Economic Growth and Wealth Inequality: The Role of Differential Fertility

Alessandro Ferrari

(Bank of Italy)

Alessandro Di Nola

(Konstanz University, Germany)

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Economic Growth and Wealth Inequality: the Role of Differential Fertility *

Alessandro Di Nola† and Alessandro Ferrari‡

Abstract

This paper provides a quantitative assessment of the impact of economic growth on wealth inequality. Using both aggregate and micro-level data, we document that a worsening of the economic outlook determines a fall in expected children income. As a consequence parents reduce fertility and increase the amount of intergenerational transfers. Wealthier households, who have ceteris paribus more offsprings, adjust their fertility decision by a greater magnitude. Therefore a persistent fall in economic growth causes an increase in wealth inequality in the subsequent generation. In order to rationalize this empirical evidence we solve the Barro-Becker model of fertility in a setup with aggregate and idiosyncratic shocks to labour income, and incomplete markets à la Bewley. We found that the slowdown in economic growth observed in the US between the 1950s and the 1970s can account for almost 40 per cent of the increase in wealth concentration since 1980s.

JEL Classifications: D19, D31, D52, E71, J11, J13.
Keywords: Wealth inequality, Fertility, Intergenerational transfers, Heterogeneous agents.

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†Department of Economics, University of Konstanz, 78457 Konstanz - Germany. Email: alessandro.di-nola@uni-konstanz.de.

‡PhD Candidate at Bocconi University, junior economist at Bank of Italy, Economic Outlook and Monetary Policy Department. Via Nazionale, 91, 00184, Rome, Italy. Email: alessandro.ferrari@bancaditalia.it.

1 Introduction

Wealth inequality has slowly but continuously increased in many developed countries in the last thirty years. In the US the Gini coefficient of wealth distribution, computed using micro-data from the *Survey of Consumer Finances*, has risen from 0.79 in 1992 to 0.85 in 2013 (Figure 1, left panel).

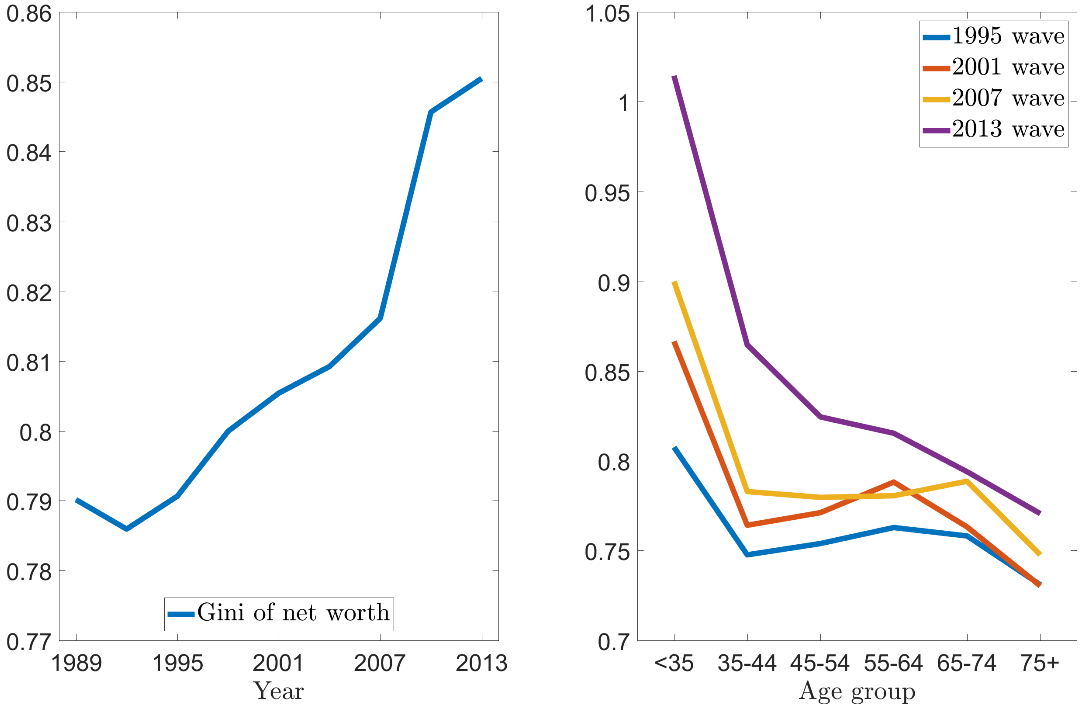


Figure 1: Gini coefficient of net wealth in the United States (left panel); intra-cohort Gini index across years in the United States (right panel). Source: Survey of Consumer Finances

A vivid debate on the possible causes and driving forces of this trend has been recently re-opened by the work of Piketty and Zucman (2014) and by new evidence and data contained in Piketty (2014). Despite the great amount of literature that has studied overall wealth inequality, intra-cohort inequality has received far less scrutiny. The right panel of Figure 1 presents the intra-age group Gini coefficient for different years: the biggest increase in inequality is concentrated in the youngest age-groups, in particular between 1995 and 2013 the Gini coefficient among households with an head younger than 35 years increased from 0.81 to more than 1[1]. **?** analyze the accumulation of wealth in a number of western countries associated with intergenerational transfers and not with life-cycle motives, and find that the share of inherited wealth increased markedly

[1]Other measures of inequality are displayed in appendix A.

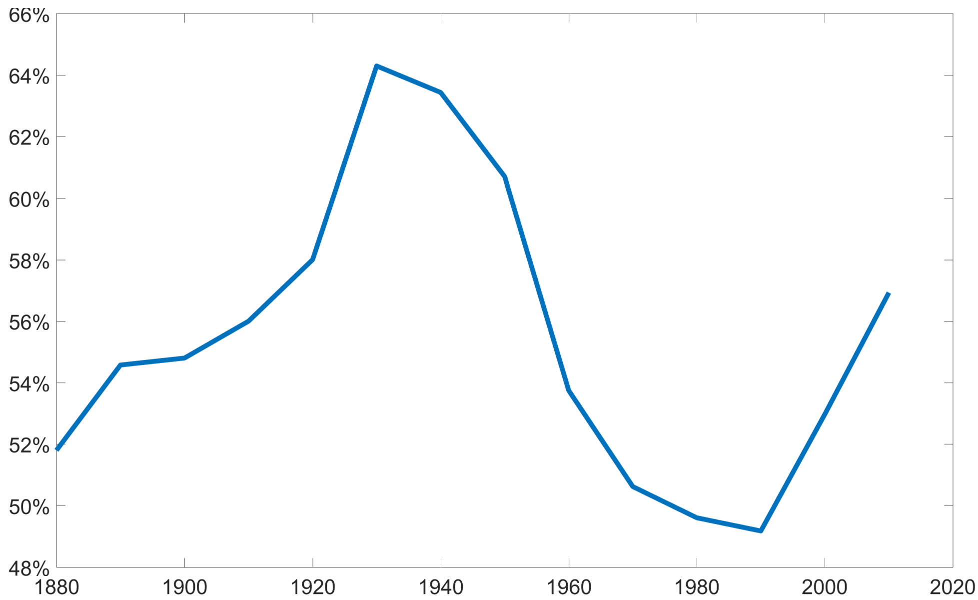


Figure 2: Percentage of aggregate wealth that came from intergenerational transfers, United States 1880-2010. Source: **?**

in the last decades (in the United States from a minimum of 50% in 1990 to 57% in 2010).

Changes in intergenerational transfers are driven by variations in the amount of resources that parents choose to give to each of their heirs and/or by changes in the number of children. While the determinants of the changes in the overall amount of resources transferred to future generations have been greatly explored in the literature[2], fertility adjustments have received much less attention.

This paper explores the links between economic growth, intergenerational transfers and fertility decision and their effects on intra-cohort wealth inequality. We find that fluctuations in total fertility rates are associated with those in total factor productivity (TFP) and that the elasticity of fertility to economic growth is higher for wealthier households. We rationalize this evidence in a framework with altruistic parents *à la* Barro-Becker: parents like having children but they also care about children prospects. Consistently with micro evidence, the number of children is increasing in the level of wealth of the housing unit, i.e. they are a normal good. In this setup a persistent decrease in TFP growth lowers expected children income. This negative shock induces a decrease in the number of children and an increase in the total amount of transfers (inter-temporal optimization let them move consumption from the present to the future). The magnitude of the wealth shock is

[2] See De Nardi (2015) and references therein.

increasing in the number of children that the household would like to have, and therefore is greater for wealthier households. To put it in another way, for the head of a dynasty a decrease in expected income tomorrow determines an increase of the optimal quantity of intergenerational transfers (an increase of savings) and a decrease of heirs (he cares about their welfare and therefore he prefers to guarantee more consumption to fewer children).

We develop the analysis through several steps. Firstly, looking at aggregate data on fertility and TFP growth we find that changes in the trend of TFP growth are positively correlated to changes in the fertility rate (detrended by its historical declining path, determined by the level of GDP reached by the economy). In particular TFP growth leads the fertility rate: the cross-correlation with a time shift of 5 years is statistically significant and is robust to different definitions of TFP and fertility rate. We document such finding for both the US and UK over the last century.

Then, the patterns observed in aggregate data are studied using a Barro-Becker model enriched with uninsurable idiosyncratic shocks and an aggregate shock on the level of income. The former is a necessary ingredient to generate wealth heterogeneity, the latter is used to study households reaction to changes in economic prospects. Under a standard calibration the model predicts that the elasticity of fertilities and bequests to expectations about children income are larger in magnitude for wealthier households, and therefore a decrease in expected economic growth determines an increase in intra-cohort wealth inequality of the next generation. This prediction confirmed by an analysis of micro-data from the *Survey of Consumer Finances* (SCF) and the *Panel Study of Income Dynamics* (PSID).

Finally, an extended version of the model with aggregate and idiosyncratic shocks is used to measure the change in wealth inequality determined by the fertility channel. We calibrate the model to match a number of cross-sectional data moments of the US economy related to fertility and income/wealth inequality, such as the income elasticity of fertility and the average fertility level. The model does a fairly good job in capturing the salient features of the fertility-income-wealth distribution (also on data moments that were not targeted) and we use it to perform some counter-factual experiments. Our main quantitative result suggests that roughly 40 per cent of the overall increase in wealth inequality over the last thirty years can be accounted for by the decline in the fertility rate (which is stronger among wealthier households) induced by a revision in parents expectations on future growth. Finally we explore the possibility that a negative shock to TFP growth can have a long lasting effect on fertility if households perceive that slow economic growth will persist for many periods in the future (biased beliefs). To this end we calibrate and solve a similar version of the quantitative model with the following modification: agents are not entirely rational and tend to overestimate the persistence of economic shocks. We find that this channel has a limited but not negligible role.

The remainder of the paper is structured as follows: in the next subsection we review the relevant literature. In section 2 we present some aggregate evidence linking TFP to fertility. Section 3 contains a detailed analysis on microeconomic data. In Section 4 we introduce a stripped-down version of the Barro-Becker model. We move on to Section 5 which contains the quantitative model. Finally, Section 6 concludes the paper.

1.1 Related literature

This paper is related to two main strands of literature. The first one studies the determinants of wealth inequality and (possibly) its evolution over time. Prominent examples in this field are the papers of De Nardi (2004), who studies the role of bequests in amplifying wealth inequality, and Kaymak and Poschke (2016), who analyse the relative importance of fiscal reforms and wage inequality behind the dramatic increase in wealth concentration in the United States. Our paper shares the quantitative methodology with the papers cited above and the importance given to intergenerational transfers but is the first to incorporate the fertility channel.

The second strand of literature to which our paper is connected focuses on fertility issues. The study of fertility decisions in economics started with the seminal work of Becker (1960) and was later embedded in a macroeconomic model by Barro and Becker (1989).

This paper is related in particular to three main works in this area: De la Croix and Doepke (2003), Jones and Schoonbrodt (2016) and Cordoba et al. (2016). De la Croix and Doepke (2003) focus on the relationship between inequality and growth: they show that an increase in income inequality leads to more fertility differential between the rich and the poor, which in turn lowers aggregate education and, hence, growth. We take inspiration from them regarding the notion of fertility differential between the rich and the poor but we adopt a different perspective: they focus on income inequality, disregarding wealth inequality, while we do exactly the opposite. Jones and Schoonbrodt (2016) study the fertility response to economic growth in a Barro-Becker model with a representative agent and find that fertility is pro-cyclical. Finally Cordoba et al. (2016) study the intergenerational persistence of wealth inequality in a Barro-Becker model with incomplete markets *á la* Bewley but without aggregate shocks[3].

This paper builds a bridge between the two previous works by looking at intergenerational transfers in a Bewley world during fertility fluctuations determined by economic growth.

2 Aggregate evidence

This section presents empirical evidence on the relationship between fertility and economic growth. Fertility and economic growth are correlated at different frequencies for different reasons, therefore we need to clarify what is the objective of our analysis.

Firstly, thwere is abundant evidence on the existence of a long-run negative relationship between TFP growth (and more generally economic growth) and fertility. This relationship has been extensively studied in demography (e.g. Heer, 1966) as well as in economics (e.g. Jones et al., 2010, Boldrin et al., 2015), it is based on the 'Quantity-Quality' trade-off and it is beyond the scope of this paper.

At the opposite of the frequency spectrum there is the short-run pro-ciclicality of fertility to business cycle fluctuations. This relationship has been extensively studied (e.g. Ben-Porath, 1973 and Sobotka et al., 2011) and it has been explained as the result of fertility postponement during economic downturns. It has proven to be short-lived (as the business cycle) and irrelevant for the life-long fertility decisions of the couple.

In this paper we study the relationship between the fertility choices of a couple over its fertile years, around twenty, and fluctuations of TFP growth in the same time span. We are therefore interested in the medium to long frequency fluctuations of fertility and TFP. This issue cannot be addressed with standard TFP[4] and fertility series that spans around 50 years. Therefore we have built an appropriate dataset that we describe in subsection 2.1. In subsection 2.2 we separate trend and cycle at different frequencies for the two series and finally in subsection 2.3 we show correlation of the two series.

[3]An early paper incorporating uninsurable income shocks in the Barro-Backer model is Alvarez (1999), who shows that wealth inequality does not persist. This lack of persistence result is confirmed also by (Bosi et al., 2010).

[4]For example the TFP series provided by (Fernald, 2012).

2.1 Data sources

We focus on the United States and the United Kingdom since they were the only two countries for which we were able to reconstruct historical series of TFP. For the analysis we do not just consider the crude total fertility rate but we adjust it for child mortality, assuming that parents internalize the survival probabilities in their fertility choices[5]. Therefore we need three time series: total fertility rate (TFR), child mortality and TFP.

TFR and child mortality are the series constructed in Roser (2017a) and Roser (2017b) respectively: their historical sources are presented in detail in appendix D.1. TFP series for the United Kingdom is taken from the Bank of England project '*A millennium of macroeconomic data*'[6]. TFP for the United States is computed using the Chari et al. (2007) methodology on updated series[7].

2.2 Trend and cycle

Our research question is related to medium to long frequencies, for this reason we want to clean our series from short-term fluctuations of TFP and from the long-term decline of fertility. On the other hand fertility exhibits very small fluctuations in the short term and TFP growth does not exhibit a long-run trend. In order to preserve simplicity we do not perform a symmetric filtering but a simple HP filtering of opposite frequencies for the two series.

For the TFP we use an HP filter[8] with $\lambda = 20$ to separate yearly TFP growth trend from TFP growth cycle and we interpret the trend as the underlying measure of growth and innovation. With respect to total fertility rate we remove the decreasing linear trend[9], that is equivalent to HP filtering with $\lambda \to \infty$.

[5] Boldrin et al. (2015) shows that the fall in mortality played an important role in the overall fall in fertility rate.

[6] The project is under development and it contains many data series starting from 1096. TFP, GDP and Central Bank balance sheet data starting in 1700.

[7] As in Jones et al. (2010).

[8] It is the optimal value for yearly data according to Mohr (2001) and an intermediate value between the 6.5 proposed by Ravn and Uhlig (2002) and the 100 proposed originally by Hodrick and Prescott (1997).

[9] According to economics of demographics it is the result of the increase in the level of income, since the aim of the model is to study fluctuations in growth we want to eliminate this trend. An alternative way of performing the same exercise, more complicated but probably with similar results, would be a cointegration between GDP and fertility assuming that, in line with the literature, any change in GDP has an effect of the same magnitude on fertility.

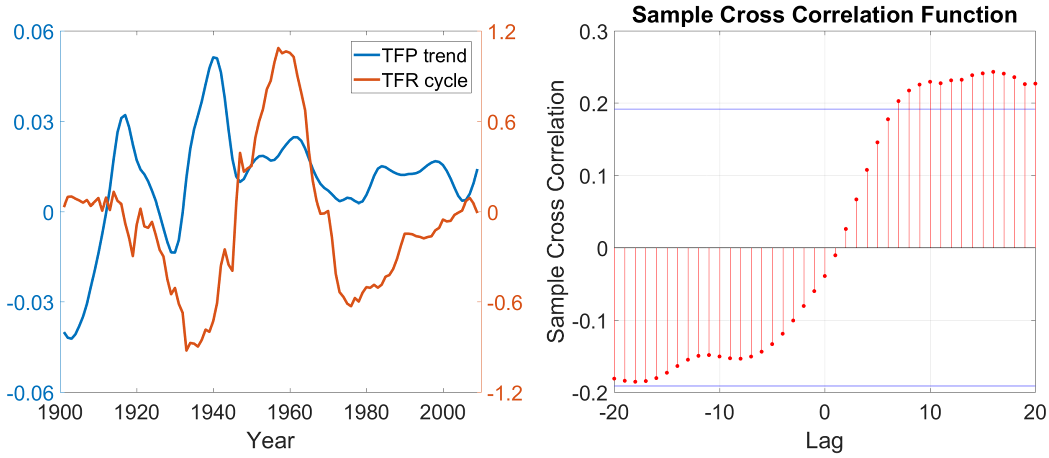


Figure 3: Fertility cycle and TFP trend in the United States and their crosscorrelogram

2.3 Aggregate correlation

We found a positive and robust correlation between the trend of TFP growth and the cycle of TFR[10].

