

# Solving the Index-Number Problem in a Historical Perspective

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## Abstract

The problem of price and quantity indexes is a typical problem of aggregation. Even when the aggregation conditions are not rejected on the basis of the observed data, there still remains a certain degree of uncertainty regarding the point estimate of the index number. Following the truly constructive Afriat's method, we can reinterpret this uncertainty by reverting the problem and asking: (i) whether the available data can be rationalized and aggregated with well-behaved "true" index functions *independently* from how they have been actually determined, (ii) if yes, what are the upper and lower numerical values of all the alternative "true" index functions aggregating the data? (iii) if the answer is no, then either the data are not generated by a rational behaviour (and in this case a correction for inefficiency can be made), or else the data are generated by a rational behaviour within a different or wider set of determinants to be considered in a different or extended accounting framework. Since any "true" price index function satisfies all Fisher's tests by construction, including the transitivity requirement at least locally, also the reconstructed upper and lower numerical values of the set of all admissible "true" price indexes must respect those tests. This solution is valid irrespective of the existence of such non-observable objects as for example utility and production functions governing the observed behaviour. The purpose of this paper is to present a full solution of the index-number problem in the perspective of the theoretical developments occurred during the last century. The proposed solution is built on Afriat's method and consist in defining for the first time the "true" bounds of the set of possible aggregating indexes which are fully consistent with all Fisher's tests, including transitivity, and invariance to the change of bases. An empirical application on Irving Fisher's data illustrates the method.

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