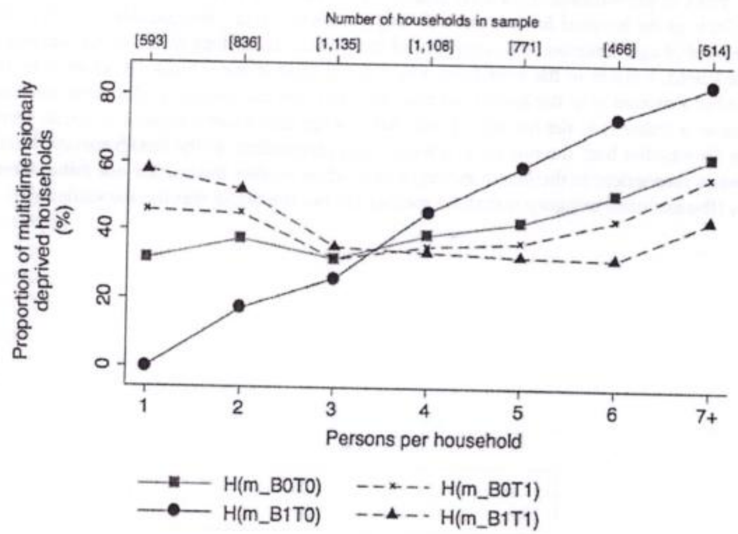
Figure 1: Proportion of multidimensionally deprived households, $H(m^{\beta,\theta})$, across household size



Source: author's calculations based on 2013 PHS.

Figure 1 stage 2

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Since differences in deprivation related to differences in needs cannot be derived from an observed deprivation profile because some of the deprivation differences may be due to other reasons, the authors constructed a counterfactual deprivation profile where the observed deprivation differences are only related to needs.

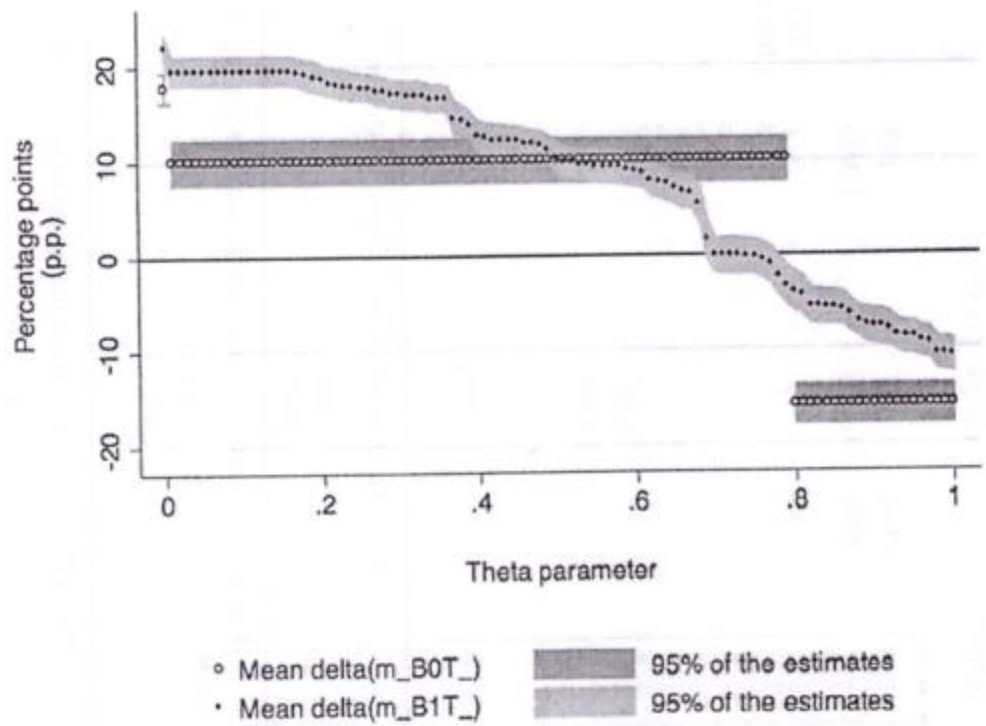
They did this by fixing the characteristics of the households that determine differences in needs (size of the household, age of the members,…) and, then for each deprivation dimension, they allocated randomly whether the member of the household is deprived or not.