The peaks of TFP trend presented in the left panel of Figure 3 coincide with the boom of the 1920s, the New Deal and the mobilization of the II World War in the 1940s. The series reached another peak in 1960 before steadily declining until 1980. Then a further period of extraordinarily high growth persisted until the beginning of 2000s. The correlation between the two series can be spotted from the plot; in particular the Baby Bust of the 1920s and 1930s is clearly associated to the Great Depression and the subsequent stagnation, similarly the Baby Boom is associated to the TFP growth of the 1940s and 1950s. The rise of TFP growth in the 1980s is then associated with an incrase of the TFR. The correlogram on the right hand side of Figure 3 shows that the correlation between the two series is statistically different from 0 when time shift between TFP and TFR is greater than 5 years.

The same analysis has been conducted for the United Kingdom, exploiting a longer time series of TFP (up until the beginning of the 19^th century) provided by the Bank of England. Results are plotted in Figure 4. Reassuringly the correlation is of the same magnitude at the same lags in the two countries, even if the time span of the United Kingdom series is much longer.

[10] The series and crosscorrelograms of all possible correlations of trends and cycle for United States and United Kingdom are presented in appendix D.2.

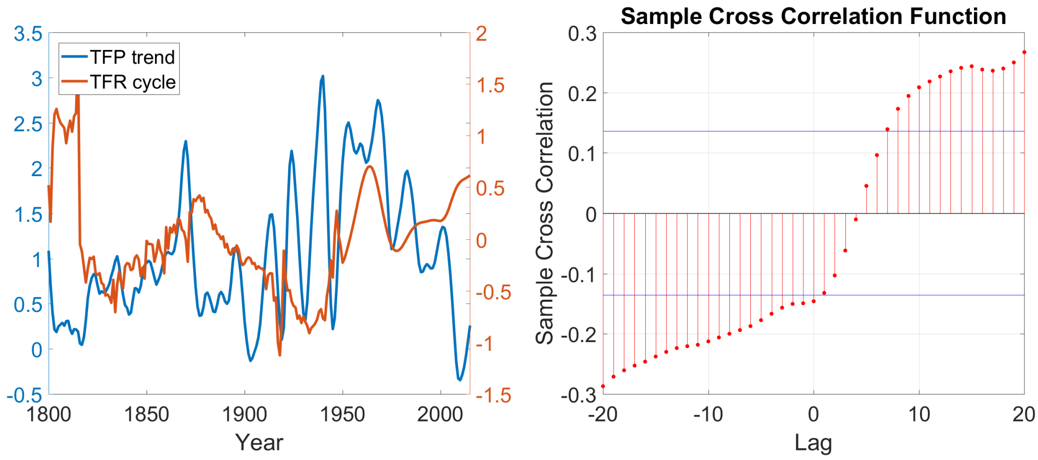


Figure 4: Fertility cycle and TFP trend in the United States and their crosscorrelogram

3 Empirical evidence on microdata

In this section we measure of the effect of TFP growth on fertility at intra-cohort deciles (or terciles) of wealth using the *Survey of Consumer Finances* (SCF) and the *Panel Survey of Income Dynamics* (PSID). We combaine / use two datasets because while the PSID contains more detailed information on demographics variables, and in particular on the number of children, the SCF contains more detailed information on wealth and it has the great advantage of over-sampling the wealthiest.

3.1 Different cohort specifications

Deciles and terciles of income and wealth are computed at the intra-cohort level and not at the population level. This allow to control for any cohort-specific shock. We use two definitions of cohort: one is "cohort of the head of family" that simply assigns a family unit to the cohort of the head of family (father), the other is "family cohort" and it is the mean between the cohort of the father and the cohort of the mother of the unit. The first measure assumes that the most important drivers of family wealth are related to the father life cycle of income, the second measure assumes that the two have an equal weight in determining the wealth of the family.

3.1.1 Deciles (and terciles) computation

In each year we count how many unit belonging to the same cohort are in the dataset. If there are at least 20 units then we extend the population to those cohort that are born two years before and two year after (thus covering a 5 year window) and the unit of the cohort get imputed the decile of wealth and income to which they belong, this should guarantee a minimal sample over which it makes sense obtain a measure of relative wealth. The procedure is repeated for each cohort and for each year. Terciles computation is based upon identical procedure but the minimal number is 10.

3.2 Model specifications

Two main model specifications are possible: the first one can be used exploiting the panel structure of PSID and can take as dependent variable the change in the number of children in the family from the previous wave of the survey, the second one can focus on the total number of children.

With respect to the first model specification the dependent variable is $\Delta n^j_{t,z} = n^j_{t,z} - n^j_{t-1,z}$, i.e. the change in the number of children in household unit z, that belongs to decile (or tercile) of wealth j inside his cohort in period t. The regression must be run on those couples that may have a children and therefore those in which the women is between 20 and 45 years old. The main model specification is:

$$\Delta n^j_{t,z} = \alpha + \beta^j \Delta TFP^{trend}_{t-l} + \gamma X_{t,z} + \varepsilon_{t,z}$$

Where ΔTFP^{trend}_{t-l} is the change in TFP trend with lag l and $X_{t,z}$ is a set of demographic and economic controls. The parameters of interest in this specification are $(\beta^j)^{10}_{j=1}$, and in particular economic growth would decrease wealth inequality if $\beta^{10} > \beta^9 > \cdots > \beta^1$.

Alternatively, it is possible to look at the completed fertility of the couple and measure how the mean of TFP trend during the fertility years affected their fertility decision. This regression is run only on those couple in which the female is above 45. In this model the dependent variable is n^j_z, i.e. the number of children that the family unit z in decile (or tercile) of wealth j has. The main specification is:

$$n^j_z = \alpha + \beta^j \left(K^{-1} \sum_{k=1}^{K} TFP^{trend}_{i+T-j} \right) + \gamma X^j_z + \varepsilon_z$$

In this specification the set of parameters of interest is $(\theta^j)^{10}_{j=1}$ and as before the relationship between growth and inequality is confirmed whenever $\theta^{10} > \theta^9 > \cdots > \theta^1$.

The former specification can be run only in the PSID where the panel structure of data allows to measure the change in family composition year after year. On the other hand the latter specification can be run also on SCF data that do not contains the age of the children but only their total number.

3.2.1 Linear specification

As specified in subsection 3.1.1 deciles are computed whenever a cohort has a minimal number of components and using a sort of rolling window computation. Nonetheless deciles are computed on a very small population that may affect results. There are two ways to address this problem: using terciles instead of deciles or making an assumption on the linearity of the coefficients. This second model specification assumes that $\frac{\beta^{10}}{\beta^{9}} = \frac{\beta^{9}}{\beta^{8}} = \cdots = \frac{\beta^{2}}{\beta^{1}}$ or alternatively that $\beta^{j} = \tilde{\beta}_{f} + j\tilde{\beta}$, that is it assumes that the elasticity of fertility with respect to growth is linearly increasing in the decile of wealth. It is a strong assumption but it can be used as a robustness check to addres the small sample problem.

3.3 Panel Survey of Income Dynamics

The PSID is a longitudinal survey of US individuals and their family units. It was started in 1968 to study the dynamics of income and poverty, for this reason it was originally designed with two different sub-samples: a specific sub-sample over-sampling poor families (2,000 family units) selected from the Survey of Economic Opportunities (SEO), a nationally representative sample (3,000 family units) designed by the Survey Research Center (SRC) at the University of Michigan. Individuals of family units in the original sample were followed also in the formation of new family units (the split-offs). The survey has annual waves until 1997 and then it became bi-annual.

The survey contains complete data on income of each family member, unfortunately it was not designed to study wealth. A specific set of questions on wealth was introduced in 1984 and it was repeated every five-years[11] until 1999 when it became part of the main set of questions and it was introduced in all waves. Details of wealth imputation are in appendix F.1.

3.3.1 Sample restriction

All the regression for model 1 are run on those couples in which both partners are present and where the female has at most 45 years and the male has at most 50 years.

[11]The set of questions on wealth was conducted also in 1988.

3.3.2 Results

Results from the main specification are shown in Table 1. The explanatory variable is the TFP trend with 5 years lag (i.e. the same explanatory variable suggested by the aggregate evidence analysis), in the first specification it is the only variable in the regression and it has the expected positive magnitude. In the second column the model contains also a set of dummies for the decile of wealth and their interaction with the TFP trend. As expected the higher deciles of income have an effect statistically different from the others. In the third column a linear trend (to control for decrease in overall fertility) and a set of dummies for the age of the head of family and his\her partners are added. In the fourth column also a set of dummies for labour income decile of the head of family are added. Finally in the last column also a set of dummies for deciles of income of the family are included in the controls. In all the specifications $0.0877 \leq \left(\hat{\beta}^{10} - \hat{\beta}^{1}\right) \leq 0.153$, and the effect is increasing in the decile of wealth even if the small sample size determines high variance and small statistical significance (limited to the highest deciles).

3.3.3 Robustness

In order to perform a robustness check and to address the small sample size that affects results in the main specification we perform some robustness checks on different specifications. In the first column of Table 2 only the interaction between the dummy of the 10^{th}decile and TFP is included and delivers a similar result. In the other two columns a linear specification of the model is estimated, in both cases the estimate of $\left(\hat{\beta}^{10} - \hat{\beta}^{1}\right)$ is around 0.10 in line with the previous estimates.

3.4 Survey of Consumer Finances

The *Survey of Consumer Finances* (SCF) is by far the best source of micro level data on household-level assets and liabilities for the United States. It is conducted every three years by the Board of Governors of the Federal Reserve System and collects detailed information on income and assets. The survey is particularly detailed on their composition of assets since it contains information on financial and non-financial assets, and capital gains. The survey includes two samples: a standard random sample of US households, and a second sample that focuses on wealthy households, identified on the basis of tax returns. Only two waves of the sample have a panel-structure and are based on the previous samples: 1986 (same sample as 1983) and 2009 (special wave after 2007 to

	Change in the number of children from the previous wave	Change in the number of children from th previous wave	Change in the number of children from the previous wave	Change in the number of children from the previous wave	Change in the number of children from the previous wave
2nd dec. of wealth X TFP tr.		0.0332	0.00766	0.00852	0.0236
		(0.0594)	(0.0581)	(0.0576)	(0.0584)
3rd dec. of wealth X TFP tr.		0.0485	0.0176	0.0108	0.0181
		(0.0453)	(0.0441)	(0.0442)	(0.0463)
4th dec. of wealth X TFP tr.		0.0776*	0.0312	0.0267	0.0170
		(0.0421)	(0.0399)	(0.0399)	(0.0428)
5th dec. of wealth X TFP tr.		0.0640	0.0317	0.0285	0.0320
		(0.0425)	(0.0394)	(0.0394)	(0.0408)
6th dec. of wealth X TFP tr.		0.0962**	0.0496	0.0469	0.0423
		(0.0395)	(0.0370)	(0.0369)	(0.0388)
7th dec. of wealth X TFP tr.		0.0988***	0.0677*	0.0602*	0.0538
		(0.0379)	(0.0361)	(0.0360)	(0.0385)
8th dec. of wealth X TFP tr.		0.0643	0.0211	0.0170	0.0179
		(0.0427)	(0.0409)	(0.0407)	(0.0429)
9th dec. of wealth X TFP tr.		0.101***	0.0693*	0.0686*	0.0665*
		(0.0384)	(0.0360)	(0.0360)	(0.0379)
10th dec. of wealth X TFP tr.		0.153***	0.0944**	0.0911**	0.0877**
		(0.0438)	(0.0419)	(0.0416)	(0.0424)
TFP tr.	0.0183***	-0.194***	-0.0755**	-0.0704**	-0.0653*
	(0.00687)	(0.0336)	(0.0318)	(0.0318)	(0.0343)
Observations	14454	3804	3804	3804	3438
R^2	0.001	0.052	0.131	0.136	0.142
Wealth decile	No	Dummies	Dummies	Dummies	Dummies
Age dummy	No	No	Head & Partner	Head & Partner	Head & Partner
Time control	No	No	Yes	Yes	Yes
labour income decile	No	No	No	Head dummies	Head & family dummies

*Table 1: Change in fertility from the previous wave, intra-cohort deciles are computed using the head of family unit as a reference, TFP trend is lagged by 5 years. Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Source: PSID*

	(1) Change in the number of children from the previous wave	(2) Change in the number of children from the previous wave	(3) Change in the number of children from the previous wave
10th decile of wealth × TFP trend 5 years ago	0.0622**		
	(0.0289)		
TFP trend 5 years ago	-0.0282***	-0.0839***	-0.0900***
	(0.00880)	(0.0219)	(0.0219)
Intra-cohort decile × TFP trend 5 years ago		0.00991***	0.0107***
		(0.00328)	(0.00331)
Constant	18.33***	18.38***	18.43***
	(1.571)	(1.571)	(1.570)
Observations	3804	3804	3804
R^2	0.080	0.081	0.078
Wealth decile	Dummies	Dummies	Linear
Time control	Yes	Yes	Yes
Head labour income decile	No	No	Linear
Family income decile	No	No	No

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Robustness check, different specifications of the model: only 10^{th} decile of wealth or linearity assumption

study the effects of the financial crisis). Both waves are excluded from the analysis.

3.4.1 The dependent variable and its measurement

The aim of the SCF is to represents the financial characteristics of a subset of the household unit referred to as the 'Primary Economic Unit' (PEU). In brief, the PEU consists of an economically dominant single individual or couple (married or living as partners) in a household and all other individuals in the household who are financially interdependent with that individual or couple. The variable 'Children' that we use in the analysis is the sum of the children included and those not included in the PEU [12](e.g. because they are financially independent). The main shortfall of this dependent variable choice is that we may be double-counting some children, indeed if the mother and the father of a children are divorced they will be in two different PEU and the children will be counted in both "families". On the other hand, this approach is necessary to extend the available data including in our sample families where the head of household is above 50. Indeed, in this case we are sure that any children even if economically independent will be counted. A different strategy would be to consider the number of children leaving in a family only for the people below a certain age. The main problem is that the life-cycle of people is dependent on their wealth: poorer people have children earlier and they leave the housing unit earlier than their wealthier counterparts. Therefore taking a snapshot of family unit of different wealth in the same moment may be misleading leading to under-estimation of the number of children of the rich if we look young ages or under-estimation of the children born in poorer family if we look older ages.

To address the issue of divorces and the potential increase in the number of children given by the fact that one may have descendants from multiple partners we exploit a specific question on the number of years spent in previous marriages. We restrict the regression on those PEU in which none of the two partners have spent years in previous marriages.

3.4.2 The effect of TFP at different ages

Ideally one would like to regress the number of children on TFP trend at different ages, nonetheless given that we are using the trend there is an high correlation between contiguous years. In Table 3 the correlation between average TFP trend at 5 years age bracket in the subsample are shown. The

[12]Question coded as X5910 is the answer to: "*Now I'd like to ask some questions about your family living elsewhere. Altogether, including children from previous marriages and adopted children, how many sons and daughters do you (or your {husband/wife/partner/spouse}) have who do not live with you?*"

correlation is above 0.70 for almost all contiguous time spans. Then, putting all spans in the same regression would result in multicollinearity. For this reason we proceed by using one time-span at a time and then we use the average between 20 and 35 years old.

Mean of TFP trend	15-20 years old	21-25	26-30	31-35	36-40
15-20 years old	1				
21-25	0.7626	1			
26-30	0.5571	0.8214	1		
31-35	0.5511	0.4934	0.7960	1	
36-40	-0.0500	-0.0972	0.1971	0.5314	1

Table 3: Correlation matrix between TFP trend at different ages of household in the subsample

3.4.3 Results

Results with deciles are shown in Table 4 for different age breaks. It can be seen that, as expected, the magnitude of coefficients is increasing in the deciles even if the difference between deciles is small, especially at the top. This is probably due to the small sample from which deciles are computed. Indeed, using terciles in Table 5 we get statistically significant results and strictly increasing in the tercile of wealth.

Finally, we use average TFP trend during fertility years (between 20 and 40 years old) as treatment variable. Results are shown in Table 6. The effect of economic growth is stronger for higher decile when using the set of dummies, results are robust when using the linearity assumption or an ordered probit specification.

