INCOME DISTRIBUTION, HOUSEHOLD HETEROGENEITY AND CONSUMPTION INSURANCE IN THE UK: A MIXTURE MODEL APPROACH

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Aim of the paper

- Assess the extent to which shocks to income appear to result in shocks to consumption
- Increase in income inequality: greater heterogeneity or more uncertainty?
- Is there evidence that ability to smooth income has increased over time?
- Using FES data

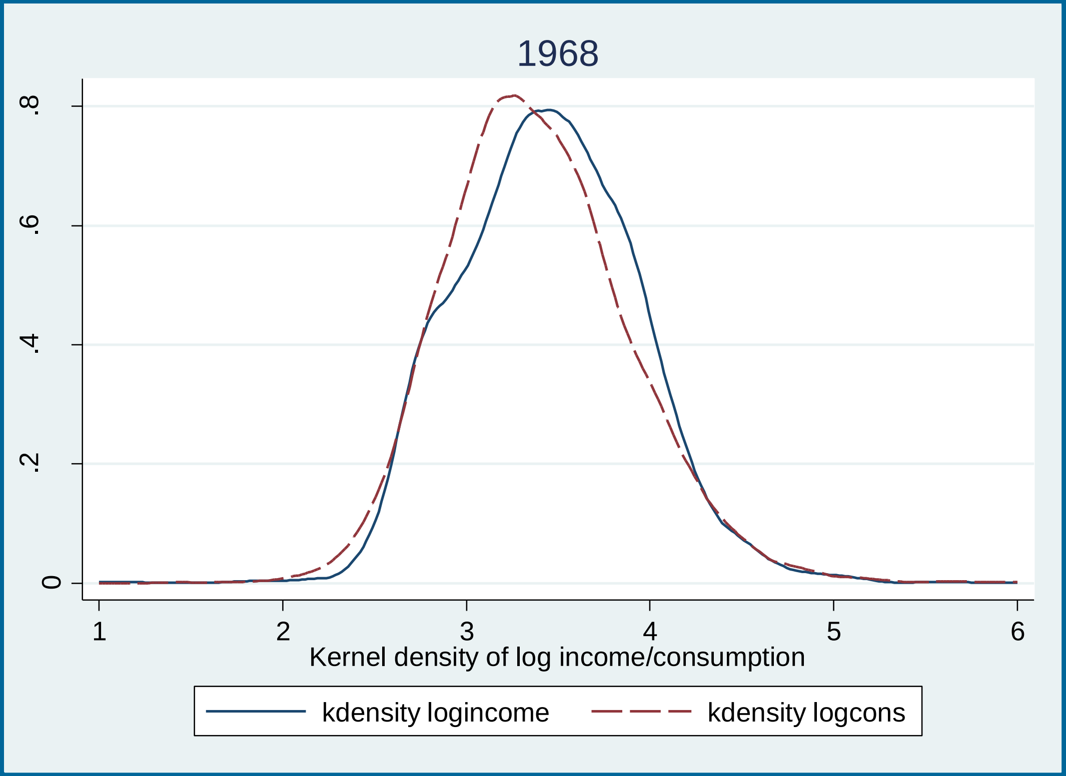
Key identification strategy

- Problem of identification: how to properly model household heterogeneity (e.g. different types of households);
  - One type: any data will reject smoothing
  - Infinite number of possible types: no data can reject smoothing
  - We assume **finite** number of types