This random allocation was implemented by sampling without replacement from the observed deprivation so that the total number of deprived people is the same in the actual and counterfactual sample.

In each counterfactual state the authors measured the correlation between the multidimensional deprivation incidence p_h and the size N_h^0 of the household needs (the sum of the needs in the different dimensions) : p_h=ρ + δ N_h^0

This random allocation was done 1000 times in order to derive confidence intervals for the coefficient δ.

Figure 2: Simulation results: distribution of the obtained δ regression coefficient in p.p. when using $m^{\beta,\theta}$ to sort and identify the most deprived households



Source: author's calculations based on 2013 Paraguayan Household Survey (PHS). Notes: Estimated population means based on a sample of 5,423 households. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed households, keeping constant the demographic configuration of the households and the societal amount of deprivation in each indicator. Shaded areas denote 95% of the obtained δ estimates. The lower limit corresponds to the δ value at the 2.5 percentile and the upper limit to the δ value at the 97.5 percentile.

- It appears that a combination of $\beta = 1$ and $0.69 \leq \theta \leq 0.77$ satisfies the desirability condition.

Some additional remarks made by the authors:

- One can naturally use different weighting systems for the various deprivation dimensions.
- The authors ignored the intra-household distribution of resources
- The authors ignored the possible correlations across dimensions.

The paper ends by a careful axiomatic derivation of the multidimensional deprivation measures $MD^{\beta,\theta}$ discussed in the paper (with specific values of β and θ).

My comments

- First of all this is a very nice paper which represents a significant Alkire-Foster approach to multi-dimensional poverty measurement because it takes into account both equivalence scales and the domains which may not be relevant for an individual or household when estimating the extent of its multi-dimensional poverty (e.g. schooling of children for a household with only one adult)
- It seems however that the authors' approach does not solve some shortcomings of the Alkire and Foster approach which were, for example, mentioned in a paper I co-authored with Gaston Yalonetzky:

- **at the identification stage** , plotting on the horizontal axis the (weighted) sum c_n of deprivations suffered by an individual (household) and on the vertical axis the probability that this individual (household) will be poor, the Alkire Foster compares c_n with the cutoff k so that de facto it is assumed that up to k the domains are substitutes and afterwards they are complements. The probability function is hence horizontal at height 0 up to k, then vertical up to the height 1 and then again horizontal.

- One could think of a more “fuzzy” approach (see, Rippin, 2010) with a convex or concave function. How does one choose between a concave and a convex function? That evidently depends on whether it is assumed that the dimensions are complements or substitutes.
- If they are substitutes, then deprivation in one dimension may be overcome by having no deprivation in another dimension so that as long as an individual is not deprived in all dimensions his overall deprivation score will be equal to zero in the case of perfect substitution (the “intersection” case) or smaller than one when dimensions are imperfect substitutes (the case of a convex identification function).
- If on the contrary the deprivation dimensions are complements, as soon as an individual is deprived in one dimension, he must suffer from some overall deprivation. If it is assumed that the deprivation dimensions are perfect complements, then one obtains the “union” case while if they are imperfect complements one gets the more general case of a concave identification function.