	(1) Between 15 and 20 years old	(2) Between 21 and 25 years old	(3) Between 26 and 30 years old	(4) Between 31 and 35 years old	(5) Between 36 and 40 years old
Average TFP trend	-0.417***	-0.397***	-0.153**	-0.0709	-0.270***
	(0.0810)	(0.0617)	(0.0675)	(0.0704)	(0.0657)
2^{nd} decile $\times$ average TFP trend	0.143**	0.0102	0.0379	0.114	0.0681
	(0.0584)	(0.0525)	(0.0606)	(0.0711)	(0.0770)
3^{rd} decile $\times$ average TFP trend	0.233***	0.118**	0.119*	0.206***	0.182**
	(0.0651)	(0.0538)	(0.0613)	(0.0714)	(0.0761)
4^{th} decile $\times$ average TFP trend	0.294***	0.146***	0.180***	0.296***	0.271***
	(0.0694)	(0.0554)	(0.0628)	(0.0729)	(0.0760)
5^{th} decile $\times$ average TFP trend	0.262***	0.128**	0.175***	0.321***	0.312***
	(0.0718)	(0.0556)	(0.0626)	(0.0722)	(0.0756)
6^{th} decile $\times$ average TFP trend	0.280***	0.171***	0.229***	0.376***	0.421***
	(0.0739)	(0.0563)	(0.0627)	(0.0723)	(0.0759)
7^{th} decile $\times$ average TFP trend	0.288***	0.201***	0.254***	0.410***	0.393***
	(0.0741)	(0.0555)	(0.0617)	(0.0707)	(0.0734)
8^{th} decile $\times$ average TFP trend	0.347***	0.259***	0.309***	0.478***	0.494***
	(0.0744)	(0.0551)	(0.0609)	(0.0697)	(0.0725)
9^{th} decile $\times$ average TFP trend	0.354***	0.278***	0.326***	0.504***	0.551***
	(0.0750)	(0.0550)	(0.0605)	(0.0694)	(0.0730)
10^{th} decile $\times$ average TFP trend	0.366***	0.215***	0.246***	0.404***	0.425***
	(0.0761)	(0.0561)	(0.0609)	(0.0694)	(0.0734)
Constant	69.53***	76.47***	45.84***	22.07***	47.78***
	(4.757)	(4.636)	(5.023)	(4.031)	(2.247)
Observations	8387	9479	9479	9479	9479
R^2	0.053	0.050	0.049	0.053	0.052
Wealth decile	Dummies	Dummies	Dummies	Dummies	Dummies
Time control	Yes	Yes	Yes	Yes	Yes
Family income decile	Dummies	Dummies	Dummies	Dummies	Dummies

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4: Total number of children, intra-cohort deciles are computed using the head of family unit as a reference, average TFP trend at different ages, source: SCF

	(1) Between 15 and 20 years old	(2) Between 21 and 25 years old	(3) Between 26 and 30 years old	(4) Between 31 and 35 years old	(5) Between 36 and 40 years old
Average TFP trend	-0.222**	-0.433***	-0.135**	0.140**	-0.126**
	(0.103)	(0.0642)	(0.0643)	(0.0661)	(0.0619)
2^{nd} tercile $\times$ average TFP trend	0.187*	0.106*	0.110*	0.226***	0.278***
	(0.107)	(0.0625)	(0.0619)	(0.0678)	(0.0752)
3^{rd} tercile $\times$ average TFP trend	0.139	0.236***	0.225***	0.288***	0.356***
	(0.0990)	(0.0576)	(0.0560)	(0.0616)	(0.0682)
Constant	76.68***	95.65***	64.40***	26.57***	56.80***
	(5.481)	(5.664)	(5.947)	(4.771)	(2.546)
Observations	8544	9673	9673	9673	9673
R^2	0.057	0.057	0.055	0.059	0.057
Wealth tercile	Dummies	Dummies	Dummies	Dummies	Dummies
Time control	Yes	Yes	Yes	Yes	Yes
Family income tercile	Dummies	Dummies	Dummies	Dummies	Dummies

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Total number of children , intra-cohort terciles are computed using the head of family unit as a reference, average TFP trend at different ages, source: SCF

	(1) Total number of children	(2) Total number of children (Linear specification)	(3) Total number of children (Ordered probit)
Average TFP trend between 20 and 40 years old	0.0822	0.0620	0.0581
	(0.139)	(0.0930)	(0.0943)
2^{nd} decile $\times$ average TFP trend	-0.0765		-0.0376
	(0.157)		(0.106)
3^{rd} decile $\times$ average TFP trend	0.151		0.110
	(0.150)		(0.0999)
4^{th} decile $\times$ average TFP trend	0.221		0.163*
	(0.143)		(0.0959)
5^{th} decile $\times$ average TFP trend	0.412***		0.288***
	(0.140)		(0.0940)
6^{th} decile $\times$ average TFP trend	0.372***		0.263***
	(0.141)		(0.0948)
7^{th} decile $\times$ average TFP trend	0.486***		0.342***
	(0.135)		(0.0911)
8^{th} decile $\times$ average TFP trend	0.546***		0.388***
	(0.132)		(0.0885)
9^{th} decile $\times$ average TFP trend	0.685***		0.500***
	(0.132)		(0.0888)
10^{th} decile $\times$ average TFP trend	0.420***		0.315***
	(0.127)		(0.0862)
Intra-cohort decile $\times$ average TFP trend		0.0651***	
		(0.00834)	
Observations	9318	9318	9318
R^2	0.054	0.049	(0.0147)
Wealth decile	Dummies	Linear	Dummies
Time control	Yes	Yes	Yes
Family income decile	Dummies	Linear	Dummies

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 6: Total number of children, intra-cohort deciles are computed using the head of family unit as a reference, average TFP trend between 20 and 40 years old, source: SCF

4 Theoretical model

In this section we develop a simple two period model that allows us to study the main mechanism of the full quantitative model.

4.1 Two periods model

The world lasts two periods: in the first one there is only an individual with labour income ω and endowment τ. She has to choose the number of children that she wants to have (n) and the amount of per-capita bequests that she wants to leave to her offsprings (b). In the second period the offsprings consume all the endowments that they receive and the inherited bequests increased by their gross rate of return R. The cost of having children is an opportunity cost in terms of time where the time requested of having children is $\Phi(n) = \left(\frac{n}{\bar{n}}\right)^{\theta}$.

Given the modelisation the cost of children is in term of time to be spent with them: therefore any change in wage has not only a wealth effect on the agent but also a substitution effect since it affects the marginal cost of having children. For this reason the modelisation with τ allows to distinguish the wealth effect from the substitution effect in comparative statics.

The problem is:

$$
\begin{aligned}
\max_{c,c',b,n} \quad & \frac{(c)^{1-\sigma}}{1-\sigma} + \alpha\,(n)^{1-\eta}\,\frac{(c')^{1-\sigma}}{1-\sigma} & \\
& s.t. & \\
& c + bn \leq \omega\left[1 - \left(\frac{n}{\bar{n}}\right)^{\theta}\right] + \tau & (\lambda) \\
& c' \leq \omega' + bR & (\lambda') \\
& c, c', b, n \geq 0 & (\phi_v)
\end{aligned}
$$

Where the interpretation of non-negativity coefficients on consumption and children is straightforward while the non-negativity on bequests is a natural assumption given a number of legal (and moral) restrictions preventing parents from imposing debt obligations on their children (as in Cordoba et al. (2016)).

Proposition 1. If $\eta \geq \sigma > 1$ and $\theta > 1$ the problem has a solution and it is unique. Then, the FOCs are not only necessary but also sufficient to characterize the equilibrium.

Proof. The proof is in appendix E. $\square$

Therefore, if $\eta \geq \sigma > 1$ and $\theta > 1$ the solution can be found by solving the system of equations together with the slackness conditions[13]

$$
\begin{aligned}
c: &\quad c^{-\sigma} = \lambda \\
c^{'}: &\quad \alpha\left(n\right)^{1-\eta}\left(c^{'}\right)^{-\sigma} = \lambda^{'} \\
b: &\quad \lambda = R\lambda^{'} + \phi_b \\
n: &\quad \alpha\left(1-\eta\right)\left(n\right)^{-\eta}\frac{\left(c^{'}\right)^{1-\sigma}}{1-\sigma} = \left[\Phi'\left(n\right)+b\right]\lambda
\end{aligned}
$$

A closed form solution cannot be computed but the four equations together with the constraints and the slackness conditions determine the optimal c^*, b^*, n^*, c'^*.

The effects of tomorrow endowment on inequality depend on the sign of $\frac{\partial^2 n}{\partial\omega'\partial\tau}$ that characterizes *differential fertility*. Under the parametrization of Jones and Schoonbrodt (2010)[14] children are complementary to their *quality* and a better prospect for their consumption leads to an increase in the optimal choice of fertility, i.e. $\frac{\partial n}{\partial\omega'} > 0$. Distributional consequences can be explored looking at $\frac{\partial^2 n}{\partial\omega'\partial\tau}$, i.e. how people with different level of wealth adjust their fertility to a change in tomorrow endowment. In particular, if $\frac{\partial^2 n}{\partial\omega'\partial\tau} > 0$ then any increase in tomorrow endowment leads to a stronger increase in fertility for the wealthier households, leading to a more wide spreading of bequests and determining a decrease in the level of inequality in the next cohort.

The first step of the analysis is to determine the elasticity of fertility with respect to tomorrow endowment.

Lemma 2. If the non-negativity constraint on bequests is not binding, i.e. $b^* > 0$ and $\phi_b^* > 0$ then the elasticity of fertility with respect to tomorrow endowment is given by:

$$\frac{\frac{\partial n}{n}}{\frac{\partial\omega'}{\omega'}} = \frac{1}{\left(\theta-1\right)}\Gamma\left(b,n\right)$$

where $\Gamma\left(b,n\right) = \left[\frac{\omega'}{\omega'+bR}\frac{b+\omega\Phi'(n)}{\omega\Phi'(n)}\right]$.

Proof. The proof is in appendix E. □

[13] With respect to the non-negativity constraints we are including only the one on bequests since it is the only one that can be non-trivially binding

[14] It is discussed in appendix B.

$\Gamma(b,n)$ measures the marginal benefits for parents from the increase in $\omega^{'}$ that allows to cut the per-capita bequest to next generation without decreasing her utility. It is composed by two terms:

- $\frac{\omega^{'}}{\omega'+bR}$: is the relative weight of tomorrow endowment on next generation consumption, the higher the amount of bequests provided by the parents the smaller is the increase in consumption coming from an increase in the next period endowment. As a result also the elasticity of fertility to future endowment is low;
- $\frac{b+\omega\Phi'(n)}{\omega\Phi'(n)}$: is the relative weight of bequests in the marginal cost of having children. The higher is b, the higher is the benefit arising from the possible decrease in b and therefore it increases $\frac{\frac{\partial n}{n}}{\frac{\partial\omega^{'}}{\omega^{'}}}$.

The overall effect of being rich on $\Gamma(b,n)$ is therefore ambiguous. On the one hand the relative weight of next period endowment on future generation consumption is lower given that an high fraction of consumption is represented by bequests, but on the other hand bequests are representing a great fraction of the marginal cost of having children and therefore even a small reduction calls for an increase in fertility.

The next proposition describes the distributional consequences of next period endowment.

Proposition 3. If the non-negativity constraints is not binding, i.e. $b^* > 0$, then $\exists\overline{\omega'}$ s.t. $\forall\omega' > \overline{\omega'}$, $\frac{\partial^2 n^*}{\partial\omega'\partial\tau} > 0$. That is, if tomorrow endowment is high enough the higher the level of initial wealth the higher the (positive) adjustment in fertility when the prospects for next period improves.

Proof. The proof is in appendix E. □

4.1.1 The effect on non-negativity constraints

Even if $\Gamma_b(b,n) < 0$, the non-negativity constraint may generate an increase of wealth inequality. Indeed, $\Gamma(b=0,n) = 1$, Then if $\forall b > 0$, $\Gamma(b,n(n)) > 1$ the existence of the non-negativity constraint generates a decrease of wealth inequality when the next period endowment is higher.

Figure 5 shows the comparative static of $\mathbb{E}[\omega']$ (when $\omega' \in \{\omega_h^{'},\omega_l^{'}\}$ and $Var[\omega^{'}]$ is kept fixed) on the differential fertility and on the differential level of bequests. More precisely, for each $\mathbb{E}[\omega^{'}]$ two individuals are considered, one with $\tau = 10$ and the other one with $\tau = 0$, this allow to study the pure wealth effect disentagling from the substitution effect generated from the opportunity cost

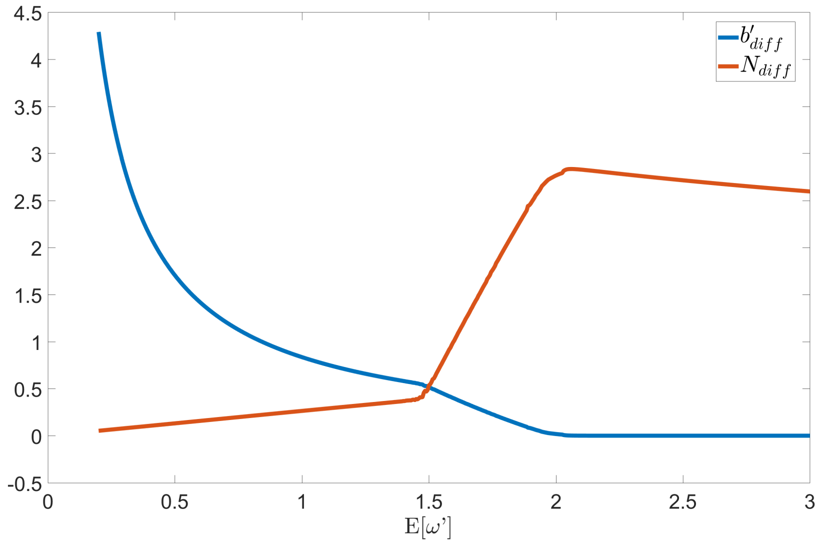


Figure 5: Comparative static on differential fertility of $\mathbb{E}[\omega']$ in absolute values and in percentage changes

of having children. In Figure 5 are represented $n^{10} - n^0$ and $b^{10} - b^0$, that is the difference in the amount of children and in the amount of per-capita bequests left.

For low levels of $\mathbb{E}\left[\omega'\right]$, an increase in the expected value of tomorrow endowment, *ceteris paribus*, decreases the amount of bequests left by both the rich and the poor agents and it increases the difference in the number of children. Notice that the fall in $b^{10} - b^0$ is decreasing with $\mathbb{E}\left[\omega'\right]$. Around $\mathbb{E}[\omega'] = 1.5$ the non-negativity constraint of the poor household becomes binding. From that moment onward the poor does not benefit anymore from the increase in tomorrow endowment given that the optimal choice would require $b^0 < 0$. For this reason children have a lower value and an higher cost that cannot be cut. This affects the Euler constraint of the agent that is prevented from borrowing and therefore $\lambda > R\mathbb{E}\left[\lambda_s'\right]$. Then they reduce the number of children today and they increase their actual consumption. When $\mathbb{E}[\omega'] > 2$ the non-negativity constraints becomes binding also for the rich household and therefore she also starts reducing the number of children (the difference in the number of children starts to decrease).

4.2 Introducing uncertainty

In this case the next-period aggregate endowment is subject to risk, $\omega' \in \left\{\omega_h', \omega_l'\right\}$). The agent has to choose the number of children that she wants to have (N) and the total amount of bequests that she wants to leave to her offsprings (B). The dynastic problem[15] therefore is:

$$\max_{C,C'_h,C'_l,B,N'} \quad \frac{(C)^{1-\sigma}}{1-\sigma} + \alpha\,(N)^{\sigma-\eta}\,\mathbb{E}\left[\frac{(C')^{1-\sigma}}{1-\sigma}\right]$$
$$s.t.$$
$$C + B' \leq \omega\left[1 - \left(\frac{N}{\bar{N}}\right)^{\theta}\right] + \tau \qquad (\lambda)$$
$$C'_s \leq N\omega'_s + BR \qquad \left(\pi_s \lambda'_s\right)$$
$$C, B, N, C'_h, C'_s \geq 0 \qquad (\phi_v)$$

Under regular parametrization of the utility function[16] the objective function is strictly quasi-concave and therefore if the feasibility set is convex the solution exists and it is unique. As a result the problem has a solution and it is unique.

Proposition 4. If $eta \geq \sigma > 1$ and $\theta > 1$ the problem has a solution and it is unique.

Proof. The proof is almost identical to the proof of Proposition 1. $\square$

Then the FOCs are not only necessary but also sufficient, therefore the solution satisfies:

$$\begin{aligned}
C: &\quad C^{-\sigma} = \lambda \\
C'_h: &\quad \alpha\,(N)^{\sigma-\eta}\,(C'_h)^{-\sigma} = \lambda'_h \pi_h \\
C'_l: &\quad \alpha\,(N)^{\sigma-\eta}\,(C'_l)^{-\sigma} = \lambda'_l \pi_l \\
B: &\quad \lambda = R\mathbb{E}\left[\lambda'_s\right] + \phi_b \\
N: &\quad \left(\tfrac{\sigma-\eta}{1-\sigma}\right)\alpha\,(N)^{\sigma-\eta-1}\left(\pi_h C'_h + \pi_l C'_l\right) + \pi_h \lambda'_h \omega'_h + \pi_l \lambda'_l \omega'_l = \omega \tfrac{\theta}{\bar{N}^{\theta}}\,(N)^{\theta-1}\,\lambda
\end{aligned}$$

Where the last condition can be written more compactly as:

$$\left(\frac{\sigma-\eta}{1-\sigma}\right)\alpha\,(N)^{\sigma-\eta-1}\,\mathbb{E}\left[C'_s\right] + \mathbb{E}\left[\lambda'_s \omega'_s\right] = \omega \frac{\theta}{\bar{N}^{\theta}}\,(N)^{\theta-1}\,\lambda$$

The effect of a reduction in the probability of high state tomorrow $\pi_h \downarrow$ is an increase in the expected marginal utility of tomorrow, i.e. $\mathbb{E}\left[\lambda'_s\right] \uparrow$. From the Euler-equation (the FOC wrt B)

[15] The problem is written as the problem of the head of the dinasty that has to choose the total amount of consumption to allocate to the generation, it is a common way to write this problem (Alvarez (1999)).

[16] $\sigma = 2$, $\eta \geq \sigma$, see Jones and Schoonbroodt (2010) for a complete discussion on the effect of this calibration as opposed to Barro and Becker (1989) calibration (with $\sigma < 1$).

it must follow $\lambda \uparrow$ and or $\phi_b \downarrow$. When the non-negativity constraint on bequests is not binding the only multiplier that must adjust is λ, and therefore we must have $\lambda \uparrow$, i.e. $C \downarrow$. From the budget constraint we have that the decrease in consumption can be compensated through an increase in the amount of bequests or an increase in the amount of children. Looking at the FOC wrt N we have that the marginal cost of children has increased due to the increase in λ and that the "marginal return" of children has decreased due to the fall of $\mathbb{E}\left[C_s^{'}\right]$ while the effect on $\mathbb{E}\left[\lambda_s^{'}\omega_s^{'}\right]$ is ambiguous. As a result

The comparative statics with respect to main parameters of the model is performed in appendix C.