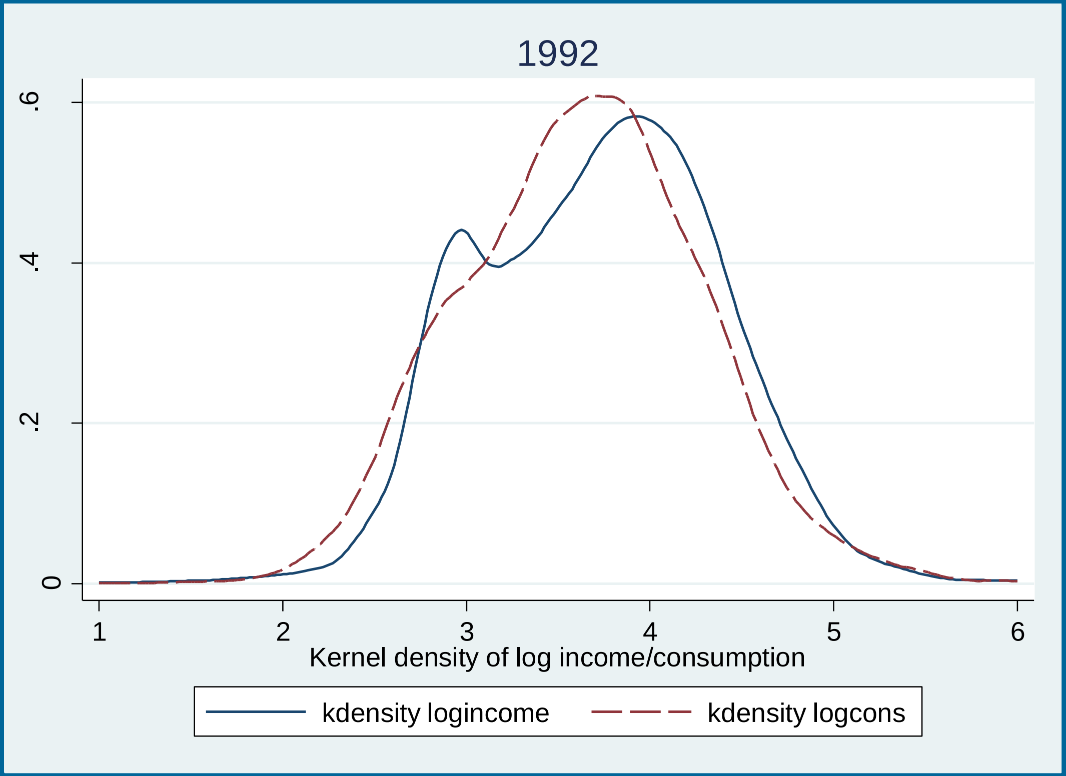
Analytical approach

- Income distribution is not unimodal (show graphs)
  - We identify the different types by fitting a mixture of normals to this distribution
  - Permanent income differs between but not within type
  - Test is essentially to see whether variance of income *within each type* is associated with variation in consumption

Kernel density estimate of income and consumption over time (1)



Kernel density estimate of income and consumption over time (2)



Income generating process

Permanent income

Temporary shock

Income of member of group 1

$$y_{it1} = p_{1t} + m_{1t} + \varepsilon_{it1}$$

Macro shock for group 1

$$y_{it2} = p_{2t} + m_{2t} + \varepsilon_{it2}$$

$$y_{it3} = p_{3t} + m_{3t} + \varepsilon_{it3}$$

Consumption model

Consumption of group j
(assuming away macro shocks)

$$c_{itj} = p_{jt} + \eta_{itj}$$

No insurance

$$cor(\varepsilon, \eta) = 1$$

Full insurance

$$cor(\varepsilon, \eta) = 0$$

Aim is to compare the fit of these two extreme models (eyeball at present, later work will develop more formal test)

Methodology

- Estimate the distribution of household income as a mixture of normals
- Take the parameters of each normal distribution and use them to predict:
  - the distribution of consumption
  - the relationship between income and consumption

Mixture models

- Mixture models useful to describe complex distributions;
- π = subgroup proportions, with $\sum\pi = 1$;
- θ = distribution parameters (e.g. mean and s.d.);
- To be estimated via ML;

$$f(x) = \sum_{j=1}^{n} \pi_j g(x; \theta_j)$$

Empirical estimation procedure

- Parameters are given by the following likelihood function for mixtures of normals:

$$L(\mu,\sigma^2,\pi)=\prod_{i=1}^{n}\left\{\sum_{j=1}^{c}\pi_j\left[\frac{1}{\sqrt{2}\sigma_j}\times\exp\left(-\frac{1}{2}\left(\frac{\left(x_i-\mu_j\right)^2}{\sigma_j^2}\right)\right)\right]\right\}$$

