Household wealth: low-yielding and poorly structured?

On the development of the real returns and the efficiency of household portfolios in the euro area

In this paper, we present a newly generated data set which allows us to address two recent debates on the development of household wealth in the euro area which have been sparked by the low interest environment. The first debate refers to the development of real yields on household wealth from 2000 to 2018, whereas the second debate deals with mean-variance efficiency of household portfolios. Contrary to widespread believe, we find that yields on total wealth, which were largely dominated by non-financial assets' yields, were mostly positive, although they exhibit a declining trend. Moreover, on average, overall real yields were significantly lower after 2008. Referring to portfolio efficiency, we find that current portfolios seem to be comparatively close to mean-variance efficiency. If households were to optimize their portfolios despite limited room for improvement, holdings of equity and investment fund shares should be reduced, contradicting common recommendations of financial advisors.

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1 Introduction

Since the outbreak of the financial crisis in 2008, with one brief exception in 2011, there has been only one direction for key interest rates in the euro zone: downwards. And there are currently no signs of a “normalisation” of key interest rates to pre-crisis levels. On the contrary: The turnaround in key interest rates has been postponed again and again. To combat the Covid-19 pandemic, central banks around the world took even more measures in 2020 to ensure the supply of liquidity and favourable financing conditions. Among others, the Eurosystem introduced an additional net asset purchase programme (Pandemic Emergency Purchase Programme (PEPP)) with a volume of at least €1,850 billion (European Central Bank, 2020a, 2020b, 2020c). Hence, it is very likely that official interest rates will remain low for the time being, also because any increases in key interest rates have been tied to numerous conditions.

Against that background, a heated debate has been sparked on the effects of the low interest rate environment on household wealth in the euro zone, where the arguments run in two different directions. First, there is great dispute respecting the impact on the development of the overall real rate of return on household wealth (hereinafter referred to as the "level debate"). While some authors speak of an “expropriation of the saver” due to low or even negative real interest rates on deposits (Heise, 2016), others emphasize the wealth- and return-increasing effects of rising asset prices for marketable assets (Bindseil et al., 2015). While this controversial debate has been initially focused on German households’ yields, it has meanwhile spread as well to other euro area countries. Second, it has been claimed that households’ portfolios are inefficiently structured in terms of risk and return, as they contain a very high proportion of safe and low-yielding assets (hereinafter referred to as the "efficiency debate"). Although the efficiency debate has been around for a long time, it has received a renewed boost from the low-interest phase, as safe assets’ returns have declined mostly further since 2008. As a result, many financial advisors have frequently called for an increase in the share of higher yielding assets like (foreign) securities in order to improve the overall household portfolio performance and efficiency (Jacobs et al., 2014).

This paper addresses both debates. For this purpose, it develops and analyses a dataset on the real rates of return on total non-financial and financial wealth of households in the four large euro area countries and in the euro area as a whole for the period 2000 to 2018. Based on this newly generated dataset, which is – to the best of our knowledge – unique to date, we deal with three core questions. First, how did real returns on total, financial and non-financial assets develop over time from 2000 to 2018 (level debate)? Second, does the evolution of average real returns in the “pre-crisis phase” (2000-2007) differ from that in the “crisis and low interest phase” (2008-2018) and, if so, how (level debate)? And third, can current portfolio allocations in 2018 be considered as efficient in terms of Markowitz’s mean-variance approach (Markowitz, 1952, 1959) and in comparison to simple common heuristic portfolio allocation strategies, or should portfolios be adjusted (efficiency debate)?

As to research question one, the results show that real returns on total household wealth were mostly positive in all jurisdictions, but followed a declining trend, and were driven mainly by real returns on non-financial assets. Referring to research question two, average real returns on total wealth were significantly lower in the crisis and low interest phase in all countries except for Germany; the reductions were mainly caused by drops in real yields (price effect), while portfolio shifts (volume effect) played a subordinate role. The negative price effects were dominated by diminishing real returns on non-financial wealth which, however, remained the mainstay of real returns on total wealth. Addressing research question three, we find that the potentials for increasing real returns and/or decreasing risk by portfolio restructuring are very limited, as household portfolios seem to be close to their mean-variance optima. If households still want to optimize their portfolios according to mean-variance framework, they should mostly follow a strategy which is against the common recommendation of financial advisors, namely to reduce the share of equities and investment fund shares. Furthermore, by and large, the potential for increasing portfolio performance when following heuristic rules as well is very limited.

The paper is organised as follows. Section 2 reviews the literature, while section 3 describes the approach used to compute the aforementioned dataset. Section 4 ties in with the level debate by presenting and analysing the data on real returns on household wealth, thereby addressing questions one and two above. Finally, section 5 addresses the efficiency debate outlined above and thus the third question. To this end, it uses the data on the real returns and applies a classical Markowitz- and a heuristic portfolio analysis to the aggregated household portfolios in all countries under consideration. Section 6 concludes.

2 Literature review and research contribution

With respect to the level debate, Deutsche Bundesbank (2015) was the first to order and weigh the arguments above by computing the real yield German households have earned on their financial wealth since the introduction of the euro. The calculation, which is described in more detail in Annuß and Rupprecht (2016), is regularly updated on a quarterly basis and published on the Bundesbank's website. In order to be able to study the topic from a broader euro area perspective, Radke and Rupprecht (2018, 2019) expanded the Bundesbank's approach by computing real yields on households' financial wealth in the major euro area countries (Germany, France, Italy, Spain), and for the euro area as a whole from 2000 to 2017. This extension required a computational approach which differed from the Bundesbank's calculations, mainly due to data limitations and access restrictions in some of the countries. Therefore, Radke and Rupprecht (2018, 2019) developed an alternative approach. Although their approach differs from the Bundesbank's pioneering work, the Bundesbank's data for Germany could be reproduced quite well, both with regard to the dynamics and levels.[1] This successful quality and consistency check was used as evidence for the appropriateness of the modified approach and applied to all other large euro area countries and the euro area as a whole. Furthermore, in 2019, the central bank of Austria published for the first time data on Austrian households' real returns on financial assets (Oesterreichische

[1] See, in particular, figure 1 in Radke and Rupprecht (2018, p. 108), for a comparison with the Bundesbank data.