4.3 Comparative statics on differential fertility and differentials bequests

In this subsection we study the effects of parameters on differential fertility. We perform the following analysis: we compute for different values of the parameter of interest the solution of the two period model, in one case we solve the model with $\tau = 0$ and in the other case we solve the model with $\tau = 10$, then we compare the optimal choice of fertility and per-capita bequests. Define for the generic variable x the solution of the problem when $\tau = t$ with x_t^*, for different values of the parameters of the model we plot $N_{diff} := N_{10}^* - N_0^*$, $b_{diff}^{'} := b_{10}^* - b_0^*$, $N_{diff}\% := \frac{N_{10}^* - N_0^*}{N_0^*}$ and $b_{diff}^{'} := \frac{b_{10}^* - b_0^*}{b_0^*}$. Results[17] are shown in Figure 6.

- $0.01 \leq \alpha \leq 0.60$: an increase in the discount factor of the parents uin absolute terms increases the difference in the number of children between rich and poors and increases the difference in per-capita bequests left to the children, nonetheless looking at the percentage differences we can see that the increase in patience leads to a reduction of per-capita bequests difference in percentage terms.

- $1.1 \leq \eta \leq 5$[18]: an increase in η reduces the differential fertility, the difference in bequests is extremely small and disappear. Indeed an increase in η moves the preference of the parents from quality children to their quantity, this lead to a reduction in the quantity of bequests also from the rich.

- $1.5 \leq \sigma = \eta \leq 3.5$: σ captures the inverse of intertemporal elasticity of substitution and the

[17] Comparative static is performed using a non-linear solver and a filter to smooth the resulting lines.

[18] In this case I let $\sigma = 1.1$, this allow to study the effect of η

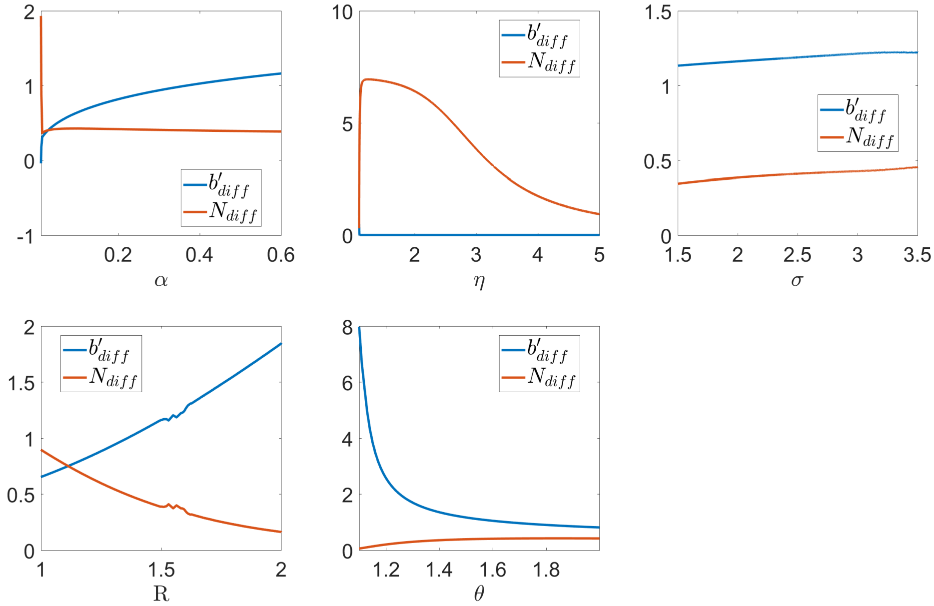


Figure 6: Effects on differential fertility and bequests of the main parameters of the model

degree of relative risk-aversion. An increase in σ determines an increase in the relative risk-aversion, given that children are the risky investment in this setup the differential fertility increases since the wealthier households will be relatively less risk-averse thus prefering children to bequests and determing a reduction in the percentage difference of bequests and an increase in the percentage difference in the number of children.

- $1 \leq R \leq 2$: the gross interest rate affects the investment opportunity in bonds instead of children. An increase in the interest rate increases the difference in bequests in absolute terms this is coming from the fact that when $R = 1.1$ the poor households has the non-negativity constraint on bequests binding. On the contrary the difference in the number of children is decreasing both in absolute and in relative terms.

- $1.1 \leq \theta \leq 2$: θ is a parameter that affects the "technology" of fertility and captures the degree of convexity of the time-cost function of children. An higher degree of concavity increases the marginal cost of having an additional child.

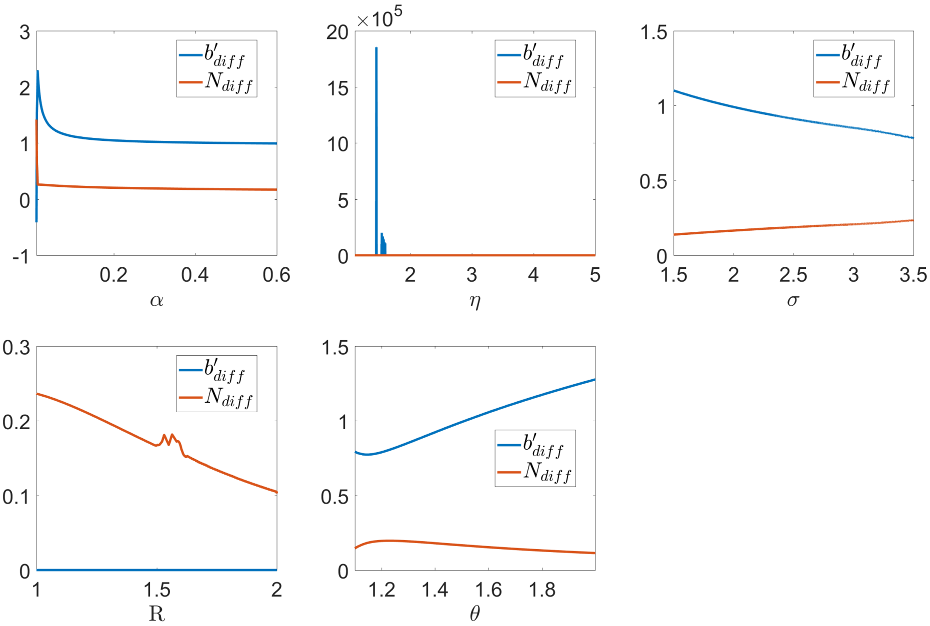


Figure 7: Effects on differential fertility and bequests of the main parameters of the model, % changes

5 Quantitative Model

The economic mechanism connecting the fertility decline to wealth inequality that we explored from an empirical point of view in Sections 2 and 3 can be summarized as follows:

1. Agents experience lower economic growth during their fertility years. Hence they revise their expectations about future growth and decrease fertility.

2. The decrease in fertility leads to an increase in per-capita bequests which in turn translates into higher wealth inequality.

In this section we extend the economic model of endogenous fertility and wealth inequality introduced in Section 4 to rationalize the empirical evidence presented in Section 3. The first channel suggests that aggregate shocks experienced earlier in life matter significantly for fertility decisions. The quantitative model allows us to measure the impact of aggregate slowdown in TFP that happened between the 1950s and the 1970s on the subsequent drop in fertility. In order to capture the second channel, we incorporate a **bequest motive** that connects each generation to the next one, along the lines of Barro and Becker (1989). The reminder of this section is organized as follows: Subsection 5.1 lays out the basic economic environment, subsection 5.2

presents the individual's maximization problem. We define the equilibrium in 5.3. After explaining the calibration procedure in subsection 5.4 we presents the quantitative results in subsection 5.5.

5.1 Economic Environment

We present now the dynastic extension of the simple two-period model of Section 4, building on Barro and Becker (1989) and Alvarez (1999). Economic agents lives for two periods: in the first period they are children and in the second period they are adults. All relevant decisions are taken in the second period. When adults, agents enjoy resources given by their parents, bequests b, and earn labour income $\omega\epsilon$. Labour income is given by the product of an aggregate shock ω (as in Section 4) and an idiosyncratic shock ϵ. The latter shock captures idiosyncratic risks such as unemployment or health risk against which the agents cannot insure themselves. We explain below how we calibrate the stochastic process for the shocks; for the time being we define the joint distribution of the shocks as $F(\omega', \epsilon' \mid \omega, \epsilon)$. After observing the shocks agents decide on fertility n. We assume that each parent behaves altruistically and attaches a weight equal to $n^{1-\eta}$ to the children. The cost of raising children is given by a factor that decreases labour income (we interpret it as an opportunity cost):

$$\Phi(n) = \left(\frac{n}{\bar{n}}\right)^{\theta}.$$

Therefore net labour income is given by:

$$\omega\epsilon\left[1 - \Phi(n)\right] = \omega\epsilon\left[1 - \left(\frac{n}{\bar{n}}\right)^{\theta}\right].$$

The available resources $\omega\epsilon\left[1 - \Phi(n)\right] + b(1 + r)$ are then spent for consumption c and to leave bequests b' to each of the n children.

Aggregate and idiosyncratic shocks. The stochastic process for TFP is assumed to be given by:

$$\log \omega_t = \rho_\omega \log \omega_{t-1} + u_t,\ u_t \sim N(0, \sigma^2_\omega). \tag{1}$$

The autoregressive process in (1) is implemented in a simple way assuming a symmetric two-state Markov chain given by $\log \omega = [-\bar{\omega}, \bar{\omega}]$ and

$$\Pi_Z = \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix}.$$

where $\bar{\omega}$ and π are function of $(\rho_\omega, \sigma^2_\omega)$ estimated on the data[19]:

$$\bar{\omega} = \sqrt{\frac{\sigma^2_\omega}{1-\rho_\omega}}, \tag{4}$$

$$\pi = \frac{1+\rho_\omega}{2}. \tag{5}$$

Regarding the idiosyncratic shock to labour income, ϵ, we assume that it follows the following first-order autoregressive process:

$$\log \epsilon_t = \rho_\epsilon \log \epsilon_{t-1} + v_t,\ v_t \sim N(0, \sigma^2_\epsilon) \tag{6}$$

where ρ_ϵ is the coefficient on the intergenerational persistence of the shock and σ_ϵ is the dispersion.

5.2 Recursive Formulation

The problem faced by each individual can be written recursively as:

$$v(b, \epsilon, \omega) = \max_{b', c, n} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \alpha n^{1-\eta} \mathbb{E}\left[v(b', \epsilon', \omega') \,|\epsilon, \omega\right] \right\} \tag{7}$$

subject to

$$\begin{aligned} c + nb' &\leq \omega\epsilon\left[1 - \Phi(n)\right] + b(1+r), \\ b' &\geq 0, \end{aligned}$$

where ω is the aggregate shock (as in Section 4) while ϵ is the idiosyncratic shock. The non-negativity constraint on desired bequests b' prevents parents from imposing debt obligation on

[19] To obtain formulas (4) and (5), simply use the method of moments and equate the variance and the serial correlation in the AR(1) and in the Markov chain:

$$\frac{\sigma^2}{1-\rho^2} = \bar{\omega}^2 \tag{2}$$

$$\rho\frac{\sigma^2}{1-\rho^2} = (2\pi - 1)\,\bar{\omega}^2 \tag{3}$$

their children and is consistent with the legal framework valid in most developed countries[20].

The model does not admit a closed form solution and will be solved numerically. We can however characterize analytically some properties of the model. To this end, let us derive the first order conditions with respect to the optimal choice of fertility and bequests:

$$n: \quad u'(c)\left[\omega\epsilon\Phi'(n)+b'\right]=\alpha\left(1-\eta\right)n^{-\eta}\mathbb{E}\left[v\left(b',\omega',\epsilon'\right)\mid\omega,\epsilon\right] \tag{8}$$

$$b': \quad u'(c)\geq\alpha n^{-\eta}\mathbb{E}\left[v_b\left(b',\omega',\epsilon'\right)\mid\omega,\epsilon\right], \text{ with equality if } b^{'}>0 \tag{9}$$

Negative relation between fertility and savings. Equation (9) makes clear that the marginal benefit of savings is decreasing in the number of children n: therefore larger households tend to leave fewer bequests. This property makes our model consistent with the quality-quantity tradeoff theroy originally proposed by Becker.

Fertility-income relation. According to the empirical evidence reported in Jones and Schoonbroodt (2010), there is a negative relationship between fertility and parents' earnings. Our model is consistent with such evidence provided that the parameter σ (the inverse of the intertemporal elasticity of substitution) is low enough[21].

5.3 Equilibrium

In this section we provide a formal definition of the equilibrium and explain how to compute the distribution of heterogeneous households in the model.

Definition of recursive competitive equilibrium. *Given[22] interest rate r, a recursive competitive equilibrium is given by value and policy functions $v(b,\epsilon,\omega)$, $g_b(b,\epsilon,\omega)$, $g_n(b,\epsilon,\omega)$, $g_c(b,\epsilon,\omega)$ and an aggregate distribution $\lambda(b,\epsilon,\omega)$ such that:*

1. *The policy functions $b'=g_b(b,\epsilon,\omega)$ for bequests, $n=g_n(b,\epsilon,\omega)$ for fertility and $c=g_c(b,\epsilon,\omega)$ for consumption solve the individual problem defined in (7).*

2. *The aggregate distribution $\lambda(b,\epsilon,\omega)$ is induced by the exogenous stochastic processes for idiosyncratic and aggregate risk, summarized by $F(\epsilon',\omega'\mid\epsilon,\omega)$, as well as the optimal policy functions for bequests and fertility.*

[20]Notice that such constraint breaks down the well-known equivalence result between OLG model with altruistic parents and the infinitely-lived agent model. See for example Blanchard and Fischer (1989), ch.3.

[21]It is not possible to derive analytical results but this is confirmed in the numerical simulations.

[22]The reader should bear in mind that the model is laid out in partial equilibrium.

We now give the explicit statement of (2) in the following. The Markov transition and the policy functions induce a transition equation for the distribution (so that the stationary distribution is simply the fixed point of such transition equation) given by:

$$\lambda_{t+1}\left(b', \epsilon', \omega'\right) = \Pr\left(b', \epsilon', \omega'\right) = \sum_b \sum_\epsilon \sum_\omega \Pr\left(b', \epsilon', \omega' \mid b, \epsilon, \omega\right) \lambda_t\left(b, \epsilon, \omega\right),$$

where

$$\Pr\left(b', \epsilon', \omega' \mid b, \epsilon, \omega\right) = \begin{cases} F\left(\epsilon', \omega' \mid \epsilon, \omega\right) \frac{g_n(b,\epsilon,\omega)}{N} & \text{if } g_b\left(b, \epsilon, \omega\right) = b' \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

and

$$N = \sum_b \sum_\epsilon \sum_\omega g_n\left(b, \epsilon, \omega\right) \lambda_t\left(b, \epsilon, \omega\right).$$

So the element:

$$\frac{g_n\left(b, \epsilon, \omega\right)}{N}$$

measures the fertility of the specific fraction of the population over the overall fertility of the population.

5.4 Calibration

We calibrate the model to match a number of cross-sectional moments related to fertility, income and wealth for the 1960-2010 U.S. economy. A detailed discussion of the moments and the identification strategy follows. See Table (8) for a summary.

	$\widehat{\rho}$	$\widehat{\sigma}$
TFP Fernald, from 1947	0.9893	0.1548
(trend, HP Filter $\lambda = 20$)	(0.0245)	(0.0186)
TFP Fernald adjusted for capacity utilitization, from 1947	0.9902	0.1616
(trend, HP Filter $\lambda = 20$)	(0.0214)	(0.0143)
TFP Chari et al. (2007) methodology, from 1900	0.9879	0.0039
(trend, HP Filter $\lambda = 20$)	(0.0130)	(0.0002)

Table 7: TFP process, different estimations

Stochastic process for aggregate productivity ω. The concept of productivity in the model corresponds to aggregate TFP in the data. Following Fernald (2012), we estimate an AR(1) process on the yearly series of total factor productivity for US economy in the period 1960-2010. In particular we estimate the following equation:

$$\Delta TFP_t = \alpha + \rho \Delta TFP_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$. We report the results in Table 7.

However, since the duration of a period in the model corresponds to 20 years in the data, we convert the yearly estimates as follows. Let $\tilde{\rho}$ and $\tilde{\sigma}^2$ be the yearly estimates; then the corresponding 20-years values $\hat{\rho}$ and $\hat{\sigma}^2$ are:

$$\hat{\rho} = \tilde{\rho}^{20},$$
$$\hat{\sigma}^2 = \left(\sum_{j=0}^{19} \hat{\rho}^{2j}\right) \tilde{\sigma}^2.$$

The child discount factor is modelled as $\alpha n^{1-\eta}$, following Barro-Becker and Alvarez. We prefer this formulation to the exponential formulation of Cordoba et al. (2016) due to its simplicity. The pure discount factor α is chosen to match the earnings-income correlation whereas the elasticity parameter α targets the income elasticity of fertility. Of course in equilibrium all parameters affects all targets but for example α affects significantly the propensity to save therefore it has a strong impact on the correlation between income (given by earnings plus rb) and earnings.

The cost of raising children in the model takes the form $\Phi(n) = \left(\frac{n}{\overline{n}}\right)^{\theta}$. The parameters θ and $\overline{n}$ are calibrated jointly with the others but in particular θ is meant to capture the cross-sectional dispersion in fertility choices whereas $\overline{n}$ targets mostly the average fertility level. We take data on fertility from Jones and Tertilt (2008). The paper reports an average fertility per household equal to 2.00. However in our model the relevant concept is fertility per person, therefore we set it equal to 1.00.

The baseline calibration is summarized in Table (8). Overall our model does a good job of capturing the salient features of the fertility-income-wealth distribution. Given the features matched in the calibration, we also analyse how well the model does on other dimensions that were not explicitely targeted. Tables (9) and (10) report these overidentifying tests. We find this encouraging as it shows that the model provides an appropriate framework to study the macroeconomic implications of changes in agents' expectations about growth on fertility and, hence, wealth.