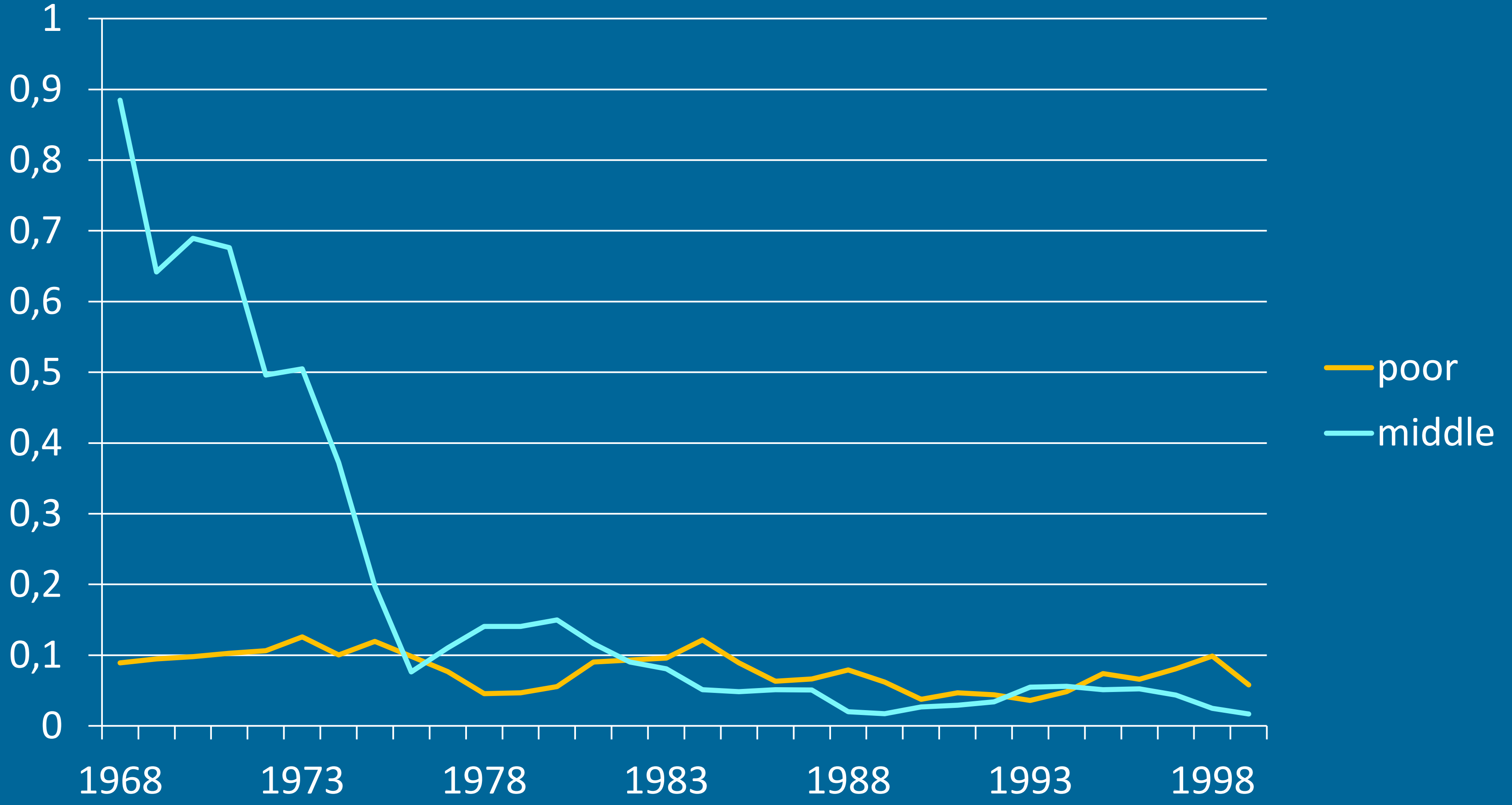
Some computational issues (1)

- In models like this the Likelihood function is not well behaved
  - In samples very different models can give very similar fit to the data
  - Different starting values can give slightly different parameters
  - The number of groups can not be directly estimated

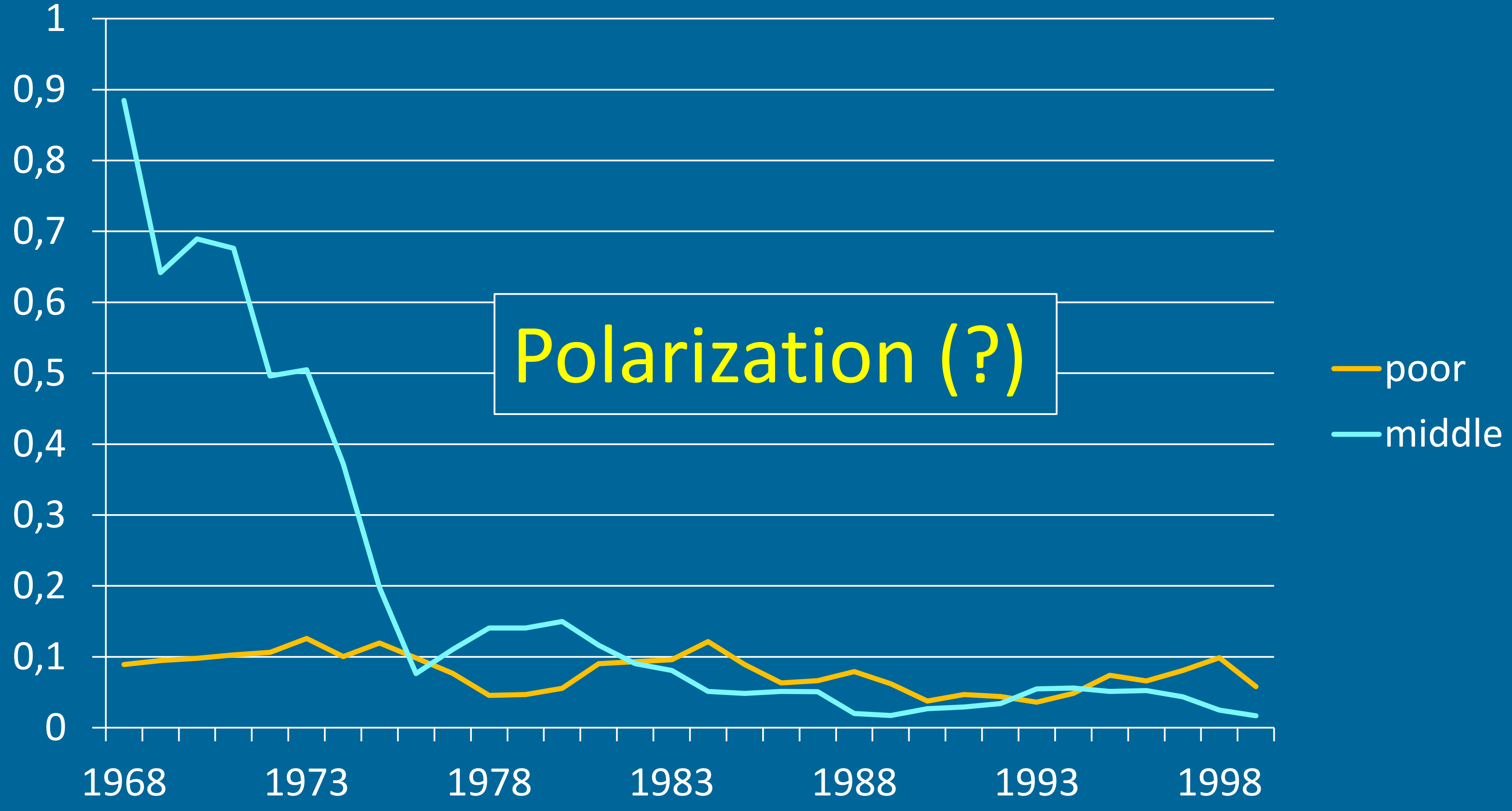
Some computational issues (2)