Nationalbank, 2019). The methodology is fairly comparable with the approaches mentioned above, and so are the results. To the best of our knowledge, other approaches for the compilation of real yields on households' financial assets currently do not exist.[2]

A major disadvantage of all approaches considered above has so far been the neglect of non-financial assets. The main reason for this was the lack of availability of comparable data both in terms of data on stocks and their respective returns. This lack is problematic, however, as non-financial assets typically account for a large part of household wealth (60-70%). A profound assessment of the returns that households achieve with all their assets in times of low interest rates therefore requires the consideration of non-financial assets. This is also true because the prices for dwellings, a key component of non-financial assets' return, have developed very differently and at times very dynamically in the respective countries since the founding of monetary union.

The efficiency debate has a long history in financial economics. It has been often argued that households' financial portfolios are poorly structured, as they contain a very high proportion of safe assets, very low or even zero shares in stocks, and that if stock investments are present, households are prone to common investment mistakes like being exposed highly to specific risk, or to be subject to mania expectations (Jacobs et al., 2014, Campbell, 2006, Goetzmann and Kumar, 2008). Rupprecht (2020) has shown that households in the countries considered here do indeed have a high preference for safe assets such as bank deposits and insurance products. This preference hardly changes over time, even with varying interest rates. Reasons for this include low levels in financial literacy (Stolper and Walter, 2017, Deuflhard et al., 2019), negative experiences with previous exposures to risky assets, such as in the form of (sometimes sudden) price losses (Ampudia and Ehrmann, 2017), as well as socio-economic factors; Badarinza et al. (2016) as well as Gomes et al. (2020) provide recent surveys. In the same vein, Rupprecht (2020) also documented a significant preference for investments in domestic assets (e.g. shares of domestic corporations), confirming the well-known "home bias" phenomenon first described by French and Poterba (1991). As a result, many financial advisors have frequently called in the past for an increase in the share of higher yielding assets like (foreign) securities in order to improve the overall household portfolio performance in terms of an optimal allocation between risk and return (Jacobs et al., 2014). In the wake of the low interest environment, however, the claim that portfolios are inefficient and should be restructured has gained an additional boost, as the yields of safe assets have mostly declined further, making portfolio performance look even worse.

In this paper, we address the above questions on both debates. With respect to the level debate, we provide a new methodology (section 3) that allowed us to create a new and – to the best of our knowledge – unique dataset on the real returns on total non-financial and financial wealth of households

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[2] Andreasch, Radke and Rupprecht (2020) combined macroeconomic information on the real returns to households' financial assets in Germany and Austria with microeconomic information on the portfolio composition of households with different wealth levels. Contrary to popular belief, they find that richer households are more affected by the low interest rate environment than poorer households, at least in relative terms. In this context, the trend is generally more pronounced in Austria than in Germany, albeit at a fundamentally lower level of returns.

in the four large euro area countries and in the euro area as a whole for the period 2000 to 2018. This data set allows us to overcome the aforementioned data problems by providing precise information on the evolution of households' real total assets returns (section 4). With respect to the efficiency debate, and based on the dataset on real returns, we test the validity of the above arguments by comparing households' portfolio performance in 2018 with efficient Markowitz portfolios and with portfolios constructed according to simple heuristic rules (section 5). Our analysis is unprecedented in three ways. First, portfolio efficiency tests are typically conducted for individual investors and individual assets. Our approach, however, tests aggregated portfolios of complete household sectors and includes all assets held by households. Second, while usual efficiency tests are mostly restricted to traded financial instruments, our approach additionally includes real estate and non-traded financial instruments like insurance and pension fund products. Third, to the best of our knowledge, no such analysis has yet been carried out for households in the major euro area countries and for the euro area household sector as a whole.

3 Computation method and data

The real return on households' total assets was calculated for each country in a multi-stage process. First, the nominal return on each individual asset *i* in country *j* at time *t*, $\theta_{ij,t}$, was determined. Then, these country-specific individual nominal asset returns were transformed into real ex-post returns at time *t*, $\theta^r_{ij,t}$, by using country-specific realized or ex-post inflation rates at time *t*, $\pi_{j,t}$, in accordance with the "exact" calculation method based on the relationship

(1)
$$\theta^r_{ij,t} = \frac{1+\theta_{ij,t}}{1+\pi_{j,t}} - 1 .$$

In the final step, the country-specific real yields of the individual assets were weighted with their respective national portfolio shares at time *t*, $\alpha_{ij,t}$, and added up to the real total portfolio yield for each country *j* at time *t*, $\Theta^r_{j,t}$,according to the relationship

(2)
$$\Theta^r_{j,t} = \sum_i \alpha_{ij,t} \cdot \theta^r_{ij,t}$$

where $\sum_i \alpha_{ij} = 1$ for all *t* and each country *j*.

The country-specific portfolio weights of the individual assets at time *t*, $\alpha_{ij,t}$, were calculated as the ratio of the market value of each asset *i* in country *j* at time *t*, $MV_{ij,t}$, to the market value of the total portfolio (sum of all national assets) of country *j* at time *t*, $MV^P_{j,t}$. Formally,

(3) $$\alpha_{ij,t} = \frac{MV_{ij,t}}{MV_{j,t}^{P}} = \frac{MV_{ij,t}}{\sum_i MV_{ij,t}} .$$

[Place Figure 1 here]

The countries considered were France, Germany, Italy, Spain and the euro area as a whole $(j = 1,\ldots,5)$. Regarding the types of assets covered, the present computation approach follows the definitions of the European System of Accounts 2010 (ESA 2010)[3] (European Commission, 2013, European Central Bank, 2014). Figure 1 lists the complete set of assets being constituents of an aggregated household sector balance sheet according to the definitions of ESA 2010. The present computational approach, however, could not take into account all ESA 2010 assets and their respective yields for the reasons mentioned below. Hence, all assets and their respective yields highlighted in italics in figure 1 were omitted in the present computation.

With regard to non-financial assets, stock data published by the OECD and the ECB were available in a consistent manner only for (tangible) fixed assets and land for Germany, France, Italy and the euro area as a whole. Therefore, data for Spain had to be estimated mainly on the basis of accumulated fixed capital information (net) in Spain; data on land was derived from ECB information on the euro area. The stocks of fixed assets officially published comprise the net capital stock valued at written-down replacement costs, i.e. at current prices purchasers would have to pay less accumulated depreciation on those goods. This valuation method, as set out in ESA 2010, serves as a measure of the current market value of most fixed assets.[4]

Regarding the computation of non-financial assets' real yields, we focussed on the yield of dwellings including land. These two asset types account for the vast majority of the non-financial assets held by households (shares in total non-financial wealth between 83% and 99%), and since there were hardly any reliable and internationally comparable yield data on the other components (other buildings and structures, machinery and equipment, and cultivated biological resources), it was assumed that the real yields on those components were identical to the real yield on housing wealth (sum of dwellings and land).