Table 8: Parameters

Parameter	Value	Target	Data	Model
<u>*Internal Calibration*</u>				
R interest rate	2	Annual interest rate of 3.5%	2.00	2.00
σ curvature utility	0.72	Gini bequests	0.82	0.80
α discount factor	0.25	Corr(earnings,income)	0.84	0.67
η child discount elasticity	0.57	Income elasticity of fertility	-0.20	-0.26
θ cost of raising children	1	Coeff. of variation fertility	0.60	1.75
$\overline{n}$ cost of raising children	10	Average fertility	1.00	1.13
ρ_ϵ persistence ability shock	0.50	Persistence wages	0.50	0.50
σ_ϵ dispersion ability shock	0.85	Gini earnings	0.64	0.63
<u>*External calibration*</u>				
ρ_ω persistence TFP shock	0.82	Annual persistence (Fernald)	0.99	0.99
σ_ω dispersion TFP shock	2.75	Annual volatility (Fernald)	0.16	0.16

Note: The child discounting takes the form $\alpha n^{1-\eta}$ and the cost of raising children is $(1-\frac{n}{\bar{n}})^\theta$.

Table 9: Calibrated moments

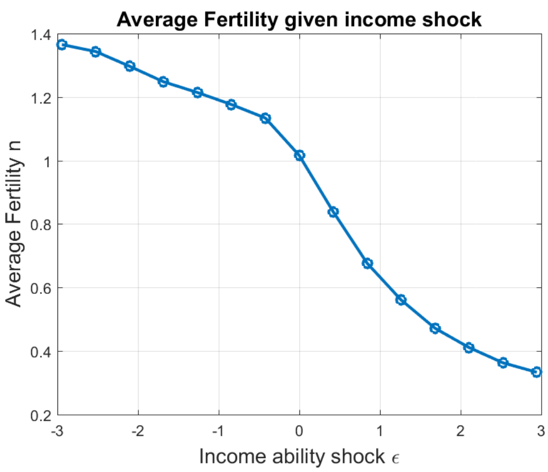
Moments	Data	Model
Gini of Wealth	**0.82**	**0.80**
Gini of Income	0.58	0.57
Gini of Earnings	0.60	0.59
Gini of Consumption	0.32	0.56

Note: Calibration targets are in boldface.

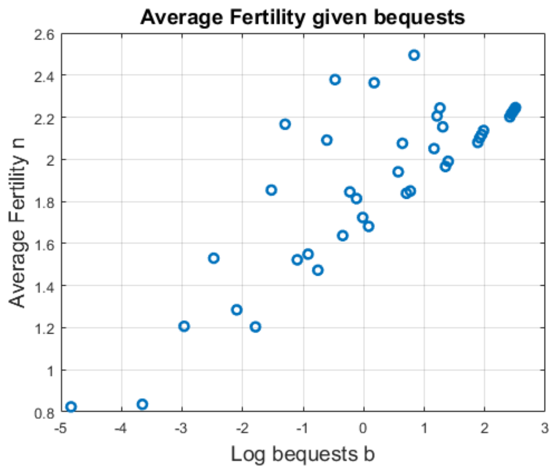
Table 10: Calibrated moments

Bequests	Gini	Bottom 40%	Top 20%	Top 10%	Top 1%
U.S. DATA	**0.82**	0.00	0.91	0.65	0.35
MODEL	**0.80**	0.00	0.83	0.64	0.19

Note: This Table compares the distribution of bequests between the U.S. data and the model. It indicates the Gini coefficient and the share of wealth owned by bottom and top percentiles of the U.S. population.

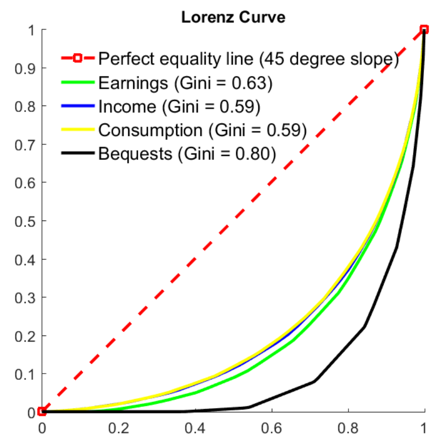


(a) Fertility vs income

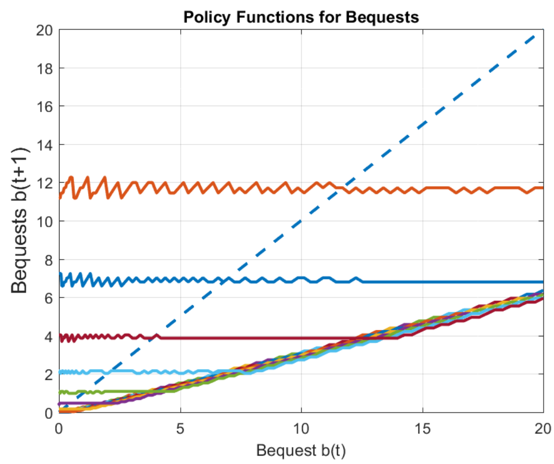


(b) Fertility vs bequests

Figure 8: Optimal choice for fertility



(a) Inequality in the model



(b) Different idiosyncratic shocks

Figure 9: Inequality and Bequests

5.5 Results

In order to highlight the importance of expectations about future growth for shaping fertility and wealth accumulation decisions, we perform two main quantitative experiments. In the last section we further stress the role of beliefs by departing from fully rational expectations and introducing overpersistence bias (see Section 5.5.3).

5.5.1 Experiment 1

First we simulate the model under two scenarios: *good* aggregate state vs *bad* aggregate state. This exercise is meant to capture the difference in the long-run distribution between periods of high and low economic growth.

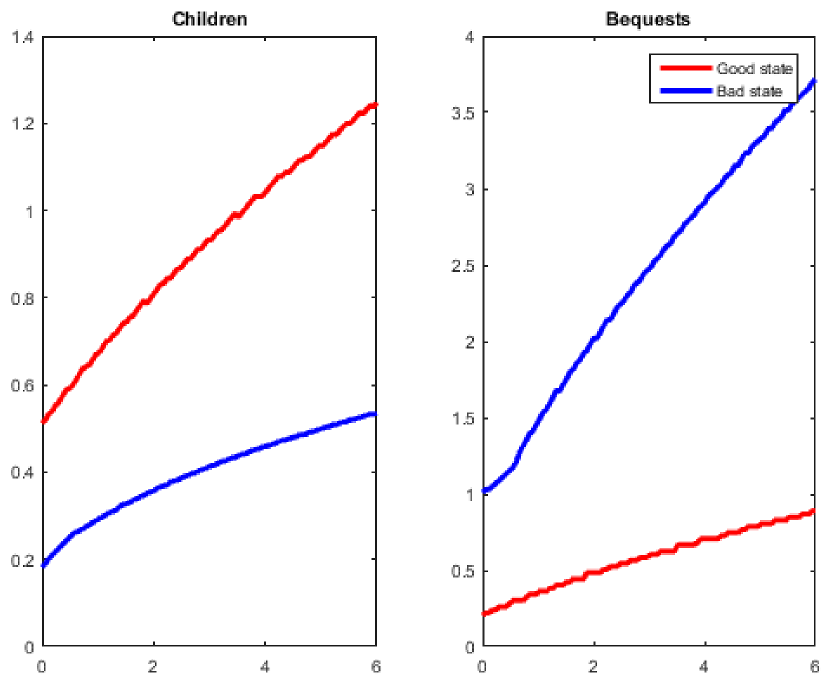


Figure 10: Policy functions for children and per capita *bequests*

We report the main findings in Figures (10)-(13). Figure (10) shows the effect of aggregate economic conditions (*good* vs *bad* times) on fertility choices and wealth accumulation through bequests. In particular, the left panel depicts the optimal choice for fertility (number of children)

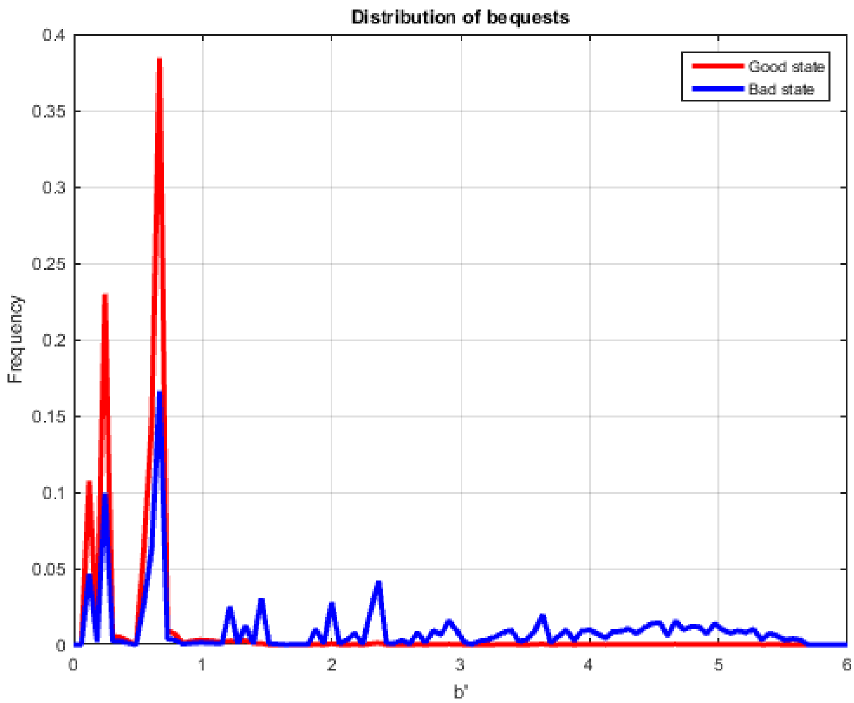


Figure 11: Bequests PDF good vs. bad state

as a function of inherited wealth, given the two possible realizations of the aggregate shock[23]. The right panel of the same figure shows instead the optimal level of bequests.

The following pattern emerges: First of all, bequests are higher in the bad state rather than in the good one for all levels of inherited wealth but the spread is higher the higher the level of initial wealth. Second, agents reduce fertility in the bad state. The figure illustrates nicely the main mechanism described in the introduction: when aggregate conditions are bad the negative perspective on future growth induces agents to leave a higher amount of bequests to a fewer number of children. Lower economic growth decreases the return of the "children" asset inducing households to accumulate more bequests due to a precautionary motive.

Figure (11) and (12) provides a comparison of the distribution of bequests in the model conditional on the two realizations of aggregate TFP: consistently with the findings reported in Figure (10) in the stationary distribution[24] there is a higher concentration of mass among high levels of

[23]More precisely, we averaged out the idiosyncratic shocks.

[24]Bear in mind that the stationary distribution is adjusted by the level of fertility of agents in each state, accordingly to equation (10).

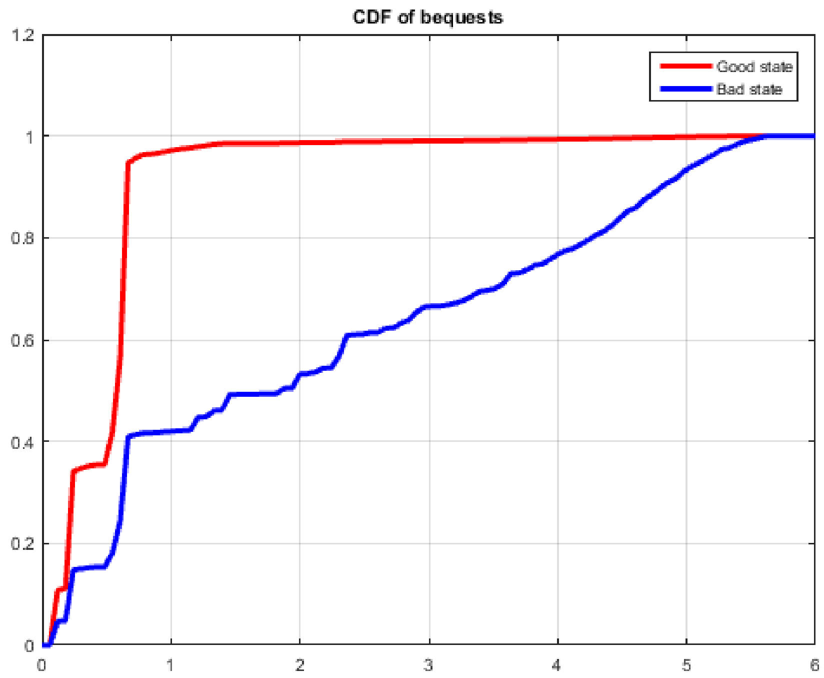


Figure 12: Bequests CDF good vs bad state

bequests when the state of the economy is bad. Under good economic conditions instead, agents never accumulate wealth higher than a certain threshold since the precuationary saving incentives are much less strong.

Finally Figure (13) reports the Lorenz curve of bequests contrasting good versus bad times and shows that inequality is higher in the bad state than in the good state.

Inequality	ω_H	ω_L
Gini of income	0.63	0.63
Gini of wealth	0.60	0.85
Var of log income	0.21	0.21
Var of log wealth	0.44	1.26
P90/P50 of income	4.40	4.40
P90/P50 of wealth	2.66	18.14
P10/P50 of income	0.16	0.16
P10/P50 of wealth	0.01	0.00

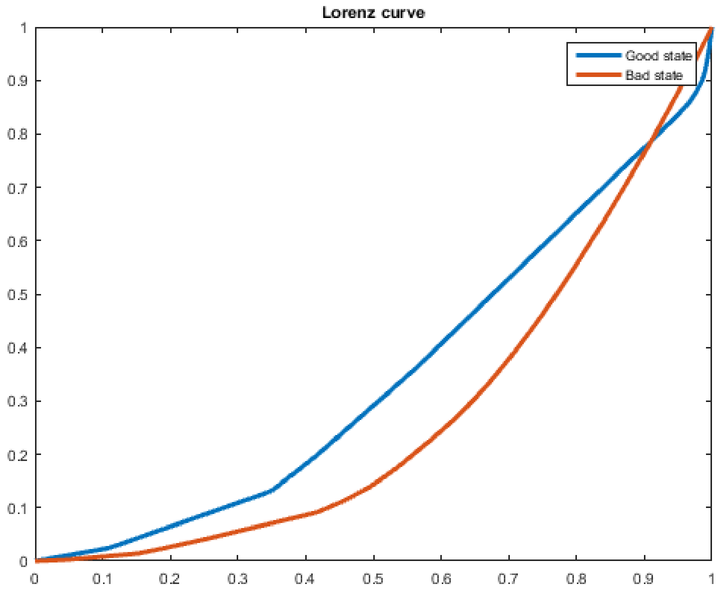


Figure 13: Lorenz curve bequests good vs bad state

5.5.2 Experiment 2

In the experiment described above we highlighted the role of aggregate shocks in shaping the incentives for fertility decisions and accumulation of wealth, showing that the mechanism we propose is *qualitatively* relevant. In the present subsection we instead ask the model to answer a specific *quantitative* question: how much of the variation in fertility and wealth concentration that occurred in the US economy between 1960 and 2010 can be explained by the fall in aggregate TFP growth. We also ask whether this mechanism rely on the fact that agents overestimate changes in TFP. The third column of Table (11) answers the first part of question, showing the effect of changes in growth on fertility and wealth concentration within a standard mechanism of beliefs. The last column reports instead the results obtained by assuming the agents in the model hold biased beliefs along the lines described in section (5).

The TFP growth rate in the US economy averaged at 1.93% in the 1960-1970 decade, slowing down to 1.23% in the 2000-2010 decade. We capture such difference in TFP growth rates by calibrating the two-state Markov process for the aggregate shock accordingly. Furthermore the total fertility rate (adjusted by child mortality) fell by almost one-third, from 2.91 children per family down to 2.03, and the wealth concentration, measured as the share of wealth owned by the top 1%, sharply increased by around 10 percentage points (see the first column of Table 11 for additional details). Our model generates a fall in fertility rate after the negative shock to TFP growth rate that is roughly comparable to the decline found in the data: in particular the model generates a decrease in fertility of 24% whereas in the data the shrinkage is of 30%. The ability of the model to capture most of the variation in fertility hinges crucially on the income elasticity of fertility, which is one of the main calibration targets. Under the assumption of biased beliefs the change in fertility is somehow more pronounced, given that parents now overestimate the persistence of the negative shock. Turning our attention to wealth inequality, we observe first that our preferred measure of wealth inequality, namely the share of wealth owned by the richest 1% in the population[25], jumped up from 27% in the 1960s to 33% in the 2010s. Our model is then able to account for 40% of such increase in the baseline version (45% in the version with overpersistence bias). The main mechanism goes through an increase in per-capita bequests. Two effects play a role here: first all agents reduce their desired number of children and increase their bequests following a negative shock to economic growth. Second, the income elasticity of fertility is higher for wealthier families: therefore the increase in per-capita bequest is stronger among wealthy households, further exacerbating the increase in wealth inequality.

We conclude observing that the fertility channel explored in this experiment a significant part of the increase in wealth inequality. However more than half is left unexplained. Other factors such as changes in tax policies and the increase in labour earnings inequality are likely to play a role.

5.5.3 Experiment 3: Overpersistence bias

The first channel suggests that agents hold wrong beliefs about future economic growth, overestimating the persistence of aggregate income shocks. In order to capture this, our model features a **small departure from the rational expectation** hypothesis, that explains why transitory shocks can have long-lasting effects on fertility and demography

Overpersistence Bias. The idea that a negative shock experienced during the impressionable years affects disproportionately your future expectations can be rationalized with the following small deviation from rational expectations: agents overestimate the persistence of the aggregate

[25]We chose this statistic for two main reasons. First it has been popularized by Piketty (2014) and features prominently in the policy debate, and, second, data on other measures (such as the Gini coefficient of wealth distribution) are not available earlier than 1989.

income process. When they are hit by a negative shock they think such shock will last more that it actually does.