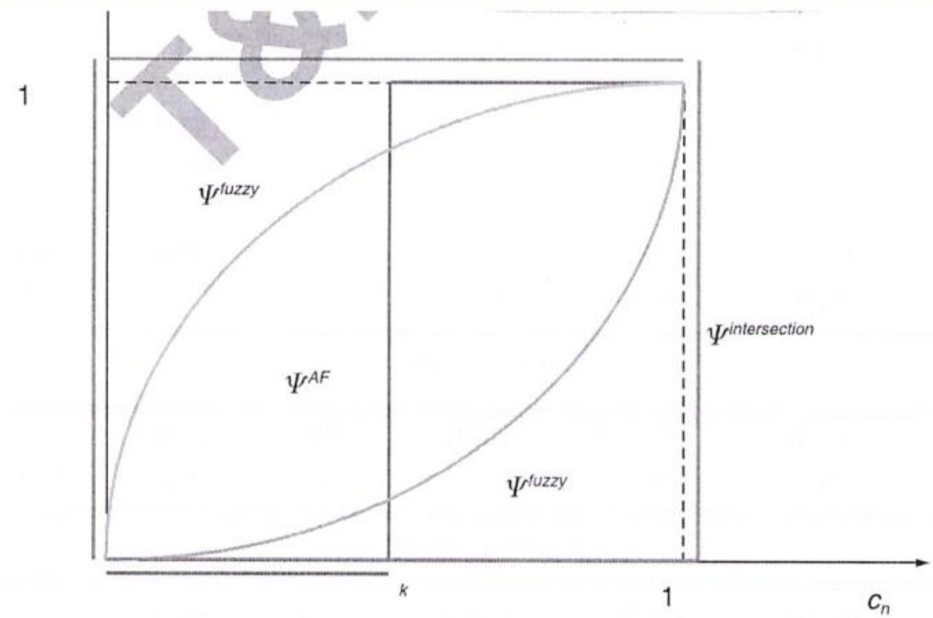


Figure 2.1 Some possible shapes of the identification function ψ.

- The stage of measuring the "breadth" of poverty:

- In the case of continuous variables, the magnitude of the poverty gap between the attainment and the poverty line is taken into account but this is impossible with ordinal variables. It is however possible to make the individual poverty function depend on the number of deprivations.
- More precisely the individual poverty function would be defined as the product of the identification function previously mentioned and a function *g* that captures the breadth of his/her poverty and will depend on the number of his/her deprivations.
- For instance, in the case of the adjusted headcount ratio from the Alkire–Foster family, $g^{AF} = c_n$. In the case of the family of deprivation scores defined by Chakravarty and D'Ambrosio (2006), $g^{CD} = h(c_n)$, where $h' > 0$ and $h'' > 0$ (convex function, as in Figure 2.2).
- We do not want to consider concave breadth functions because we want overall social poverty not to increase whenever inequality in deprivation counts among the poor decreases, an issue which precisely is ignored by the Alkire and Foster approach.