Non-financial assets' real yields were compiled in a two-stage process. First, nominal yields on housing wealth were computed as the sum of nominal percentage real estate price changes and nominal rental

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[3] ESA 2010 is the European counterpart of the System of National Accounts 2008 (SNA 2008) (United Nations, 2010) which is the worldwide accounting standard for the compilation of national accounts data.

[4] It must be noted, however, that the net capital stock's current replacement costs, as reported in the statistics and used as weights in the present compilation, can be quite different from the actual market prices for those goods. The ratio between the net capital stock's current market price *MP* and its replacement cost *RP* defines Tobin's q as $q = MP / RP$, determining the desired rate of growth of the capital stock, i.e. net investment (Tobin 1969, 1977, Tobin and Brainard 1968, 1977). If $q > 1$, net investment is positive as buying new capital goods at replacement costs is cheaper than buying already existing capital goods in markets for tradable capital goods (as, for example, at the stock exchange). If $q < 1$, net investment is negative as buying new capital goods at replacement costs is more expensive than buying already existing capital goods in markets for tradable capital goods.

yields[5]. Data on nominal house price changes and changes in rental yields were taken from the OECD. Absolute values of the national nominal rental yields for specific years, which were updated by using the OECD series on changes in rental yields, were drawn from private data providers. Second, the nominal real estate yields were deflated by national inflation rates (CPI based) published by the OECD, and thereby transformed in real ex post yields on non-financial assets as defined by equation (1).

Data on the stocks of financial assets were taken from national and euro area financial accounts statistics published by the respective national central banks, Eurostat, and the ECB. As outlined in Figure 1, some financial asset categories were not considered. First, as households do not hold monetary gold and special drawing rights, this asset class was neglected. Second, since only households in France and Italy hold tiny amounts of loans (around €10 bn or 0.2% (France) and 0.3% (Italy) of total financial assets each on average), this asset category was not considered.[6] Third, though stock data on other equity, financial derivatives and employee stock options, and other accounts receivable/payable were available, there were no reliable sources which could have allowed a compilation of their respective real returns. Also, there is anecdotal evidence that the calculation of these assets in some cases suffers from a lack of adequate data, which is why the information on these components is less reliable compared to the other financial assets (Rupprecht, 2020). Since these asset types contribute less than 8% to the overall portfolio of financial assets, we therefore excluded them from the computation. All other asset categories illustrated in Figure 1 were included and valued with their current market prices.

Regarding the computation of financial assets' nominal yields, the approach used here follows the method outlined in Radke and Rupprecht (2018). In brief: Currency was assigned a nominal yield of zero, nominal deposit yields were derived from euro area interest rate statistics published by the ECB (broken down by maturity), and nominal returns on securities (i.e. on debt securities, equities and investment fund shares) were computed on the basis of a variety of performance or total return indices published by private data providers. In this context, bond and equity returns were also broken down by issuer sector and country of issue. Nominal yields on claims against insurance companies and pension funds were computed by using accounting data of stock market listed insurance companies in the respective countries published by private data providers.[7] In a final step, all nominal yields were converted into real ex post yields as described in equation (1).

In summary, the asset categories *i* covered in the current compilation approach include six types: (1) non-financial assets (fixed assets and land), (2) currency and deposits, securities (i.e. (3) debt securities,

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[5] The nominal rental yield is defined as the rent-to-price ratio, i.e. by the ratio of rental prices to nominal house prices, and serves as an indicator of the profitability of house ownership. According to OECD statistics, the rental yield includes actual rents for rented housing, imputed rents for owner-occupied housing, and maintenance and repair costs of the respective dwellings.

[6] In the euro area as a whole, the total amount of loans stood as well at a negligible amount of €75 bn or 0.3% of total financial assets on average.

[7] Though international organisations like the OECD publish data on the returns of holdings of claims against insurers and pension funds, the data are only available for specific years and specific countries, and suffer from limited international comparability. Therefore, a new, consistent, and internationally comparable computation method had to be applied.

(4) equity holdings in the form of listed and unlisted shares, and (5) investment fund shares), as well as (6) claims against insurances (insurance, pension and standardised guarantee schemes).

4 Reviewing the "level debate" - developments of real yields on household wealth from 2000 to 2018

4.1 Real returns on total, financial and non-financial assets

Addressing the first research question (how did real returns on total, financial and non-financial assets develop over time from 2000 to 2018?), we first look at the entire period of real returns on total assets (Figure 2). It is striking that these were predominantly positive in all countries, despite very different dynamics in some cases. Only in the crisis year of 2008 real returns turned negative – and even then, Germany saw positive returns. Spain and Italy also achieved consistently positive returns, with the exception of the period 2010 to 2013, which marked the peak of the euro crisis. This result is also reflected in positive average total returns for the entire period (Table 1), although levels did diverge. Spain and France, with average yields of 5.68% and 4.61% respectively, were above, while Germany (3.34%) and Italy (2.74%) were below the euro area average of 3.73%. The differences in levels were also accompanied by discrepancies in volatility (an approximation of risk), although – with the exception of Italy – average returns and risk were positively correlated throughout. Behind this initially positive overall picture, however, there has been a tendency for real returns on total assets to fall; it will be discussed in more detail below.

[Place Figure 2 here]

[Place Table 1 here]

Breaking down the returns on total assets into returns on non-financial and financial assets clearly shows that returns on total assets have been largely dominated and driven by returns on non-financial assets over time (Figure 3). There are two reasons for this. The first is high portfolio shares of non-financial assets in all countries, as mentioned earlier. Second, the average returns on non-financial assets were significantly higher than those on financial assets in all countries over the long term (Table 1). Over the entire period, for example, the return advantage of non-financial assets over financial assets averaged 2.50 percentage points (pp) in Germany, 2.78 pp in Italy and 4.27 pp in France. In Spain, it was 5.93 pp, and for the eurozone as a whole the average was 3.33 pp.