Formally, agents believe that the aggregate TFP shock is given by

$$\log \omega_t = \widehat{\rho}_\omega \log \omega_{t-1} + u_t, \quad u_t \sim N\left(0, \sigma_\omega^2\right)$$

whereas the true process is given by

$$\log \omega_t = \rho_\omega \log \omega_{t-1} + u_t, \quad u_t \sim N\left(0, \sigma_\omega^2\right),$$

with $\widehat{\rho}_\omega > \rho_\omega$. (They have correct beliefs about the variance σ_ω^2). We implement these non-rational expectations in a simple way assuming that the TFP process is governed by a symmetric[26] two-state Markov chain given by $log(\omega) = [-\omega, \omega]$ and

$$\Pi_Z = \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix},$$

where ω and π are given by $(\rho_\omega, \sigma_\omega^2)$ estimated on data[27]:

$$\omega = \sqrt{\frac{\sigma_\omega^2}{1-\rho_\omega}} \tag{11}$$

$$\pi = \frac{1+\rho_\omega}{2} \tag{12}$$

However agents have incorrect beliefs and think that the transition matrix is given by:

$$\widehat{\Pi}_Z = \begin{bmatrix} \widehat{\pi} & 1-\widehat{\pi} \\ 1-\widehat{\pi} & \widehat{\pi} \end{bmatrix},$$

where $\widehat{\pi} > \pi$. In the quantitative section we will show that the presence of biased beliefs improves

[26] A *symmetric* Markov chain implies however that the duration of the two states (high and low) is the same, which is not a good assmption for recessions and booms.

[27] To obtain formulas (11) and (12), simply use the method of moments and equate the variance and the serial correlation in the AR(1) and in the Markov chain:

$$\frac{\sigma^2}{1-\rho^2} = \omega^2$$

$$\rho\frac{\sigma^2}{1-\rho^2} = (2\pi - 1)\,\omega^2$$

the ability of the model to explain the data with respect to the baseline with rational expectations.

Regarding the numerical implementation we follow the procedure outlined in Rozsypal and Schlafmann (2017) with minimal modifications.

Table 11: How a fall in growth expectations affects fertility and wealth concentration: Comparing the rational expectations model to the biased model

	U.S. data	**Rational Expectations**	**Biased Beliefs**
Delta TFP	-0.69%	-0.69%	-0.69%
Delta Fertility	-30.28%	-24.22%	-25.74%
Delta Wealth concentration	$+6.00\%$	$+2.40\%$	$+2.70\%$

Note: The source for US data on TFP is Fernald (2012). Here we use data on TFP corrected by capacity utilization. The data on fertility are adjusted by mortality rate and taken from Roser (2017a,b). The data on wealth concentration are taken from Rios-Rull and Kuhn (2016) and from the World Income Database (Alvaredo et al. 2017) . Wealth concentration is measured here as the top 1 percent share. See the appendix for alternative measures of wealth inequality.

6 Conclusions

The findings of our paper emphasize the role of decreasing total factor productivity growth and, in turn, of fertility as a relevant channel for higher wealth inequality in the US economy over the last 50 years.

Using the Barro-Becker model of fertility in a setup with aggregate and idiosyncratic shocks and with incomplete markets we show that the slowdown in total factor productivity growth determines a fall in the fertility rate and an increase in intergenerational transfers by worsening parents expectations about children future income. The magnitude of this effect is proportional to the number of children that parents plan to have. Wealthier parents have more children, fertility behave as a 'normal' good, as a result they decrease fertility and they increase per-capita transfers more than poorer households. As a result the intra-cohort inequality in the next generation increases.

Using a calibrated extended version of the baseline model we are able to explain roughly 40 per cent of the increase in wealth inequality observed in the US between 1980 and 2010. The ability of the model to explain the increase in wealth inequality is somewhat enhanced when we relax the rational expectations hypothesis and assume that agents overestimate the persistence of the shocks.

The analysis of micro-data on wealth from SCF and PSID confirm the main predictions of the model.

A final caveat is in order. The model used in the quantitative analysis is set out in partial equilibrium for simplicity. We are aware that by doing so we neglect many endogenous effects, including the impact of bequests accumulation on the interest rate. We leave the general equilibrium extension for future research.

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Appendix

A Additional evidence on the change in intra-cohort inequality

The increase in wealth inequality among households of all ages is robust to the exclusion of households with negative net worth from the sample. Figure 14, together with the standard Gini coefficient, contains the evolution of Gini coefficient computed excluding households with negative net worth. The Gini index by construction takes value between 0 and 1 when only null or positive values are used. If it is equal to 0 everyone has exactly the same amount of wealth, if it is equal to 1 then only one households owns the all wealth of the economy. When households with negative net worth are included the index can take also values bigger than one and thus it has a less obvious interpretation, for this reason sometimes they are excluded from the computation.

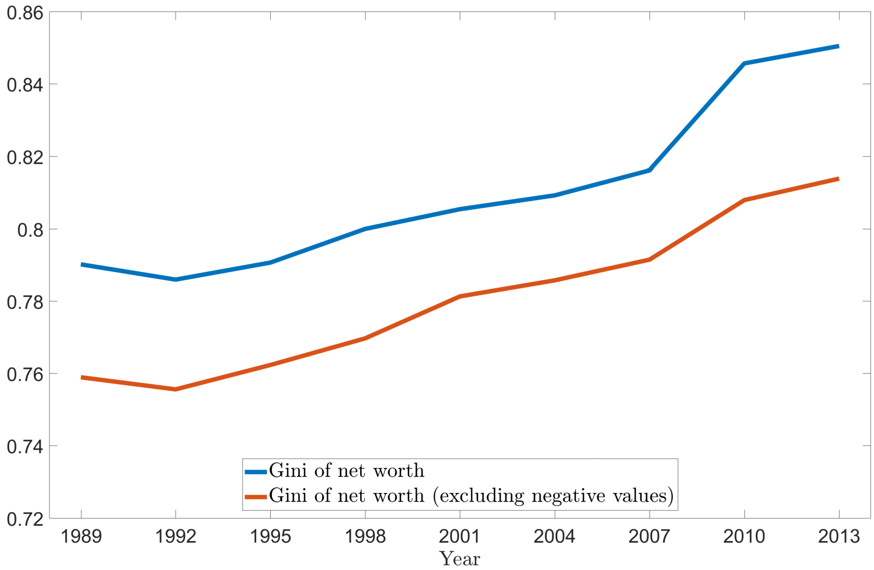

Figure 14: Gini coefficient of net wealth in the United States (including and excluding negative values), source: Survey of Consumer Finances.

The increase in wealth inequality in youngest cohorts is robust to different measures of inequality. In Figure 15 is plotted the 90-50 interpercentileand we can see that there has been a substantial increase in the last 20 years, despite the fall in 2013 after the Great Recession the index for under-35 is around 15 and it has almost doubled since 1995. The interpercentile at the bottom (i.e. the p50-p10 ratio) displayed in Figure 16 displays a similar pattern. We also consider the share of wealth owned by the richest 1 percent as a measure of wealth inequality. Figure 17 plots the evolution of the top 1 percent share over time for the US economy, whereas Figure 18 decomposes this statistic among cohorts with different ages.

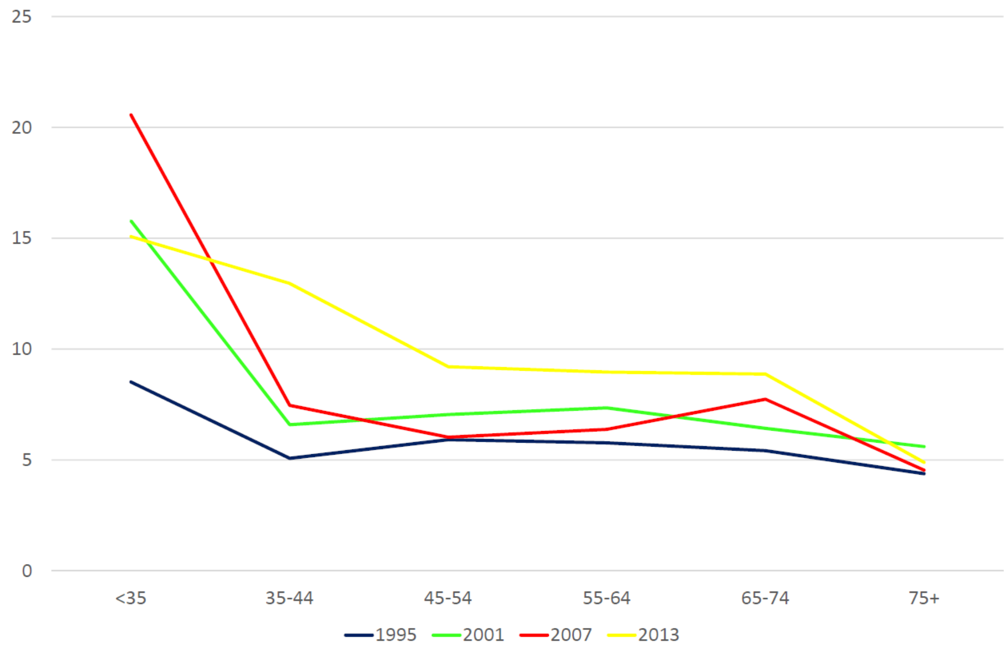

Figure 15: Intra-cohort interpercentile p90/p50 across years, source: Survey of Consumer Finances, source: Survey of Consumer Finances

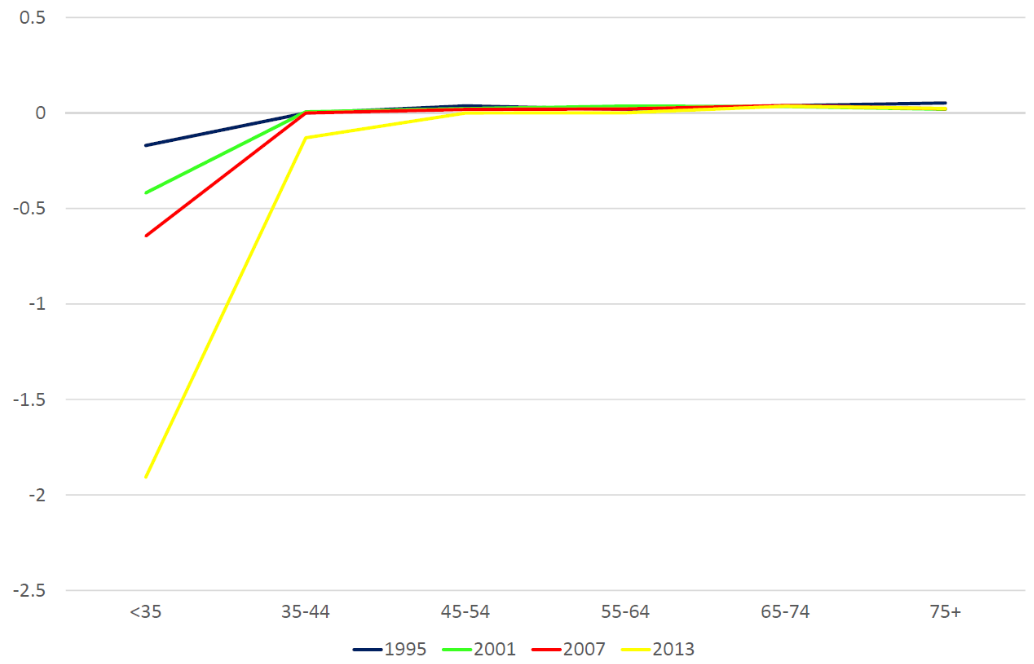

Figure 16: Intra-cohort interpercentile p10/p50 across years, source: Survey of Consumer Finances

Figure 17: Share of wealth owned by the richest 1 percent, source: Survey of Consumer Finances

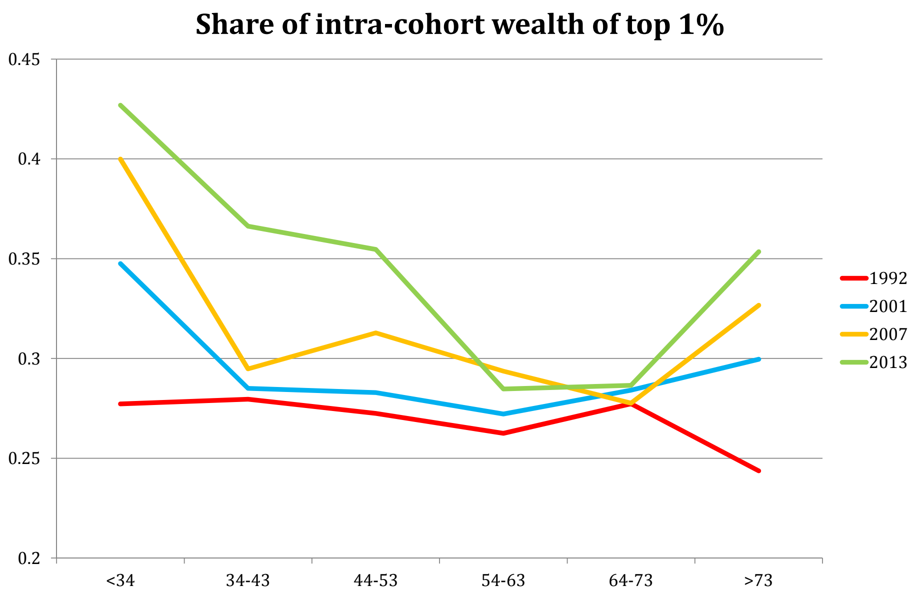

Figure 18: Intra-cohort share of wealth owned by the richest 1 percent, source: Survey of Consumer Finances

B Barro-Becker model and its calibration

The household preferences for fertility are represented using the model proposed by Barro and Becker (1989), the model of endogenous fertility most used in the literature. In this section I will explain what are the main features of the model and make references to other papers that have analyzed is fit to aggregate data.

In the Barro-Becker world each cohort lives one period: decides consumption, savings (bequests) and fertility and then dies. Consider a simple case with an endowment economy. The value function associated to an agent of today generation is the following:

$$\begin{aligned} V(\omega, b) &= \max_{b', c, n' \geq 0} u(c) + n'a(n')\mathbb{E}\left[V(\omega', b')\right] \\ & s.t. \\ & c + n'b' + B(n') = \omega + b(1+r) \end{aligned}$$

Where $u(c)$ is a standard utility function and measures the utility from her own consumption, n' is the number of children that today cohort decides to have,$\mathbb{E}\left[V(\omega', b')\right]$ is the expected utility of each children (that enters the problem since agents are altruistic), $n'a(n')$ is the discount factor on which I will focus later on, $B(n')$ is a cost-function from having children, r is the real interest rate that the new generation gets from the endowment left from the past cohort and ω is the endowment of each agent of today cohort. The endogenous state variables are b and n'.

The discount factor $n'a(n')$ must have the following properties:

- An additional children, *ceteris paribus*, increases the utility of the parents;
- Having children increases utility at a marginally decreasing rate.

If we use CRRA functional form for utility from consumption good, i.e. $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and an exponential form for the discount factor, i.e. $a(n') = \alpha(n')^{-\eta}$ as it has been deeply discussed in Jones and Schoonbroodt (2010) there are two alternatives calibration of σ and η that guarantees that the modelization satisfies the two assumptions above and the concavity of the value function:

- Assume that $u(c) \geq 0\ \forall c \geq 0$, that $u(\cdot)$ is strictly increasing and strictly concave, and that $0 < 1-\eta < 1$. Under this calibration the value function is increasing and concave in c and n if and only if $0 < 1-\sigma \leq 1-\eta < 1$

- Assume that $u(c) \leq 0 \; \forall c \geq 0$, that $u(\cdot)$ is strictly increasing and strictly concave, and that $1-\eta < 0$. Under this calibration the value function is increasing and concave in c and n if and only if $0 > 1-\sigma \geq 1-\eta$

While the first calibration is the most used in the fertility literature the second calibration is in line with macro-literature calibration of the intertemporal elasticity of substitution (IES¡1) and implies the negative relationship between income and fertility that we have in data. Indeed the two calibrations have also different implications on the relationship between children and consumption goods: if $0 < 1-\sigma \leq 1-\eta < 1$ the consumption goods and children are complements on the other hand if $0 > 1-\sigma \geq 1-\eta$ they are substitutes. This implies that if children have a cost in terms of time the higher the wage the lower the number of children that the household will have but it will guarantee to each of them an higher level of consumption (consistent with both individual and aggregate data on fertility[28]).

B.1 Inter-temporal trade off

The main difference of this model with respect to the standard model with a representative agent comes from the inter-temporal trade off. Usually the Euler-equation relates the marginal utility of consumption today with the marginal utility of consuption tomorrow, in this setup the inter-temporal optimality condition relates the marginal utility from consumption today with the marginal utility of the next generation and the marginal utility of having children. Therefore the number of children affects the discount factor making the interpretation of the Euler-equation less intuitive since children are, from a certain point of view, also "consumption" of the current generation (that indeed have children for having an higher utility).

For this reason the next subsections will try to shed a light on the mechanism of the model and on the effects that different parameters have on the optimal choice of children and bequests.

[28]At the individual level fixing a country and a year households with higher labour income have less children, at the aggregate data comparing country with different levels of income or a country across years we get the same result. The full discussion with data is in Jones and Schoonbrodt (2010)

C Comparative statics of the two period model

A comparative statics exercise in order to fully understand the role of different parameters in driving the results.

- $0.01 \leq \alpha \leq 0.6$: an increase in α, *ceteris paribus*, increases the discount factor and therefore the weight that the agent gives to the next generation. Therefore, when α increases the agent reduces consumption today ($C \downarrow$) and increases both bequests and children ($N \uparrow$, $B^{'} \uparrow$). Clearly also per-capita bequests increase.