- Thus we impose
  - The number of groups to be the same in each year
    - 3 groups found to be the maximum that could be estimated
  - That the proportion in each group does not experience wide year on year variation but may (and does) change gradually over time
  - That the relative ranking in income of each group does not change
    - Groups are best understood as skill groups
- These restrictions are imposed by taking the best fit unrestricted estimates of each years proportions, smoothing them using a MA(5) process and then re-estimating the means and the variances using these restricted proportions

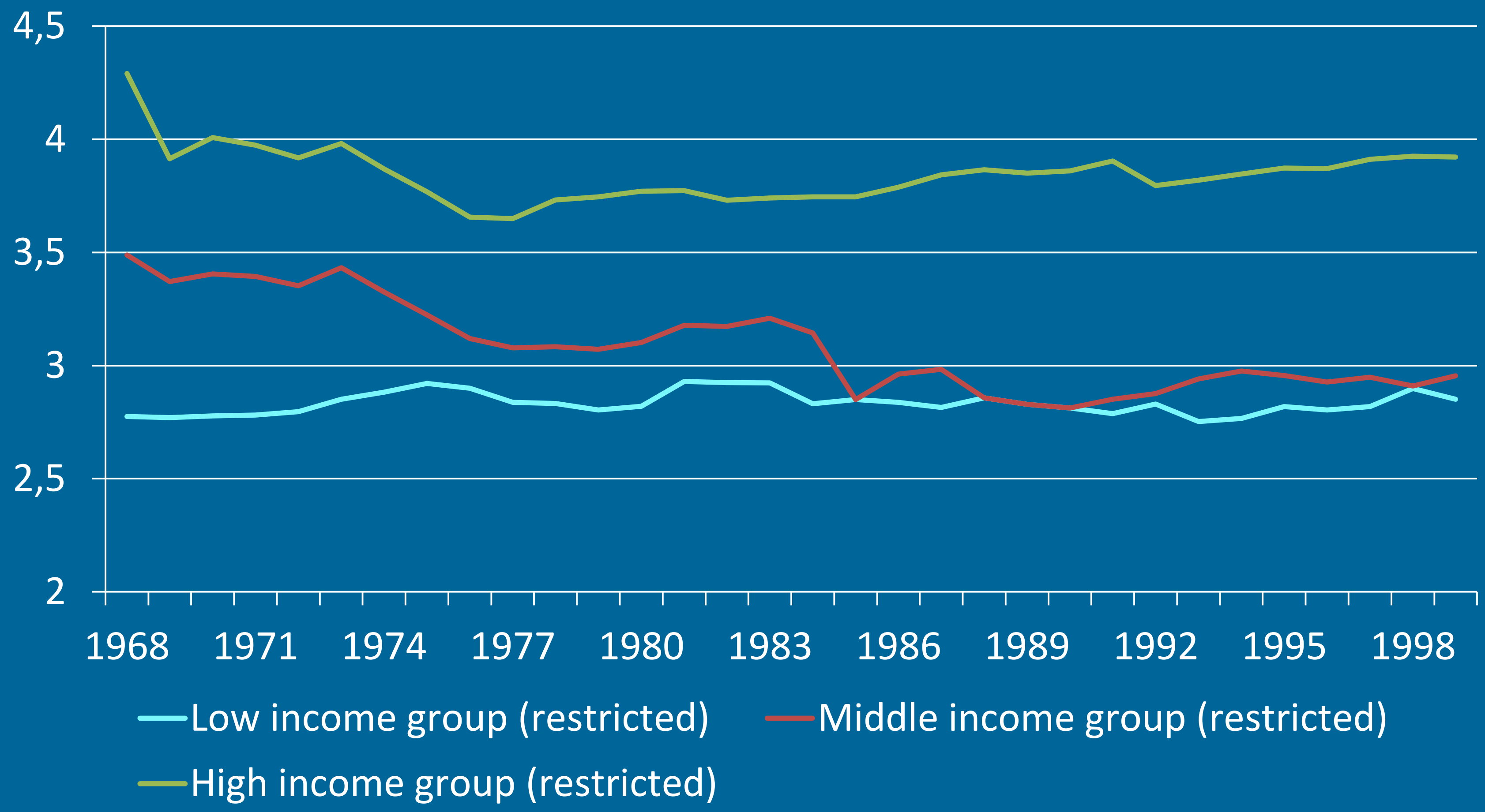
Trends in smoothed proportions of each income groups π