The results so far thus show that taking non-financial assets into account is of central importance for assessing the return situation of households in Europe. Analyses (and the criticism based on them) that consider only financial assets clearly fall short. The fact that non-financial assets are of such great importance is not, in principle, a new insight. Survey-based datasets at the microeconomic level, such as the German Socio-Economic Panel (SOEP) or the Eurosystem's Household finance and consumption survey (HFCS), have long shown this (Household Finance and Consumption Network, 2020; Schröder et al., 2020). Taking these assets into account at the macroeconomic level for the determination of the real returns, on the other hand, is new.

[Place Figure 3 here]

Turning to research question two of the paper (does the evolution of average real returns in the "pre-crisis phase" (2000-2007) differ from that in the "crisis and low interest phase" (2008-2018)?), the declining trend of the real return on total assets observed above becomes even more obvious. In Spain, average returns on total assets collapsed from 10.86% to 1.90%, in France from 8.05% to 2.10%, in Italy from 5.71% to 0.58% and in the euro area as a whole from 5.14% to 2.70%. In contrast, Germany saw an increase from 2.05% to 4.28%. Germany's special role is put into perspective when taking a closer look at the "crisis and low interest" phase, as the trend of falling total returns since 2015 can be clearly observed in all countries – including Germany. In addition, real returns on total assets in 2018 were below their long-term averages in all jurisdictions, although they were still in positive territory despite zero official interest rates.

Considering non-financial and financial assets separately, the yield advantage of non-financial assets observed above was significantly larger in the pre-crisis period in Spain (12.93 pp), Italy (7.47 pp) and France (9.90 pp); in Germany, on the other hand, the yield advantage was almost zero. This was mainly due to the development of the German real estate market, which was much less dynamic than in other European countries during this period for various reasons (economic slowdown, high unemployment, lack of structural reforms, etc.) (Geiger, Muellbauer, Rupprecht, 2016). However, the differences reversed during the "crisis and low interest" phase, with Germany leading the way with a yield advantage of 4.28 pp, Spain and France almost completely eroding their yield advantage to 0.85 pp and 0.18 pp respectively, and Italy reversing to a yield disadvantage of -0.63 pp. The reasons for this were both the significant declines in average real returns on non-financial assets (with the exception of Germany) and an – albeit less pronounced – reduction in average returns on financial assets in all countries. However, since 2015 the German real return on non-financial assets has also followed a downward trend. In the last year of observation, 2018, real returns on financial assets were negative across the board and could only be offset by positive returns on non-financial assets.

As an interim conclusion, real returns on total household wealth fell almost everywhere over time, but their level was consistently positive up to and including 2018. The latter is, however, exclusively attributable to non-financial assets, as financial assets have recently shown negative returns across the board. By contrast, it remains to be seen whether households have reacted to the different developments in returns by changing their portfolio structure and how this, in turn, has affected returns. This aspect is addressed in the next section.

4.2 Portfolio shifts and individual assets' contribution to total real return

According to the definition of the real return on total assets in equation (2), changes in total wealth returns are either caused by (i) changes in the respective portfolio shares (volume effects) and/or by (ii) changes in the respective individual real returns (price effects). Moreover, owing to classical portfolio theory, volume effects result from changes in yields and risks of all assets contained in a portfolio (Markowitz, 1952, Tobin, 1969). Against this background, we analyse how the investment behaviour of households during the "pre-crisis" phase differed from that during the "crisis and low interest" phase, and how changes in returns and the resulting portfolio regroupings affected overall real returns.

As far as price effects are concerned, average real returns have fallen for a substantial share of the assets in the countries considered here. Figure 4 and Table 2 show that this is the case in 50% of all instances (15 out of 30 asset type/country combinations). In some of them, the decline was very sharp; in a few cases, it was even accompanied by a change in sign (e.g. deposits in Germany or equities in Italy). Nevertheless, 50% may be less than generally expected, especially by critics of the ECB's low interest rate policy (see above). Moreover, average real returns on safe (and thus low-yielding) assets were in some cases surprisingly even higher in the low interest rate environment than in the "pre-crisis" phase (e.g. currency and deposits in Spain or Italy). Finally, it may also come as a surprise that, contrary to what is often claimed, securities did not consistently and everywhere yield higher real returns in the low interest rate environment than before. For example, while equities in Germany, France and the euro area as a whole exhibited higher real returns on average in the "crisis and low interest" phase, the opposite was true in Italy and Spain. Price effects thus did not in themselves have a negative impact on total returns. In countries such as France and Italy, however, this was mostly the case.

[Place Figure 4 here]

[Place Table 2 here]

Against this background, what happened to the structure of household portfolios (volume effects)? Figure 5 shows that average portfolio shares of all security types declined across the board in the "crisis and low interest" phase, with exposure to equities declining the most on average. This is in line with the findings of Rupprecht (2018), who found similar developments for earlier periods. It is also true if one takes into account the change in the share of the portfolio resulting solely from (relative) price effects, i.e. if one attributes the changes in the share of a particular asset class exclusively to purchases/sales (transactions) of this class.[8] In contrast, the portfolio shares of almost all other forms of investment increased. As a result, the proportions of lower-yielding asset types or of assets with declines in average returns (see above) increased everywhere. From a risk-return perspective, this result could be explained by a simultaneous reduction in the risk (i.e. standard deviation as a measure of volatility) of these forms of investment. However, the results show that the risks of almost all forms of investment have increased

[8] This consideration is often referred to as "notional stocks". For a detailed discussion, see Rupprecht (2020).

in all countries (exceptions were claims against insurances in Germany and France, and non-financial assets in France). Overall, the quantitative extent of most portfolio share shifts was relatively small, such that the portfolio structures in all countries changed only slightly.

[Place Figure 5 here]

With regard to the effects on the total real return or the return contributions of individual assets (Figure 6), the relatively weak changes in volume were thus offset by comparatively dramatic price changes. Consequently, all of the aforementioned portfolio shifts ultimately played a subordinate role, since the yield effect dominated everywhere. Although the reduction in the return contributions of non-financial assets dominated everywhere in quantitative terms (decreases between -2.43 pp and -9.01 pp) – with the exception of Germany (increase of +2.34 pp) – tangible assets nevertheless remained the mainstay of total real returns in all jurisdictions. The often-cited bank deposits, which have often been described as suffering from a “moment of expropriation”, were insignificant for the total real returns. This was the case even despite recent negative real returns across the board – with the exception of France.

[Place Figure 6 here]

To sum up, the results so far indicate that the duration of the current phase of low interest rates and its influence on the evolution of individual yields have been rather decisive for the level and development of real total returns, although the composition of household portfolios also plays a major role. Previous public debates that were limited to the returns of individual asset classes had thus far not been able to capture the “big picture” and thus have all fallen short.