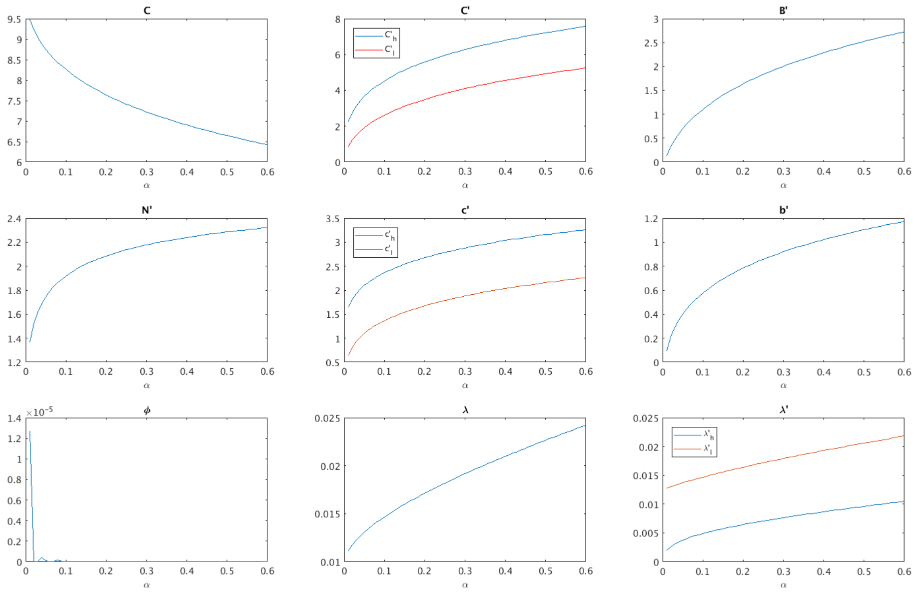

Figure 19: Comparative statics on α

- $1.5 \leq \sigma = \eta \leq 3.5$[29]: an increase in σ determines a reduction in intertemporal elasticity and a decrease in the risk aversion. In this setup children are the risky investment (their endowment depends on the state of the world tomorrow) while bequests are the safe asset, therefore an increase in σ determines a switch from children to bequests, leading to an increase in per-capita bequests.

[29] A calibration with $\eta < \sigma$ does not guarantee the existence of a solution, for this reason the comprative static on σ is conducted adjusting η in any step.

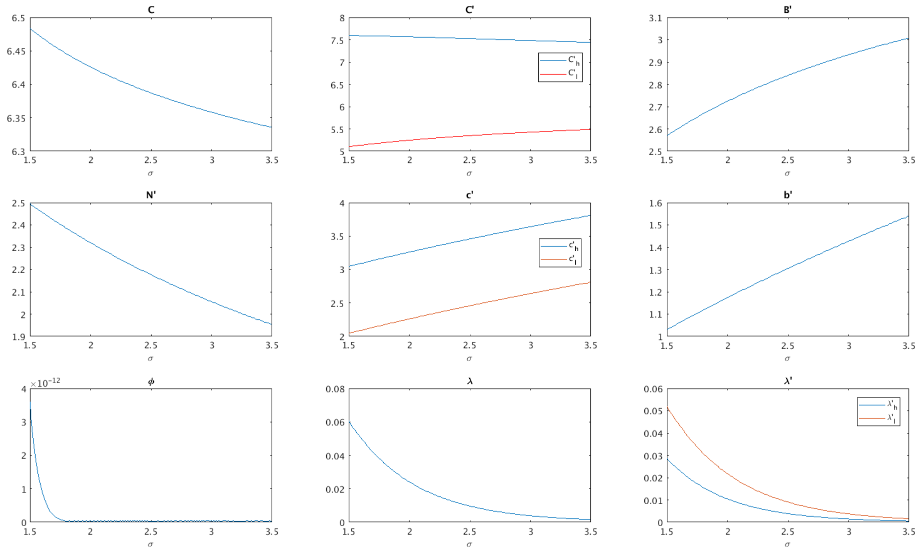

Figure 20: Comparative statics on σ

- $2 \leq \eta \leq 3.5$[30] determines the degree of concavity of the children component in the discount factor, when $\eta = \sigma$ the relative importance of the quality of children is maximal and it decreases when η increases. As a result an increase in η determines a reduction of the amount of bequests and an increase in the number of children and a decrease in consumption. Here there are two effects: on the one hand the quantity becomes more important than quality and therefore the agent increases the number of children as much as possible at the expenses of current consumption and bequests, at a certain point the increase in η determines also a decrease in the discount factor. For this reason the agent starts decreasing also children and slightly increase consumption. So the effect of η on consumption, children and bequests is not monotonic, notice that also the multiplier corresponding to the amount of bequests decreases.

- $1.1 \leq \theta \leq 2$: this affects the degree of convexity of the time cost of children. The higher the degree of convexity of the children cost function, the higher the number of children that the agents has.

[30] In this case I let $\sigma = 2$, this allow to study the separate effect of the parameter η

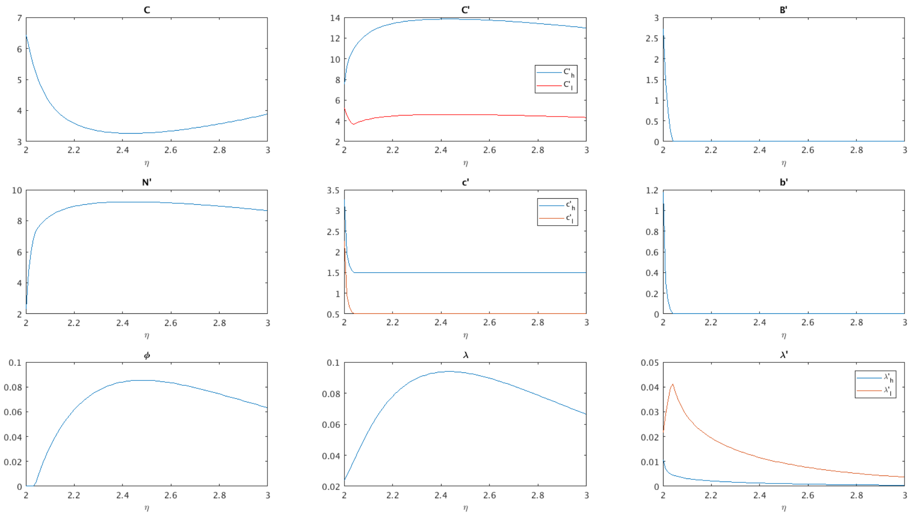

Figure 21: Comparative statics on η

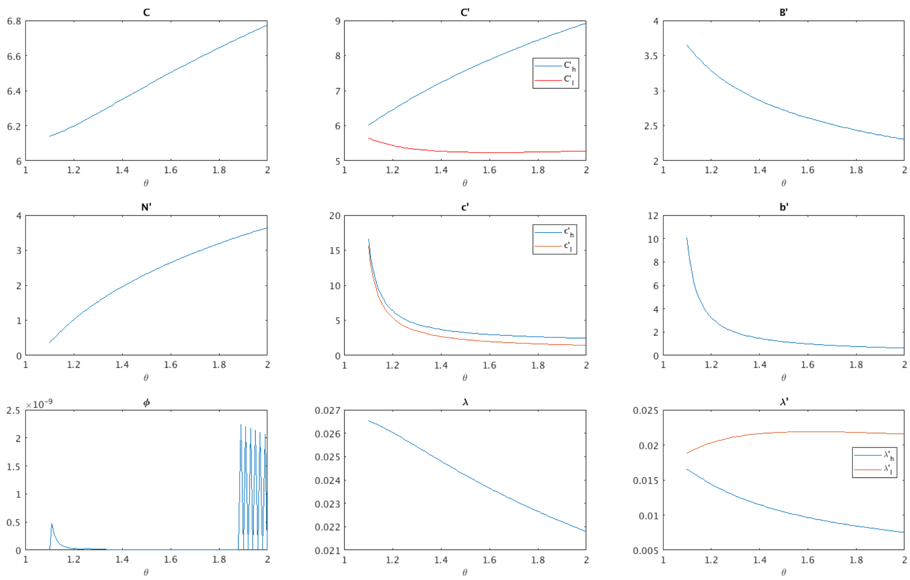

Figure 22: Comparative statics on θ

D Aggregate evidence

D.1 TFR and child mortality: historical data sources

All the historical sources used to reconstruct the TFR of the United States are listed in Table 12. For some periods it has been reconstructed using the crude birth rate (CBR) and life tables provided by the Statistical office of the US.

Years	Source of TFR	Source of CBR	Frequency
1900	Computed from CBR	Statistical office of the US	1 year
1903-1908	Chesnais (1992)	(Statistical office of the US)	5 years
1909-1912	Computed from CBR	Statistical office of the US	1 year
1913	Chesnais (1992)	(Statistical office of the US)	1 year
1914-1916	Computed from CBR	Statistical office of the US	1 year
1917-1939	Heuser	(Statistical office of the US)	1 year
1940-1949	Statistical office of the US	(Statistical office of the US)	1 year
1950-2010	HFD (2013)	(UN Pop.)	1 year

Table 12: Data sources for fertility in the United States.

Data sources for child mortality in the United States, with which the TFR is adjusted, are listed in Table 13.

Years	Source	Frequency
1900-1923	Model based on Life Expectancy	1 year
1924-1932	-	-
1933-1949	HMD	1 year
1950-2010	CME (2014)	1 year

Table 13: Data sources for child mortality in the United States.

Sources for UK data on TFR are listed in Table 14 (together with the corresponding CBR source), those for child mortality are listed in Table 15.

Years	Source of TFR	Source of CBR	Frequency
1800-1871	Computed from CBR	Wrigley and Schofield (1989)	1 year
1873-1908	Chesnais (1992)	(Chesnais (1992))	5 years
1911-1949	Festy (1979)	-	1 year
1950-2010	UN Population Division (2013)	(UN Population Division (2008))	1 year

Table 14: Data sources for fertility in United Kingdom.

Years	Source	Frequency
1800-1831	Model based on Life Expectancy	1 year
1832-1840	-	-
1841-1921	HMD England	1 year
1922-1949	HMD	1 year
1950-2010	CME (2014)	1 year

Table 15: Data sources for child mortality in United Kingdom.

D.2 Trends and cycles in fertility and in TFP

Using the trend-cycle decomposition for TFP and TFR we get four different series: TFP cycle and TFP trend, TFR cycle and TFP trend. The analysis of the correlation among the two of interests have been already discussed in Section 2. For completeness we plot all the possible combinations of the series and their correlogram. Results for the United States care plotted in Figure 23 and those for the United Kingdom are plotted in Figure 24.

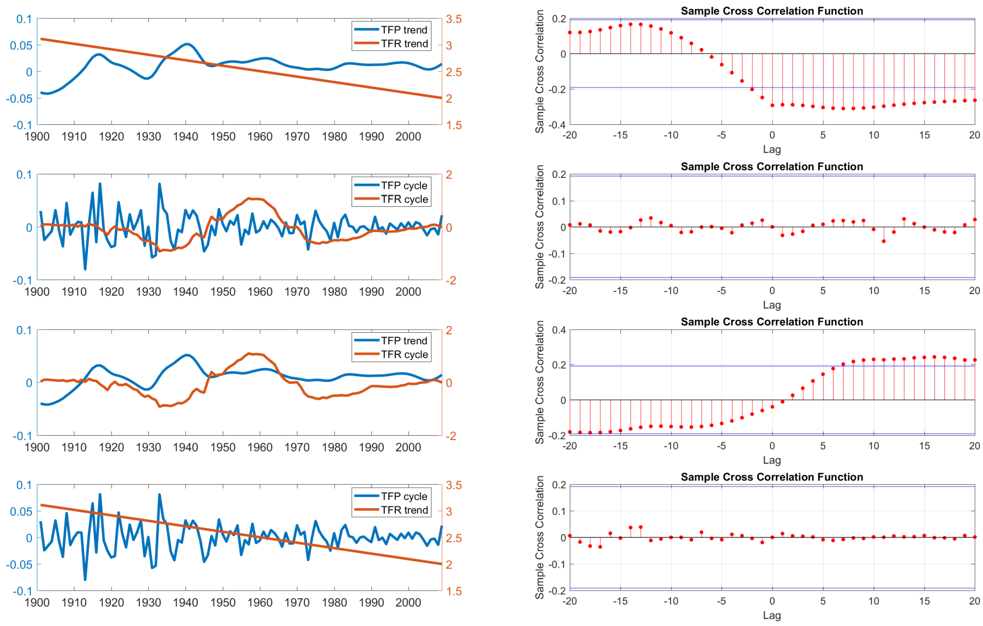

Figure 23: Fertility and TFP trend and cycle decomposition for the United States

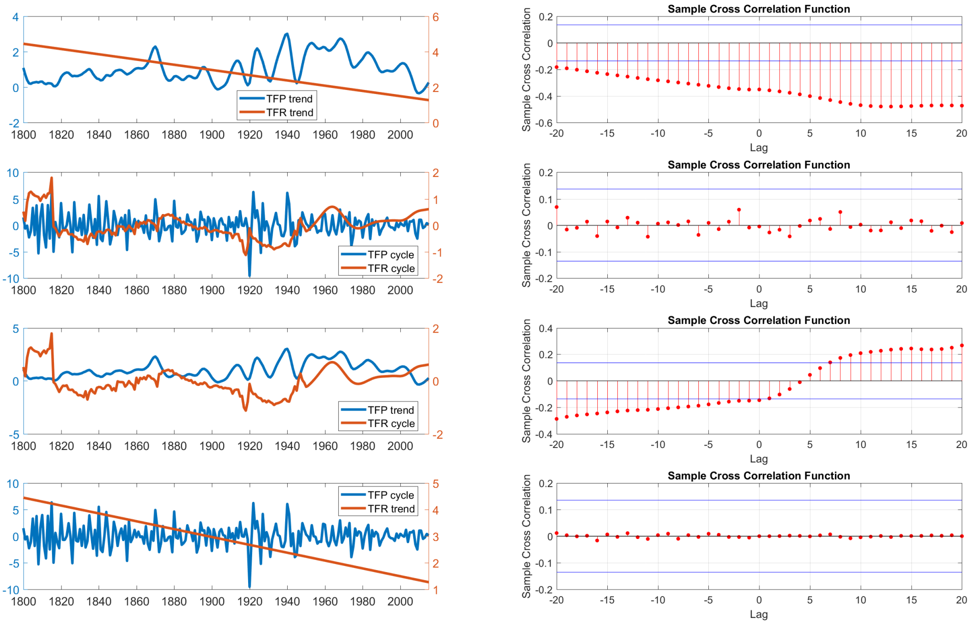

Figure 24: Fertility and TFP trend and cycle decomposition for the United Kingdom

D.3 Different time spans

As an additional check we perform the same experiment on different time spans. Firstly, in Figure 26 we plotted the TFP trend and TFR cycle for the United States after 1920 and 1950. Results are plotted in Figure 25 for the post-1920 period and in Figure 26 for the post-1950 period (that excludes the II World War period which may be problematic for several reasons). The correlation between the two series is similar to the one observed in subsection 2.3 and it only differs the lag at which they are maximally correlated: the recent period of the series is correlated at lower lags.

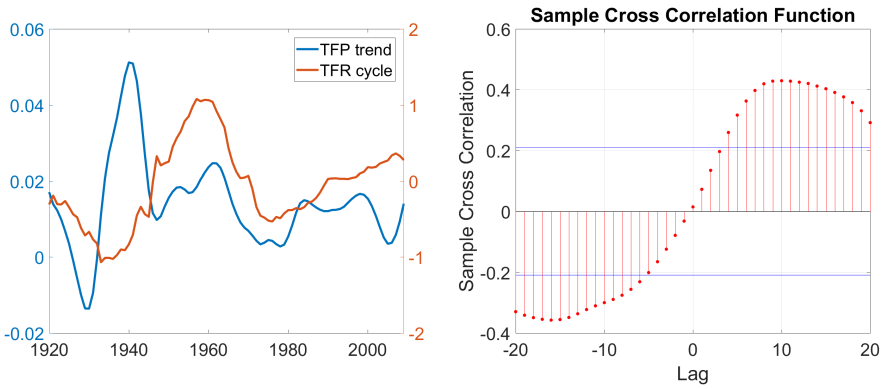

Figure 25: United States, post 1920 data

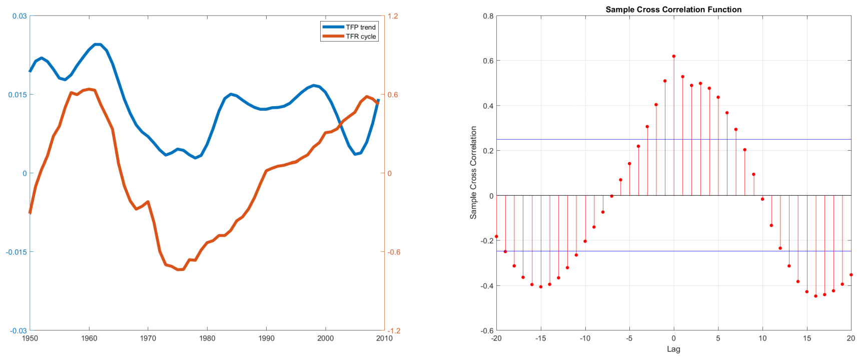

Figure 26: United States, post 1950 data

In Figure 27 we plot the series and the crosscorrelogram for UK starting from 1900, therefore discarding $19th$ century data on TFP that maybe more problematic. Results are similar to those previously found.

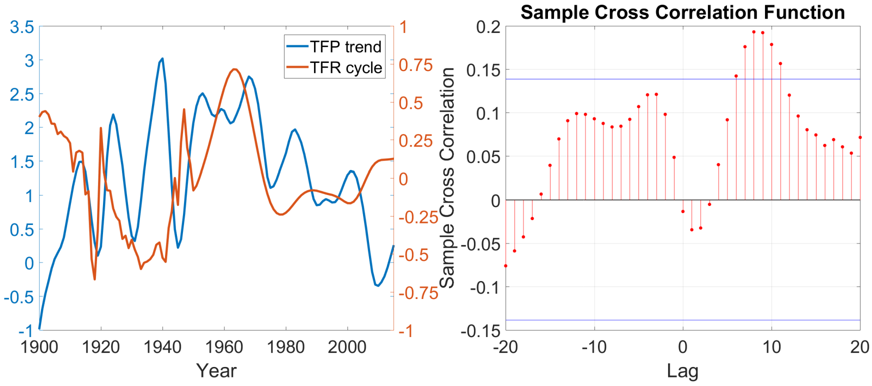

Figure 27: United Kingdom, post 1900 data

E Mathematical appendix

Proof of Proposition 1

Proof. The proof can be split in two parts. Firstly we show that the problem is defined on a convex set, then that the objective function is strictly quasi-concave.