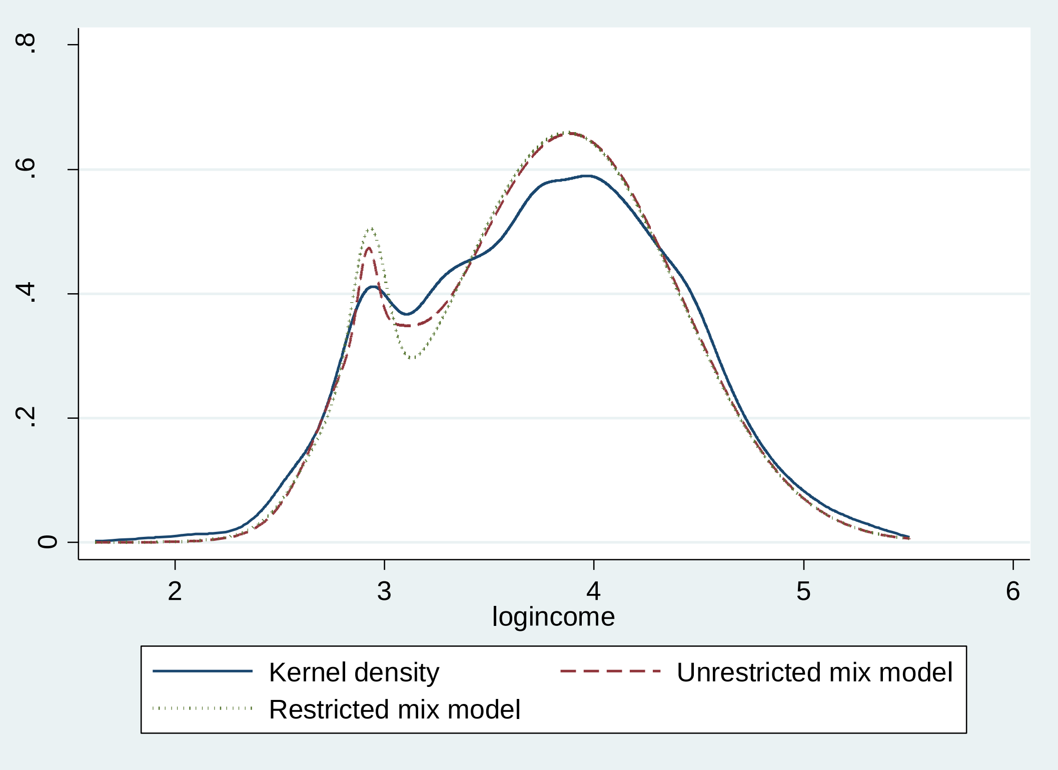
Trends in smoothed proportions of each income groups π



Trends in mean income over time



Income density 1996



Predicted distribution of consumption

Consumption with perfect smoothing

$$f(c_i) = \pi_1 \frac{\phi}{\sigma_\varepsilon^C}\left(\frac{c_i - \bar{y}^{P_1}}{\sigma_\varepsilon^C}\right) + \pi_2 \frac{\phi}{\sigma_\varepsilon^C}\left(\frac{c_i - \bar{y}^{P_2}}{\sigma_\varepsilon^C}\right) + (1 - \pi_1 - \pi_2)\frac{\phi}{\sigma_\varepsilon^C}\left(\frac{c_i - \bar{y}^{P_3}}{\sigma_\varepsilon^C}\right)$$