5 Reviewing the “efficiency debate” – assessment of households’ portfolio performance in 2018

With regard to euro area households’ investment behaviour, the preceding analysis has brought to light three key observations that lead to research question three (can portfolio allocations in 2018 be considered as efficient in terms of Markowitz’s mean-variance approach and in comparison to simple common heuristic portfolio allocation strategies?). First, the reaction of households to the low interest rate environment in terms of portfolio shifts was, on the one hand, rather hesitant and, on the other hand, in parts even counterintuitive as they primarily increased the shares of assets that were characterised by declining returns and a simultaneous increase in risk. Second, as queried by many authors, households’ portfolios in all jurisdictions contain a comparatively high proportion of low-yielding assets like deposits, while the share of higher-yielding securities is rather low (and even declined). Third, household portfolios in all jurisdictions contain very high shares of non-financial assets making them highly vulnerable to specific or idiosyncratic portfolio risk. In summary, these observations may lead to the hypothesis that household portfolios were and are not well-diversified or efficient in terms of an optimal allocation between risk and return.

In the following, the hypothesis of inefficient portfolio structures will be tested by using the newly generated data set in two ways. First, actual portfolio allocations in all countries will be compared to efficient portfolio structures derived from Markowitz's classical mean-variance portfolio optimization approach. Second, alternative portfolio allocation strategies based on simple "heuristic" rules which have been found relevant by the empirical literature for individual financial decision making will be compared as well to both actual portfolios and efficient Markowitz portfolios.

5.1 The mean-variance optimization approach

Markowitz's (1952, 1959) classical mean-variance approach comprises a two-stage optimization procedure. First, a set of efficient portfolios is determined via a minimization of risk for given levels of expected returns. These optimal portfolios, also called "efficient frontier portfolios", offer a menu of choice for investors, as they define optimal portfolio allocations and the associated risks for different target returns set by the investor. Second, given those efficient frontier portfolios, one optimal portfolio is determined by an optimization of expected utility summarizing investor's preferences regarding risk and return.

The present analysis follows Markowitz's approach by determining the efficient frontier of household portfolios in the last observation year 2018 for all countries based on the newly generated dataset described and presented above, where three different model variants are studied. It deviates, however, with regards to the maximization of expected utility due to limited information regarding aggregate household sectors' risk-return preferences. Instead, it uses a different approach which does not define one optimal portfolio, but two alternative portfolios, so-called "corner portfolios", based on investors' expressed preferences in 2018. It would have been desirable to study optimal portfolios not only in 2018, but to compare as well pre-crisis from post-crisis portfolios. This, however, would have reduced the robustness of the results significantly, in particular for the pre-crisis phase due to a further reduction of the already limited number of data points.

To assess and compare the performance of those optimal household portfolios, the present approach introduces a modified metric which differs from traditional measures introduced by Treynor (1965), Sharpe (1966), and Jensen (1968). The departure was necessary, as the traditional measures are all based on the capital-asset pricing model (CAPM) whose assumptions cannot be applied to the present case.

5.1.1 Efficient frontier analysis

5.1.1.1 Model definitions

Three model variants are studied in the following which differ with respect to the constraints imposed. *Model 1 (classical model)*, the "original" and unconstrained mean-variance model allowing for short sales, serves primarily as a theoretical reference point for the present analysis. It assumes that there are *n* assets which are combined in a portfolio. The return of each asset *i* is defined as θ_i, and its portfolio share as α_i, where $i = 1, \ldots, n$. Using matrix notation, the asset returns $\boldsymbol{\theta}$ and the respective assets' weights $\boldsymbol{\alpha}$ in the portfolio are given by

$$\boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \ldots \\ \theta_n \end{pmatrix} \quad \text{and} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \ldots \\ \alpha_n \end{pmatrix}.$$

The individual asset weights α_i are calculated as the ratio of the market value of asset *i*, MV_i to the market value of the total portfolio (sum of all assets) $MV_P = \sum_{i=1}^{n} MV_i$, i.e. by

$$\alpha_i = \frac{MV_i}{MV_P} = \frac{MV_i}{\sum_{i=1}^{n} MV_i}.$$

The portfolio weights must add up to one, i.e. it must hold that $\boldsymbol{\alpha}^{\mathbf{T}}\, \mathbf{i} = 1$, where $\mathbf{i}$ represents the unit vector which is of dimension *n* x 1. In case of short-selling[9], some asset weights can be negative, but still have to add up to one. Portfolios for which $\boldsymbol{\alpha}^{\mathbf{T}}\, \mathbf{i} = 1$ are called feasible portfolios.

The actual portfolio return θ_P is given by

$$\theta_P = \boldsymbol{\alpha}^{\mathbf{T}}\, \boldsymbol{\theta}. \tag{4}$$

The expected portfolio return μ_P is defined as

$$\mu_P = \boldsymbol{\alpha}^{\mathbf{T}}\, E[\boldsymbol{\theta}] = \boldsymbol{\alpha}^{\mathbf{T}}\, \boldsymbol{\mu} \tag{5}$$

where the expected yields of the individual assets *n,* μ_i, are given by

$$\boldsymbol{\mu} = E[\boldsymbol{\theta}] = \begin{pmatrix} E[\theta_1] \\ \vdots \\ E[\theta_n] \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}.$$

The portfolio variance σ_P^2, or portfolio risk, is given by

$$\sigma_P^2 = Var[\theta_P] = E\left[(E[\boldsymbol{\theta}] - \boldsymbol{\theta})^2\right] = E\left[(\boldsymbol{\mu} - \boldsymbol{\theta})^2\right] = \boldsymbol{\alpha}^{\mathbf{T}}\, \mathbf{Y}\, \boldsymbol{\alpha} \tag{6}$$

[9] Short-selling defines the act of borrowing an asset, as for example a bond, which is sold immediately and whose proceeds are invested in another asset, as for example in a stock. At maturity date the bought asset (stock) has to be sold in order to re-purchase a bond which has to be given back to the lender of the bond. Consequently, short-selling is only profitable if the price of the borrowed asset declines.

where **Y** denotes the variance-covariance matrix (dimension n x n, and positive definite) of assets' individual returns. **Y** is defined as

(7)
$$\mathbf{Y} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

where $\sigma_{ij} = Cov[\theta_i, \theta_j] = E[(\theta_i - E[\theta_i])(\theta_j - E[\theta_j])] = E[(\theta_i - \mu_i)(\theta_j - \mu_j)]$ denotes the covariance between assets i and j, and $\sigma_{ii} = Cov[\theta_i, \theta_i] = Var[\theta_i] = E[(\theta_i - E[\theta_i])^2] = E[(\theta_i - \mu_i)^2]$ the variance of asset i.