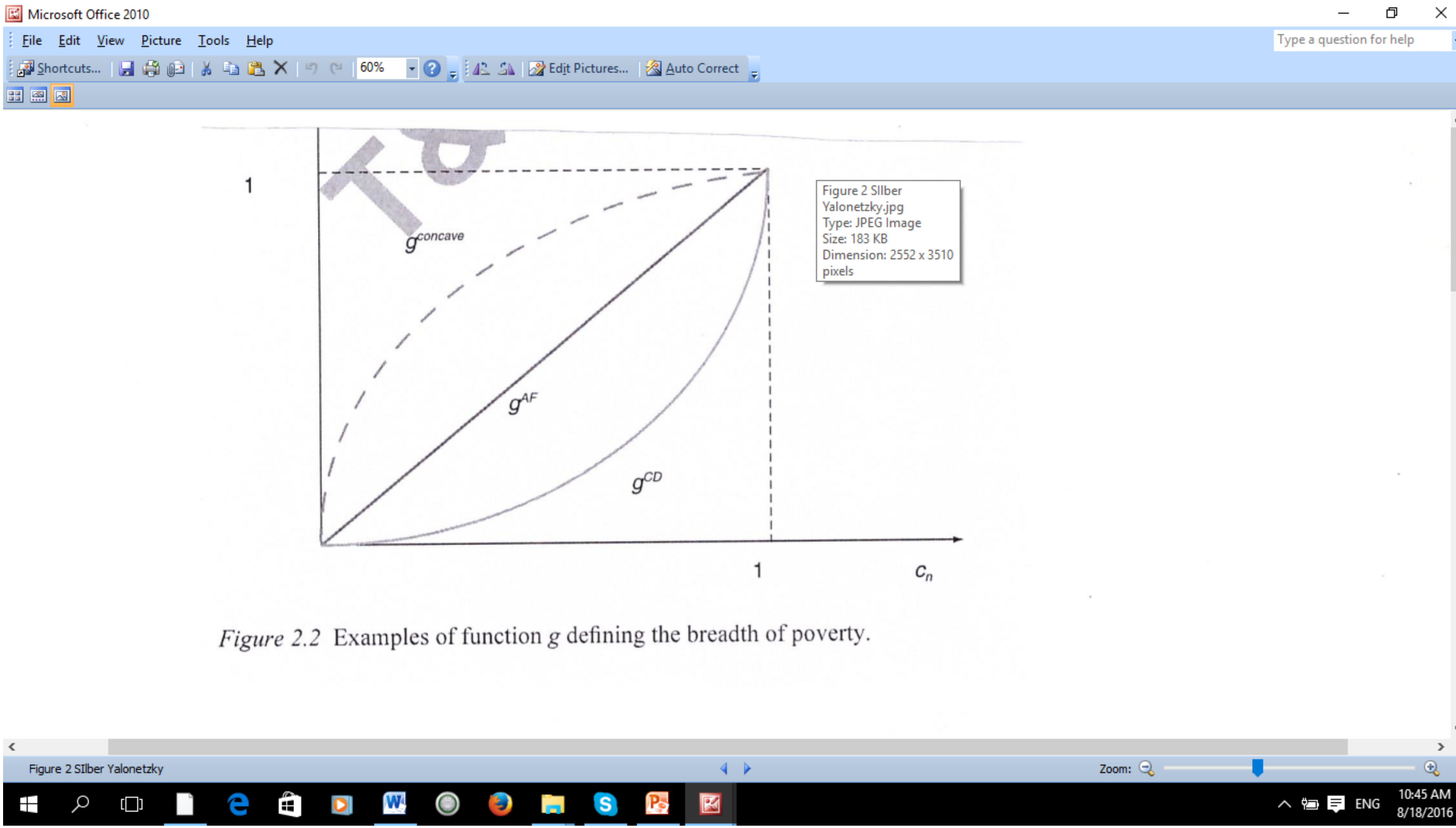


Figure 2.2 Examples of function g defining the breadth of poverty.

A simple illustration:

Individual	Deprivation in domain 1	Deprivation in domain 2	Deprivation in domain 3	Deprivation in domain 4
A	1	1	1	1
B	0	0	1	1

Assume the cutoff *k* is equal to 2. The value of c_n for A is hence (4/4)=1 and for B (2/4)=0.5. For Alkire and Foster the "breadth" of poverty will hence be 1 for A and 0.5 for B. Since the identification for A and B is 1 (both are poor) the product of identification by breadth (individual poverty function) will be 1 for A and 0.5 for B and the poverty at society's level will be (1+0.5)/2=0.75.

With a convex function like $g = c^2$ the "breadth" of poverty for A is 1 and for B 0.25. The poverty in society will hence be (1+0.25)/2=0.625

A concave function like $c = \sqrt{c}$ would give a "breadth" of poverty of 1 for A and $\sqrt{0.5}$ =.707 and poverty in society will be 1.707/2=0.854

Now assume an alternative situation:

Individual	Deprivation in domain 1	Deprivation in domain 2	Deprivation in domain 3	Deprivation in domain 4
A’	1	1	0	1
B’	1	1	1	0

Assume the cutoff *k* is equal to 2. The value of c_n for A and B is hence (3/4)=0.75. For Alkire and Foster the “breadth” of poverty will hence be 0.75 for A and B. Since the identification for A and B is 1 (both are poor) the product of identification by breadth (individual poverty function) will be 0.75for A and 0.5 for B and the poverty at society’s level will be (0.75+0.75)/2=0.75, like in the previous case.

With a convex function like $g = c^2$ the ”breadth” of poverty for A and B is $(0.75\times 0.75) = 0.5625$. The poverty in society will hence be (0.5625)/2=0.5625 and hence smaller than before.

A concave function like $c = \sqrt{c}$ would give a “breadth” of poverty of $\sqrt{0.75}$=0.866 for A and B and poverty in society will be 0.866, a higher value than in the previous case, although inequality in deprivation between A and B decreased, an issue that Alkire and Foster’s approach also ignores.