The feasibility set (i.e. the set of points that satisfies the constraints of the problem) is:

$$\mathcal{C} = \left\{ (c, c', b, n) \in \mathbb{R}^4_+ : \; c + bn \leq \omega \left[1 - \left(\frac{n}{\overline{n}} \right)^{\theta} \right] + \tau \; \wedge \; c' \leq \omega' + bR \right\}$$

The set is convex if and only if $\forall \breve{x}, \tilde{x} \in \mathcal{C} \;\; \Rightarrow \;\; \alpha \breve{x} + (1 - \alpha) \tilde{x} \in \mathcal{C}$. Then, we have to check if a generic convex combination of two points belongs to the feasibility set. The non-negativity constraints are trivially satisfied. With respect to the other constraints, define $\hat{x} \equiv \alpha \breve{x} + (1 - \alpha) \tilde{x}$ we have to check if the following inequalities are satisfied:

$$\begin{aligned} \hat{c} + \hat{b}\hat{n} \;&\leq\; \omega \left[1 - \left(\frac{\hat{n}}{\overline{n}} \right)^{\theta} \right] + \tau \\ \hat{c}' \;&\leq\; \omega' + \hat{b}R \end{aligned}$$

Applying the definition of $\hat{x}$:

$$\begin{aligned} \alpha \breve{c} + (1 - \alpha) \tilde{c} + \left[\alpha \breve{b} + (1 - \alpha) \tilde{b} \right] \left[\alpha \breve{n} + (1 - \alpha) \tilde{n} \right] \;&\leq\; \omega \left[1 - \left(\frac{\alpha \breve{n} + (1 - \alpha) \tilde{n}}{\overline{n}} \right)^{\theta} \right] + \tau \\ \alpha \breve{c}' + (1 - \alpha) \tilde{c} \;&\leq\; \omega' + \left[\alpha \breve{b} + (1 - \alpha) \tilde{b} \right] R \end{aligned}$$

With respect to the first equation, applying the definition of $\hat{x}$:

$$\alpha \breve{c} + (1 - \alpha) \tilde{c} + \left[\alpha \breve{b} + (1 - \alpha) \tilde{b} \right] \left[\alpha \breve{n} + (1 - \alpha) \tilde{n} \right] \;\leq\; \omega \left[1 - \left(\frac{\alpha \breve{n} + (1 - \alpha) \tilde{n}}{\overline{n}} \right)^{\theta} \right] + \tau$$

Given that $\breve{x}, \tilde{x} \in \mathcal{C}$ we have:

$$\begin{aligned} \alpha \breve{c} + \alpha \breve{b} \breve{n} \;&\leq\; \alpha \omega \left[1 - \left(\frac{\breve{n}}{\overline{n}} \right)^{\theta} \right] + \alpha \tau \\ (1 - \alpha) \tilde{c} + (1 - \alpha) \tilde{b} \tilde{n} \;&\leq\; (1 - \alpha) \omega \left[1 - \left(\frac{\tilde{n}}{\overline{n}} \right)^{\theta} \right] + (1 - \alpha) \tau \end{aligned}$$

Summing side by side we get:

$$\begin{aligned}
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \alpha\omega\left[1-\left(\frac{\breve{n}}{\overline{n}}\right)^{\theta}\right]+(1-\alpha)\,\omega\left[1-\left(\frac{\tilde{n}}{\overline{n}}\right)^{\theta}\right]+\tau \\
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \omega\left\{\alpha\left[1-\left(\frac{\breve{n}}{\overline{n}}\right)^{\theta}\right]+(1-\alpha)\left[1-\left(\frac{\tilde{n}}{\overline{n}}\right)^{\theta}\right]\right\}+\tau \\
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \omega\left\{1-\left[\frac{\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}}{(\overline{n})^{\theta}}\right]\right\}+\tau \\
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \omega\left\{1-\left[\frac{\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}}{(\overline{n})^{\theta}}\right]\right\}+\tau \\
\hat{c}+\hat{n} &\leq \omega\left\{1-\left[\frac{\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}}{(\overline{n})^{\theta}}\right]\right\}+\tau \\
\hat{c}+\hat{n} &\leq \omega-\frac{\omega}{(\overline{n})^{\theta}}\left[\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}\right]+\tau
\end{aligned}$$

The using the fact that $\theta > 1$:

$$\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta} \geq [\alpha\breve{n}+(1-\alpha)\,\tilde{n}]^{\theta}\,\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta} \geq [\hat{n}]^{\theta}$$

Then:

$$\begin{aligned}
\omega-\frac{\omega}{(\overline{n})^{\theta}}\left[\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}\right]+\tau &\leq \omega-\frac{\omega}{(\overline{n})^{\theta}}\,[\hat{n}]^{\theta}+\tau \\
\omega-\frac{\omega}{(\overline{n})^{\theta}}\left[\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}\right]+\tau &\leq \omega\left[1-\left(\frac{\hat{n}^{\theta}}{\overline{n}^{\theta}}\right)^{\theta}\right]+\tau
\end{aligned}$$

And then:

$$\hat{c}+\hat{n} \leq \omega\left[1-\left(\frac{\hat{n}^{\theta}}{\overline{n}^{\theta}}\right)^{\theta}\right]+\tau$$

With respect to the second equation, applying the definition of $\hat{x}$ and rearranging:

$$\begin{aligned}
\alpha\breve{c}'+(1-\alpha)\,\tilde{c} &\leq \omega'+\left[\alpha\breve{b}+(1-\alpha)\,\tilde{b}\right]R \\
\alpha\breve{c}'+(1-\alpha)\,\tilde{c} &\leq \alpha\omega'+(1-\alpha)\,\omega'+\alpha\breve{b}R+(1-\alpha)\,\tilde{b}R
\end{aligned}$$

That is satisfied since $\breve{x}, \tilde{x} \in \mathcal{C}$. Secondly, we have to prove quasi-concavity of the objective function. Fix a generic level t. Take two points from the upper contour set $\forall \breve{x}, \tilde{x} \in UCS(t)$, then we want to show that $f(\overline{x}) > t$, where $\overline{x} = \alpha \breve{x} + (1-\alpha)\tilde{x}$. We need to show:

$$\frac{(\beta \breve{c} + (1-\beta)\tilde{c})^{1-\sigma}}{1-\sigma} + \alpha(\beta \breve{n} + (1-\beta)\tilde{n})^{1-\eta}\left[\frac{(\beta \breve{c}' + (1-\beta)\tilde{c}')^{1-\sigma}}{1-\sigma}\right] > t$$

Using the hypothesis $\sigma > 1$ we can easily show:

$$\frac{(\beta \breve{c} + (1-\beta)\tilde{c})^{1-\sigma}}{1-\sigma} > \beta\frac{(\breve{c}+)^{1-\sigma}}{1-\sigma} + (1-\beta)\frac{(\tilde{c}+)^{1-\sigma}}{1-\sigma}$$

Using $\eta \geq \sigma > 1$ the end of the proof is trivial. □

Proof of Lemma 2

Proof. The utility function of the first generation is:

$$\overline{U}(c,b,n) = \frac{(c)^{1-\sigma}}{1-\sigma} + \alpha(n)^{1-\eta}\frac{\left(c'\right)^{1-\sigma}}{1-\sigma}$$

The partial derivative with respect to today consumption is:

$$\frac{\partial \overline{U}}{\partial c} = c^{-\sigma}$$

The partial derivative with respect to children is:

$$\frac{\partial \overline{U}}{\partial n} = \alpha(1-\eta)(n)^{-\eta}\frac{\left(c'\right)^{1-\sigma}}{1-\sigma}$$

And finally the partial derivative with respect to offsprings consumption is:

$$\frac{\partial \overline{U}}{\partial c'} = \alpha(n)^{1-\eta}\left(c'\right)^{-\sigma}$$

On the other hand the the intertemporal budget constraint can be computed plugging the second onder period budget constraint of each children inside the first period budget constraint:

$$\begin{aligned} c+n\left[\frac{c_s^{'}-\omega^{'}}{R}\right] &\leq \omega\left[1-\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\right]+\tau \\ c+\frac{n}{R}c_s^{'}-n\frac{\omega^{'}}{R} &\leq \omega\left[1-\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\right]+\tau \\ c+\frac{n}{R}c_s^{'}-n\frac{\omega^{'}}{R}+\omega\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta} &\leq \omega+\tau \end{aligned}$$

As a result, the relative price between $c^{'}$ and c is $\frac{n}{R}$. Then, the optimality condition between consumption today and consumption tomorrow can be derived as follow:

$$\frac{\frac{\partial \overline{U}}{\partial c^{'}}}{\frac{\partial \overline{U}}{\partial c}}=\frac{\alpha\left(n\right)^{1-\eta}\left(c^{'}\right)^{-\sigma}}{c^{-\sigma}}=\frac{n}{R}=\frac{p_{c^{'}}}{p_c}$$

Rearranging it we obtain:

$$\begin{aligned} \frac{\left(c^{'}\right)^{-\sigma}}{c^{-\sigma}} &= \frac{n}{\alpha R}\left(n\right)^{\eta-1} \\ c &= \left[\frac{1}{\alpha R}n^{\eta}\right]^{\frac{1}{\sigma}}c^{'} \end{aligned}$$

And using the second period budget constraint:

$$c=\left[\frac{1}{\alpha R}n^{\eta}\right]^{\frac{1}{\sigma}}\left(\omega^{'}+bR\right)$$

Applying the logarithm we have:

$$\begin{aligned} \log c &= \frac{1}{\sigma}\log\left[\frac{1}{\alpha R}n^{\eta}\right]+\log\left(\omega^{'}+bR\right) \\ \log c &= -\frac{1}{\sigma}\log\alpha-\frac{1}{\sigma}\log R+\frac{\eta}{\sigma}\log n+\log\left(\omega^{'}+bR\right) \end{aligned}$$

And therefore applying the partial derivative we get:

$$\frac{\partial c}{c}=\frac{\eta}{\sigma}\frac{\partial n}{n}+\frac{\omega^{'}}{\omega^{'}+bR}\frac{\partial \omega^{'}}{\omega^{'}}+\frac{bR}{\omega^{'}+bR}\frac{\partial b}{b}$$

On the other hand, looking at the ratio between marginal utilities and relative prices of c and n we have:

$$\frac{\frac{\partial \overline{U}}{\partial n}}{\frac{\partial \overline{U}}{\partial c}} = \frac{\alpha\left(1-\eta\right)\left(n\right)^{-\eta}\frac{\left(c^{'}\right)^{1-\sigma}}{1-\sigma}}{c^{-\sigma}} = b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n} = \frac{p_n}{p_c}$$

Rearranging it we get:

$$\begin{aligned}
\alpha\left(\frac{1-\eta}{1-\sigma}\right)\left(n\right)^{-\eta}\left(\omega^{'}+bR\right)\frac{\left(c^{'}\right)^{-\sigma}}{c^{-\sigma}} &= b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n}\\
\alpha\left(\frac{1-\eta}{1-\sigma}\right)\left(n\right)^{-\eta}\left(\omega^{'}+bR\right)\frac{1}{\alpha R}\left(n\right)^{\eta} &= b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n}\\
\left(\frac{1-\eta}{1-\sigma}\right)\left(\omega^{'}+bR\right)\frac{1}{R} &= b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n}\\
\left(\frac{1-\eta}{1-\sigma}\right)\left(\omega^{'}+bR\right)\frac{1}{R} &= b+\omega\theta\left(\overline{N}\right)^{-\theta}\left(n\right)^{\theta-1}
\end{aligned}$$

Applying the logarithm on both sides, using $\Phi'\left(n\right)=\theta\left(\overline{N}\right)^{-\theta}\left(n\right)^{\theta-1}$ to simplify the notation and taking the partial derivatives we have:

$$\begin{aligned}
\log\left[b+\omega\Phi'\left(n\right)\right] &= \log\left(\frac{1-\eta}{1-\sigma}\right)+\log\left(\omega^{'}+bR\right)-\log R\\
\frac{\omega\Phi'\left(n\right)}{b+\omega\Phi'\left(n\right)}\left(\theta-1\right)\frac{\partial n}{n} &= \frac{\partial\omega^{'}}{\omega'+bR}\\
\frac{\partial n}{n} &= \frac{1}{\left(\theta-1\right)}\frac{\partial\omega^{'}}{\omega^{'}}\left[\frac{b+\omega\Phi'\left(n\right)}{\omega\Phi'\left(n\right)}\frac{\omega^{'}}{\omega'+bR}\right]\\
\frac{\partial n}{n} &= \frac{1}{\left(\theta-1\right)}\frac{\partial\omega^{'}}{\omega^{'}}\Gamma\left(b,n\right)\\
\frac{\frac{\partial n}{n}}{\frac{\partial\omega^{'}}{\omega^{'}}} &= \frac{1}{\left(\theta-1\right)}\Gamma\left(b,n\right)
\end{aligned}$$

$\square$

Proof of Proposition 3

Proof. Given that $\frac{\partial b}{\partial \tau} > 0$ we can derive Γ by b and then look at the sign of the derivative. Then:

$$\begin{aligned}
\frac{\partial}{\partial b}\Gamma(b,n) &= \frac{1}{\omega\Phi'(n)}\frac{\omega'}{\omega'+bR} - \frac{b+\omega\Phi'(n)}{\omega\Phi'(n)}\frac{R}{(\omega'+bR)^2} \\
&= \frac{\omega'\left(\omega'+bR\right) - R\left(b+\omega\Phi'(n)\right)}{\omega\Phi'(n)\left(\omega'+bR\right)^2}
\end{aligned}$$

Given that $\omega\Phi'(n)\left(\omega'+bR\right)^2 > 0$ we have that $\frac{\partial}{\partial b}\Gamma(b,n) > 0$ if and only if:

$$\begin{aligned}
\omega'\left(\omega'+bR\right) &> R\left(b+\omega\Phi'(n)\right) \\
\omega'^2 + \omega' bR - R\left(b+\omega\Phi'(n)\right) &> 0
\end{aligned}$$

That can be seen decomposed as:

$$\begin{aligned}
&\left(\omega' - \frac{-bR+\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}\right)\times \\
&\left(\omega' - \frac{-bR-\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}\right) > 0
\end{aligned}$$

That has solution for $\omega' < \frac{-bR-\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}$ and $\omega' > \frac{-bR+\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}$. The first solution has no economic meaning, while the second one is:

$$\begin{aligned}
\omega' &> \frac{-bR+\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2} \\
\omega' &> \frac{-bR+bR\sqrt{\frac{1+4R\left(b+\omega\Phi'(n)\right)}{(bR)^2}}}{2} \\
\omega' &> \frac{bR\left[\sqrt{1+\frac{4R\left(b+\omega\Phi'(n)\right)}{(bR)^2}}-1\right]}{2}
\end{aligned}$$

It exists a threshold of ω' above which $\frac{\partial}{\partial b}\Gamma(b,n) > 0$, and therefore:

$$\begin{aligned}
\frac{\partial n}{n} &= \frac{1}{(\theta-1)}\frac{\partial \omega'}{\omega'}\Gamma_b(b,n)\,\partial b > 0 \\
\frac{\partial n}{n}\frac{b}{\partial b} &= \frac{1}{(\theta-1)}\frac{\partial \omega'}{\omega'}\Gamma_b(b,n)\,b > 0 \\
\frac{\partial^2 n}{\partial \omega' \partial b} &= \frac{n}{(\theta-1)\,\omega'}\Gamma_b(b,n) > 0
\end{aligned}$$

And therefore: $\frac{\partial n}{\partial \omega' \partial b} > 0$. And given that $\frac{\partial b}{\partial \tau} \geq 0$ by construction[31], i.e. the amount of per-capita bequests left is increasing in the level of wealth and $\frac{\partial b}{\partial \tau} > 0$ when the non-negativity constraint is not binding we have $\frac{\partial^2 n}{\partial \omega' \partial \tau} > 0$. $\square$

[31] Look at appendix B for a discussion on the possible calibrations of Barro-Becker model and its implications.

F Micro-evidence

F.1 Wealth imputation in the PSID

Given that before 1999 wealth data are available only every 5 years we decided to impute them in order to extend the number of waves in which the analysis can be conducted. We use the following rule:

$$\tilde{W}_t = \begin{cases} W_t & if\ W_t \neq \cdot \\ W_s & where\ s \in \arg\min_s |t-s|\,, W_s \neq \cdot, |t-s| \leq 5 \\ \cdot & elsewhere \end{cases}$$

That is:

1. Maximum 5 years of lag imputation (i.e. we take wealth data from not 5 years ahead as a maximum)

2. Choose the closer data among the one available

How data are imputed year-by-year can be seen in Table 16.

Wave	Wealth data available	Imputed from	Wave	Wealth data available	Imputed from	Wave	Wealth data available
1979	No	1984	1989	Yes		2001	Yes
1980	No	1984	1990	No	1989	2003	Yes
1981	No	1984	1991	No	1989	2005	Yes
1982	No	1984	1992	No	1994	2006	Yes
1983	No	1984	1993	No	1994	2007	Yes
1984	Yes		1994	Yes		2009	Yes
1985	No	1984	1995	No	1994	2011	Yes
1986	No	1984	1996	No	1994	2013	Yes
1987	No	1988	1997	No	1999		
1988	Yes		1999	Yes			

Table 16: Available data on wealth in PSID wave and imputations