Given the characteristics of the available assets above, the goal of every investor is to construct a portfolio by an optimal selection of the shares of the individual assets which generates minimum risk for a given expected portfolio return μ_P, or, put differently, which generates a maximum portfolio return for a given risk level. All portfolios which fulfil these conditions are called "efficient". Mathematically, the optimal composition of the portfolio, i.e. the optimal shares of each asset in the portfolio α_n representing the optimal portfolio weights vector $\boldsymbol{\alpha}_{opt}$, of an investor who wants to achieve a target portfolio return of μ_P with minimum risk can be determined by solving the following minimization problem:

(8)
$$\underset{\boldsymbol{\alpha}}{\text{Minimize}} \quad \frac{1}{2}\sigma_P^2 = \frac{1}{2}\boldsymbol{\alpha}^\mathbf{T}\ \mathbf{Y}\ \boldsymbol{\alpha}$$

subject to

(i) $\boldsymbol{\alpha}^\mathbf{T}\ \boldsymbol{\mu} = \mu_P$ (expected return equals target return μ_P)
(ii) $\boldsymbol{\alpha}^\mathbf{T}\ \mathbf{i} = 1$ (portfolio is feasible)

The solution to this minimization problem determines for each given target portfolio return μ_P, to be formulated by the investor, efficient portfolio shares $\boldsymbol{\alpha}_{opt}$ which guarantee a minimum variance σ_P^2. Illustrating these combinations of different expected portfolio returns and their corresponding minimum portfolio standard deviations in the $\mu_{P,}\sigma_P$-space in a graph gives rise to the so-called "optimal frontier".

Model 2 (classical model with non-negativity constraints) uses the same relationships like model 1, but assumes additionally that short sales are not allowed, i.e. all asset shares must be positive. These model assumptions can be justified by the fact that "normal" households cannot engage in complex financial transactions that would allow them, for example, to sell a house they do not own and use the proceeds to invest in the stock market and later reverse the transaction. All model equations presented above still hold, and the minimization problem (8) is supplemented by the non-negativity condition (see condition

(iii) below, where $\mathbf{0}$ is the null vector which is of dimension *n* x 1) as to the optimal asset shares $\boldsymbol{\alpha}_{opt}$ as follows:

$$\text{(9)}\quad \underset{\boldsymbol{\alpha}}{\text{Minimize}} \quad \frac{1}{2}\sigma_P^2 = \frac{1}{2}\boldsymbol{\alpha}^{\mathbf{T}}\ \mathbf{Y}\ \boldsymbol{\alpha}$$

subject to

(i) $\boldsymbol{\alpha}^{\mathbf{T}}\ \boldsymbol{\mu} = \mu_P$ (expected return equals target return μ_P)
(ii) $\boldsymbol{\alpha}^{\mathbf{T}}\ \mathbf{i} = 1$ (portfolio is feasible)
(iii) $\boldsymbol{\alpha} \geq \mathbf{0}$ (no short-selling, $\alpha_n \geq 0$ for all n)

The solution to this minimization problem gives rise as well to an "optimal frontier" in the $\mu_{P,}\sigma_P$-space, defining for each given target portfolio return optimal portfolio shares $\boldsymbol{\alpha}_{opt}$ which guarantee a minimum portfolio risk while not allowing for short sales.

Model 3 (classical model with non-negativity and non-financial assets constraint) uses the same equations like model 2 and adds an additional constraint on asset shares by stipulating that the share of non-financial assets is set to a pre-defined fixed value and cannot be changed (see condition (iv) below). As a consequence, only the financial portfolio can be restructured. This model assumption results from the fact that most households cannot easily diversify non-financial assets; once a house is bought, it will be either held or sold completely, but it cannot be separated in pieces which can be traded individually. Consequently, the minimization problem as to the optimal asset shares $\boldsymbol{\alpha}_{opt}$ reads as follows:

$$\text{(10)}\quad \underset{\boldsymbol{\alpha}}{\text{Minimize}} \quad \frac{1}{2}\sigma_P^2 = \frac{1}{2}\boldsymbol{\alpha}^{\mathbf{T}}\ \mathbf{Y}\ \boldsymbol{\alpha}$$

subject to

(i) $\boldsymbol{\alpha}^{\mathbf{T}}\ \boldsymbol{\mu} = \mu_P$ (expected return equals target return μ_P)
(ii) $\boldsymbol{\alpha}^{\mathbf{T}}\ \mathbf{i} = 1$ (portfolio is feasible)
(iii) $\boldsymbol{\alpha} \geq \mathbf{0}$ (no short-selling, $\alpha_n \geq 0$ for all n)
(iv) $\alpha_{NFA} = c$ where $0 < c < 1$ (non-financial assets' share predetermined)

This model's solution defines for each given target portfolio return optimal portfolio shares $\boldsymbol{\alpha}_{opt}$ which guarantee a corresponding minimum variance, while short sales are not allowed, and the share of fixed assets is predetermined.

All three models define optimal portfolio strategies for households, but differ significantly with regards to their practical applicability. While model 1 serves as pure theoretical reference point which cannot be realized in practice mainly due to the short-selling assumption, model 2 represents a more medium-run optimum for households (time perspective one to five years) which can be practically achieved, though it may be subject to frictions because of a time-consuming and costly adjustment of non-financial assets. Model 3, by way of contrast, represents a short-run portfolio optimum (time perspective up to one year), which can be realized quickly, as there are only financial portfolio adjustments which are less costly and time-consuming.