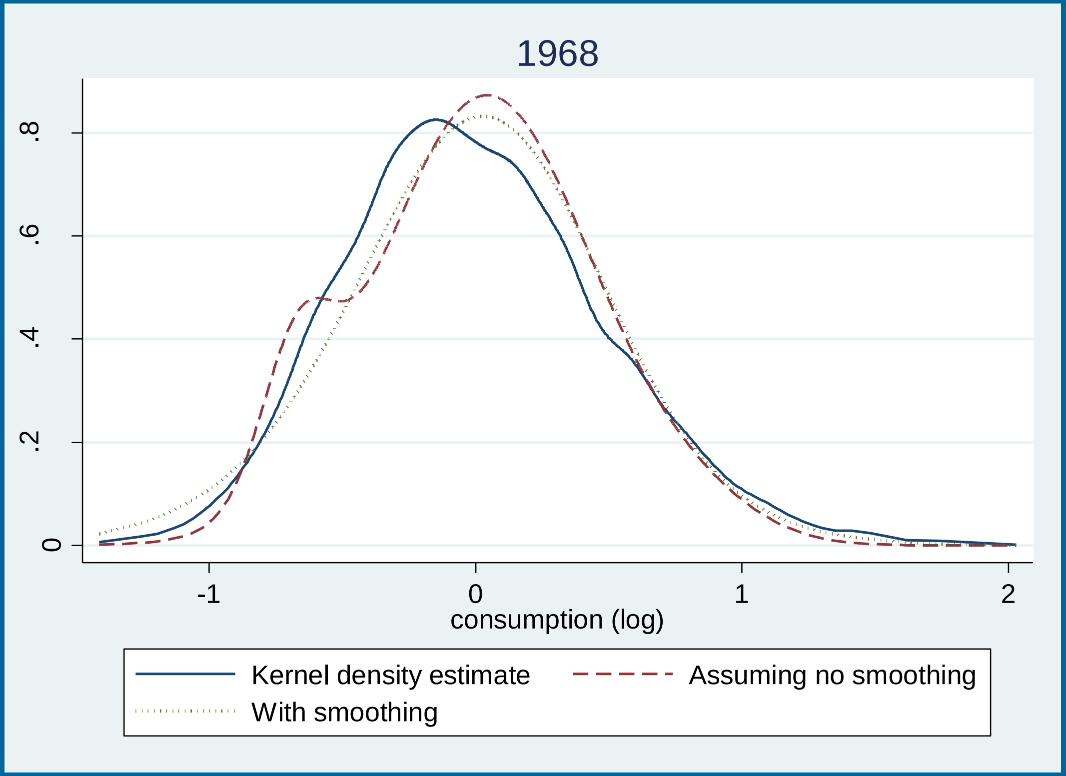
$$\sigma_\varepsilon^C = \sqrt{\mathrm{var}(c) - \mathrm{var}(y_i^P)}$$

Consumption with no smoothing

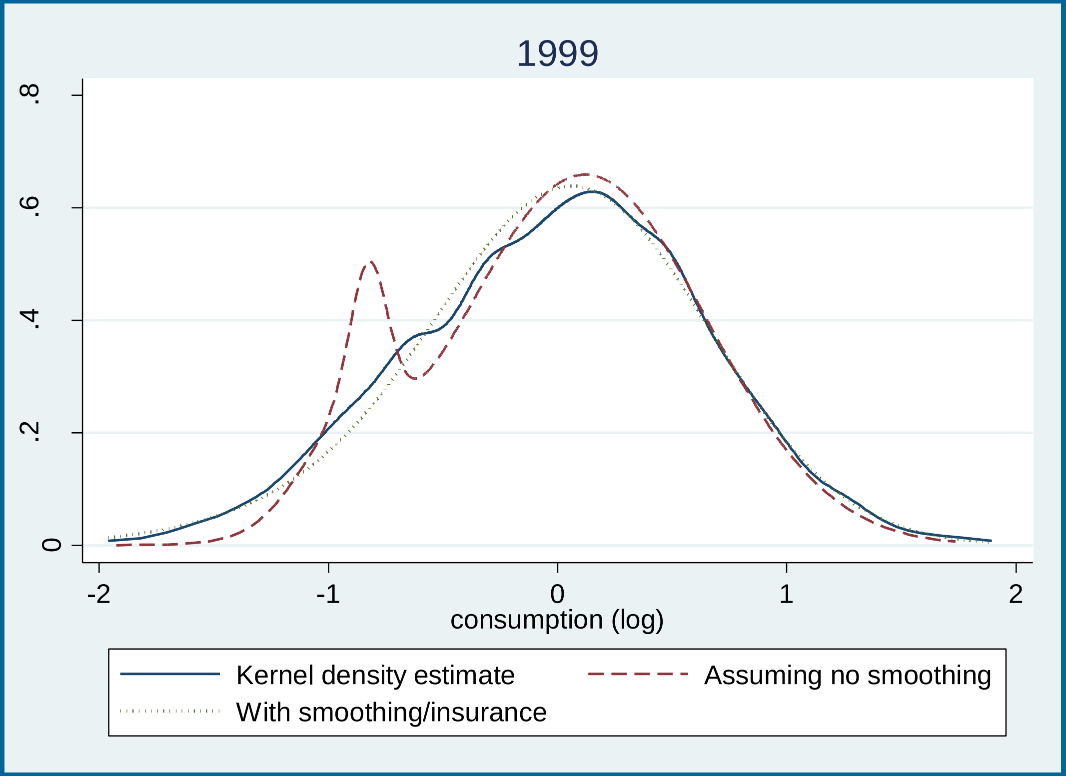
$$f(c_i) = \pi_1 \frac{\phi}{\sigma_{1\varepsilon}^C}\left(\frac{c_i - \bar{y}^{P_1}}{\sigma_{1\varepsilon}^C}\right) + \pi_2 \frac{\phi}{\sigma_{2\varepsilon}^C}\left(\frac{c_i - \bar{y}^{P_2}}{\sigma_{2\varepsilon}^C}\right) + (1 - \pi_1 - \pi_2)\frac{\phi}{\sigma_{3\varepsilon}^C}\left(\frac{c_i - \bar{y}^{P_3}}{\sigma_{3\varepsilon}^C}\right)$$