5.1.1.2 An alternative portfolio performance measure

To compare the performance between different efficient portfolios on the one hand, and between the actual household portfolios in 2018 and several optimal portfolio choices on the other hand, a metric had to be chosen. Today, there are three common measures to assess portfolio performance which combine return and risk in one single measure, namely the Treynor measure (Treynor, 1965), the Sharpe ratio (Sharpe, 1966), and Jensen's alpha (Jensen 1968). The three measures are very similar and compute an excess return, i.e. a portfolio's additional return over a risk-free asset (such as a short-term AAA government bond), and relate that excess return to the portfolio's risk, either measured by the portfolio's standard deviation or its beta.[10] The three measures are based on or closely linked to the capital-asset pricing model (CAPM) which assumes that investors only hold a combination of a so-called market portfolio including all risky assets (a mutual fund or an exchange trade fund (ETF) tracking an underlying stock market index) and a risk-free asset. The only choice investors face is how much of the risk-free asset they want to hold. As a result, investors can either lend (long position) or borrow (short position) funds at the risk-free rate.[11]

The three performance measures' investment view, however, is not suitable to the present approach due to two differences. First, a household sector's "market" portfolio does not consist of only stocks, but includes many different assets as outlined above, and cannot be split up in identical purchasable shares like an ETF, where each share mirrors exactly the composition of the market portfolio. Therefore, it is not possible for a household to buy only a share of the market portfolio and to hold the remaining part in a risk-free asset. Second, as households are treated as a single entity, the household sector as a whole must always hold the complete market portfolio. As a consequence, borrowing and lending at the risk-free rate, allowing to hold a mixed portfolio of risky assets and a risk-free asset, is not possible.

Summing up, another portfolio performance measure combing a portfolio's return and risk while excluding the risk-free return had to be defined. One candidate considered appropriate, being very similar to the traditional measures discussed above, is the coefficient of variation (CV), being also known as the relative standard deviation. A portfolio's coefficient of variation, CV_P, is defined as the ratio of

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[10] The Sharpe ratio, *SR*, for example, is defined as $SR = \frac{\theta_P - r_{rf}}{\sigma_P}$, where θ_P denotes the portfolio return, r_{rf} the risk-free rate, and σ_P the portfolio's standard deviation.

[11] If investors lend funds at the risk-free rate, a share of $x\%$ is held in the risk-free asset (where $0\% < x\% < 100\%$), and a share of $100\% - x\%$ is held in the market portfolio. If investors borrow funds at the risk-free rate, a share of $100\% + x\%$ is held in the market portfolio, and a share of $-x\%$ in the risk-free asset (where $-x\% < 0\%$), allowing them to hold more than 100% of their wealth in the market portfolio.

the portfolio's standard deviation σ_P to the portfolio's expected return μ_P, and is a dimensionless number. Formally, the CV_P is given by

$$CV_P = \frac{\sigma_P}{\mu_P} = \frac{\sqrt{\boldsymbol{\alpha}^T \mathbf{Y} \boldsymbol{\alpha}}}{\boldsymbol{\alpha}^T \boldsymbol{\mu}} \quad (11)$$

measuring the amount of risk per unit of return. The lower the CV_P, the better the portfolio performance in terms of its risk-return trade-off. The CV can also be computed for individual assets accordingly by the ratio of an asset's standard deviation to the asset's expected return. An asset's CV also measures the amount of risk per unit of return. The lower an asset's CV, the more efficient it is regarding its risk-return mix. Note that both an asset's or a portfolio's CV can be negative in case the expected return is negative.[12] A negative CV is worse than any positive CV value.

5.1.1.3 Model solutions and characteristics

To solve the three efficient frontier models, expected returns and covariances had to be estimated first for each asset category in each country as model inputs, showing up in the respective national variance-covariance matrices $\mathbf{Y}$. While there exist many methods to estimate expected returns, including highly sophisticated econometric techniques, the present approach is based on simple (arithmetic) sample means from 2000 to 2018 for each asset category in the respective countries, being presented in Table 2 and summarized as well in Tables 4 and 5. This procedure can be justified by the fact that an "average" household does not possess both knowledge and methods to carry out complex estimation procedures, but relies on simple heuristic rules when predicting future returns. Having compiled expected returns, the sample variance-covariance matrices and the asset correlation coefficients, being illustrated in table 3, were determined.

[Place Table 3 here]

[Place Figure 7 here]

In the next step, the three quadratic minimization problems (8), (9), and (10) were solved by using numerical optimization techniques to compute the three optimal frontiers which are shown in Figure 7 as solid lines for each country for the last data observation year 2018.[13] In the case of Spain, France, Italy, and the euro area, the model 3 frontiers contain next to the solid parts, which represent efficient portfolios, as well dashed lines which do not define efficient portfolios, but which had to be included, as

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[12] An asset's variance and standard deviation are always positive. While a portfolio's variance, as defined by equation (6), can be theoretically negative, implying a non-existing standard deviation, this case is excluded by requiring that the variance-covariance matrix, defined by equation (7), must be positive definite.

[13] The construction of model 1's efficient frontier required only two arbitrarily chosen efficient portfolios according to the two-fund theorem which is only valid for the unconstrained model allowing for short sales. The two-fund theorem stipulates that the portfolio weights of any efficient portfolio are a weighted average of two other arbitrarily chosen efficient portfolios. Moreover, the classical model 1 minimization problem possesses an analytical, closed-form mathematical solution, whereas other models with additional constraints, as e.g. models 2 and 3 in the present approach, do not possess closed-form solutions and, hence, must be solved by numerical optimization techniques.

some of those inefficient portfolios represent possible portfolio choices in case some additional restrictions regarding households' preferences are imposed (see section 5.1.2.2). The starting points of each efficient frontier in the southwest always represent the minimum variance portfolio, whereas the end points of the solid drawn frontiers in the northeast always represent the portfolio with the maximum expected real return.[14] The asset structure and the performance of the respective minimum variance and maximum expected real return portfolios are illustrated in Table 3.

[Place Table 4 here]

The actual household portfolios in 2018, being illustrated as rectangles in Figure 7 (and broken down by instruments in Tables 4 and 5), all lie below the respective efficient frontiers in all countries. As a result, none of these portfolios is efficient. However, an initial graphical inspection indicates that the 2018 portfolios in all countries are "quite close" to the model 2 and 3 efficient frontiers, but "far away" from the model 1 efficient frontier. From this simple graphical analysis, it can already be deduced that the more realistic restrictions are allowed, the closer the actual portfolio performance comes to the optimal one. To better understand how and why household portfolios in 2018 differ from efficient frontier portfolios and what kind of portfolio restructuring is needed in order to improve portfolio efficiency, it is helpful to study first the characteristics of the minimum variance and the maximum expected return portfolio in more detail, and second, their differences compared to the actual 2018 portfolios by using the data from Table 4.