$$\sigma_{j\varepsilon}^C = \sqrt{\mathrm{var}(y|\ j) + \mathrm{var}(c) - \mathrm{var}(y)}$$

Comparing consumption (1)



Comparing consumption (2)



Joint distribution of consumption and income with smoothing

$$E(c|\ y) = p_1 \Pr(G=1|\ y) + p_2 \Pr(G=2|\ y) + p_3 \Pr(G=3|\ y)$$

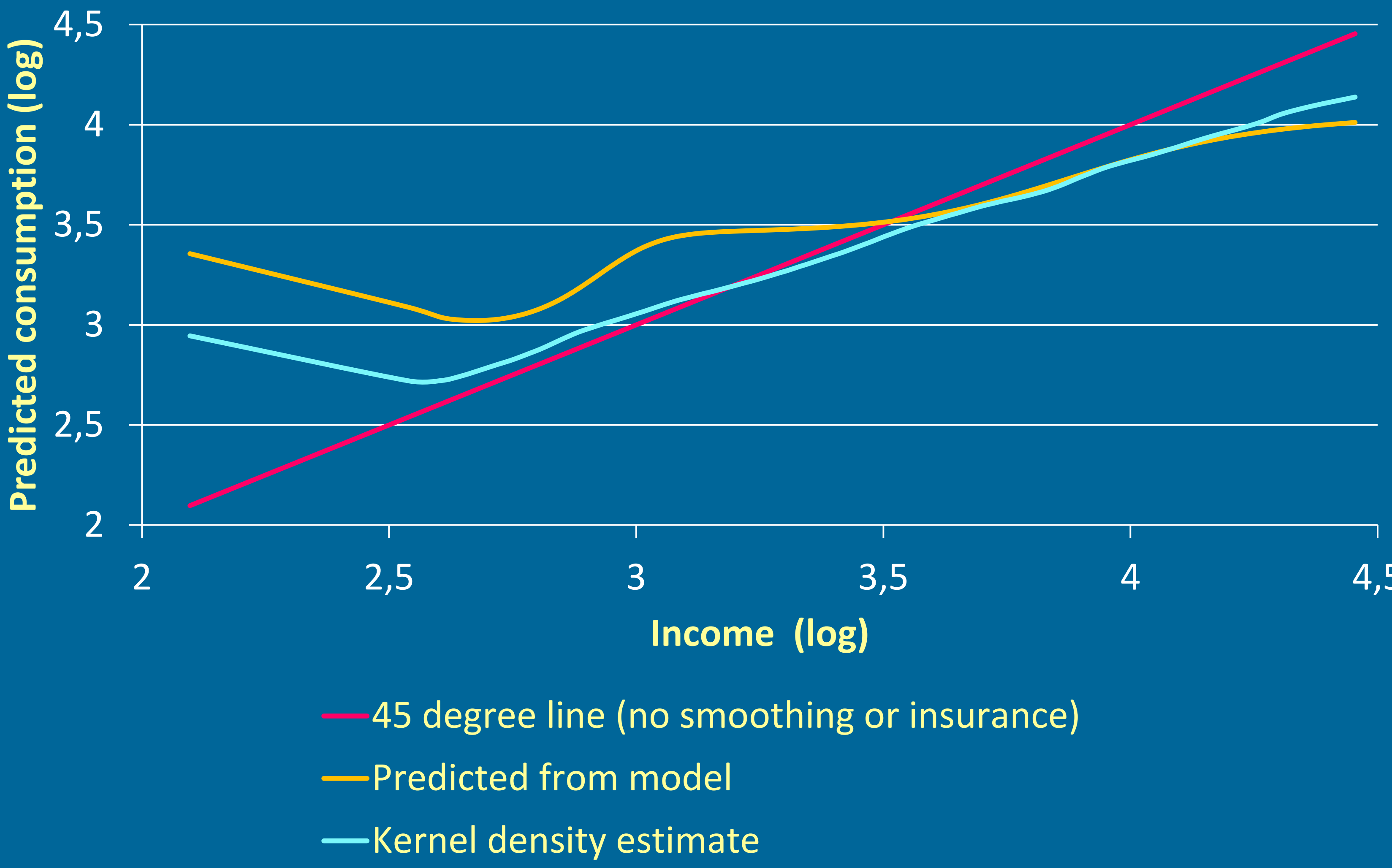
where

$$\Pr(G=1|\ y) = \frac{\Pr(Y=y|\ G=1)\pi_1}{\Pr(Y=y)} \quad \text{using Bayes}$$

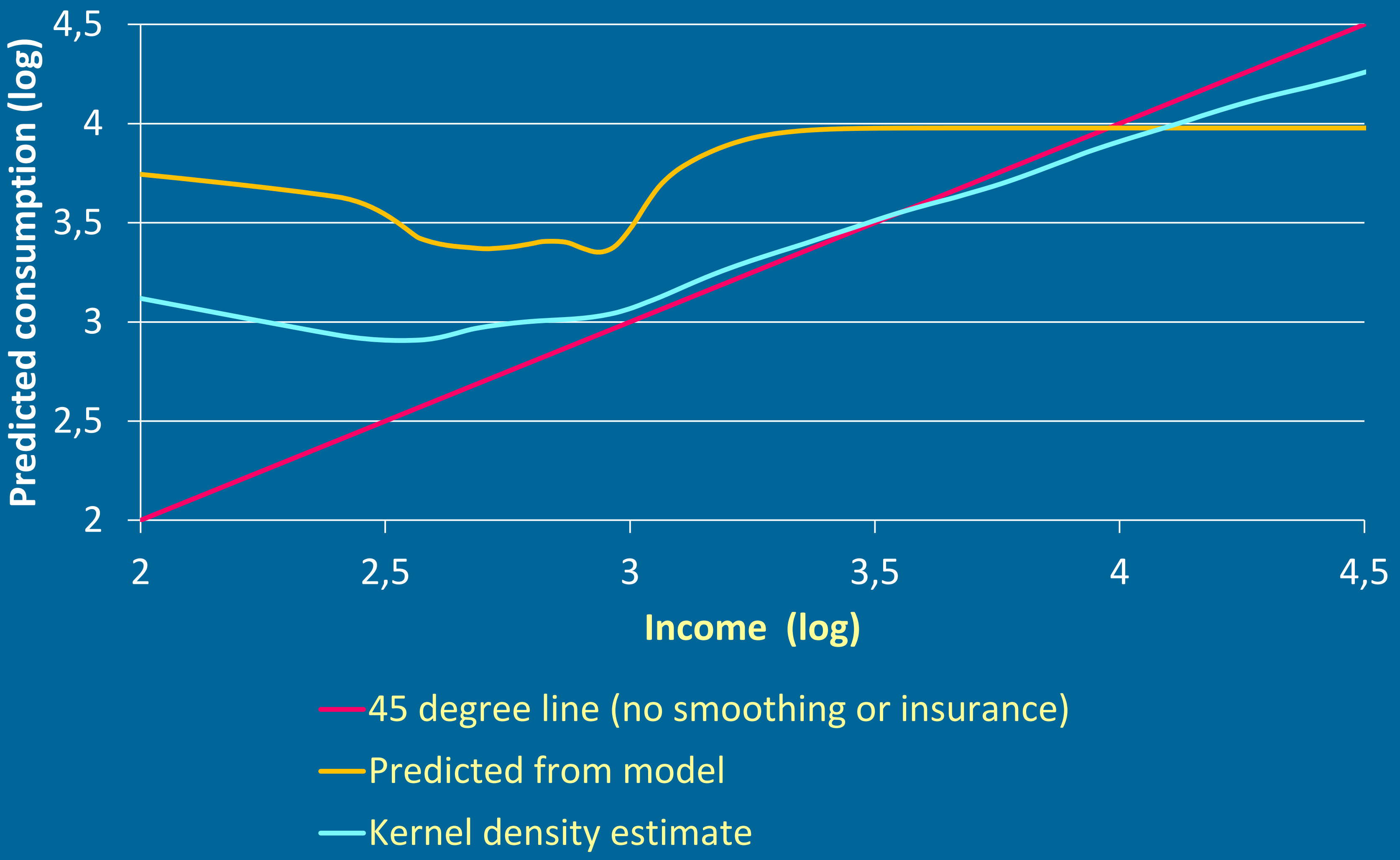
Joint distribution of consumption and income with no smoothing

$$E(c|\ y) = y + E(c) - E(y)$$

1970


Predicted consumption (log)
4,5
4
3,5
3
2,5
2
2
2,5
3
3,5
4
4,5
Income (log)
45 degree line (no smoothing or insurance)
Predicted from model
Kernel density estimate

1999



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Income (log)
45 degree line (no smoothing or insurance)
Predicted from model
Kernel density estimate

Concluding remarks

- Mixture model seems to fit income density reasonably well
- Eyeballs of data reject both extreme models
- Evidence of both growing risk and of the ability of markets to insure agents against this
- Model gives a good intuition as to why consumption should fall with income at lower ranges
  - Some of those currently poor may have high expected life incomes
- Likely that more types will be needed to fit the relationship between income and consumption at higher levels to some extent