Regarding the maximum expected return portfolio, the model 1 maximum achievable expected real portfolio return can be chosen freely and can exceed the expected real return of the asset with the highest individual return, as – in theory – any return is achievable due to short selling. As a result, the optimization problem given by equation set (8) does not change and requires just a choice of another arbitrary expected portfolio return as defined by constraint (i). Moreover, as the portfolio variance is still minimized, the optimal solution does not result in short sales of the asset with the lowest return and investment of all funds in the asset with the highest yield, but still in a balancing of all asset shares to meet all constraints. For the current estimation process, the choice of the model 1 maximum expected real portfolio returns was largely determined by the resulting model 1 efficient frontier's graphical proximity to the model 2 and 3 efficient frontiers in Figure 7. The following values were chosen: For Germany and the euro area 10 %, for Spain 35 %, and for France and Italy 15 %. As outlined in Table 4, the model 1 maximum return portfolios are characterized in all countries by significant short sales in deposits mainly due low and predominantly negative returns resulting in negative CVs (and in a high CV value in France), and heavy investments in insurance products conditioned by relatively high returns in relation to risk, as expressed by the low CV values. Regarding the remaining assets, there are as well short shales in all countries, very often, but not exclusively, in assets with comparatively high CV values.

[14] Both the starting (minimum variance portfolio) and the end points (maximum expected return portfolio) are indicated by asterisks, and labelled by the respective portfolio standard deviations and expected returns.

In contrast to model 1, the maximum achievable expected real portfolio returns in models 2 and 3 cannot be chosen freely and are determined on the one hand by the expected return of the asset with the highest individual return, and on the other hand by the restrictions regarding the non-negative portfolio shares. Therefore, the maximum achievable expected real total portfolio return can never be higher than the real return of the asset with the highest individual return, as short sales are not allowed.[15] Since only non-negativity restrictions apply to model 2, the maximum expected real total portfolio return must be identical to the return on the asset with the highest individual return. Hence, to maximize the expected portfolio return, only the highest yield asset can be held in the portfolio. As non-financial assets are the assets with the highest yields in all countries considered, the maximum expected return portfolios consist of only non-financial assets as outlined in Table 4. According to model 3, containing an additional non-negativity restriction regarding the proportion of non-financial assets, however, the maximum expected real total portfolio return can be only lower than the return of the asset with the highest individual return, since at least one other asset with a lower return, i.e. the asset with the second-highest yield, must be held. Table 4 indicates that in addition to the 2018 holdings of non-financial assets, German households must hold investment fund shares, Spanish and French households must hold insurance products, and Italian and euro area households need to hold debt securities to maximize the expected portfolio return. Summing up, the more restrictions apply, the lower the maximum expected real portfolio return will be. Moreover, this result holds as well for the efficient frontiers as a whole: The more restrictions are imposed, the less "favourable", i.e. the flatter, the efficient frontier becomes, as less and less return can be expected for a given level of risk. Consequently, the efficient frontiers in all countries become flatter when moving from model 1 over model 2 to model 3.

Turning to portfolio performance measured by the portfolios' CVs, the model 1 maximum return portfolios all exhibit a better performance than the 2018 portfolios, as the rise in expected returns overcompensates the simultaneous increase in the portfolios' standard deviations. By way of contrast, the model 2 maximum yield portfolios' change in performance is mixed. While in Germany and the euro area, performance improves compared to 2018 portfolio holdings as the rise in returns overcompensates the increase in risk, performance in all other countries is worse because the rise in risk overcompensates the increase in return. The performance of the model 3 maximum return portfolios compared to the 2018 portfolios is worse in Germany, but better in all other countries; while there is a rise in return which falls short of the simultaneous rise in risk in Germany, all other jurisdictions exhibit next to the rise in return a simultaneous decrease in risk.

If one abstracts from the changes in risk and asks only for the changes in return when comparing 2018 portfolios with maximum return portfolios, then substantial return increases are possible under model 1,

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[15] Consequently, for the computation of the maximum return portfolio, the optimization problems for models 2 and 3 have to be modified. Respecting model 2, the optimization problem reads as follows: $\underset{\boldsymbol{\alpha}}{\text{Maximize}}\ \boldsymbol{\alpha}^{\mathsf{T}}\boldsymbol{\mu} = \mu_P$ subject to (i) $\boldsymbol{\alpha}^{\mathsf{T}}\mathbf{i} = 1$ (portfolio is feasible), and (ii) $\boldsymbol{\alpha} \geq \mathbf{0}$ (no short-selling, $\alpha_n \geq 0$ for all n) ; regarding model 3, the additional constraint (iii) $\alpha_{NFA} = c$ where $0 < c < 1$ (non-financial assets' share predetermined) must be added.

ranging from +6.30 pp in the euro area up to +29.60 pp in Spain. However, as pointed out earlier, model 1 serves primarily as a theoretical reference point. When allowing for a more realistic medium-run investment environment, the possible return increases under model 2 are far smaller, ranging from +1.08 pp in Germany up to +2.05 pp in Spain. Turning to the potential yield gains in the short-run, model 3 results show even smaller increases in real yields, from +0.44 pp in France up to +1.05 pp in Spain. Summing up, though there is potential for return improvement, it seems to be limited and, as a consequence, that households' actual portfolios perform quite well despite a limited knowledge as to portfolio optimization techniques.

The computation of the minimum variance portfolios requires that constraint (i) in equation sets (8) to (10) is left out. As the data from Table 4 show, minimizing the portfolio variances is not achieved by simply holding the asset with lowest standard deviation while reducing all other asset shares with higher standard deviations to zero or even engage in short sales according to model 1. This due to the fact that according to equation (6) the portfolio variance is, in contrast to the portfolio return, non-linear in portfolio weights, and consists of asset share weighted variances and covariances, where the weight of the covariance terms in the total portfolio variance increases with a rising number of assets. Moreover, the more assets which are subject to negative correlations are added, the lower the portfolio's variance will be. As a result, when minimizing portfolio variance, the number, the sign, and the absolute size of the covariance terms in relation to number and the absolute size of the variance terms are the relevant determinants for optimal portfolio re-balancing.[16]

The model 2 minimum variance portfolios are most appropriate as starting points to study the relative importance of the various influencing factors. It can be seen from Table 4 that households in all countries hold more than 85% of their wealth in deposits, as they exhibit the lowest variances of all assets. In addition to that, households in Germany, France and the euro area hold additionally a small fraction in non-financial assets. As regards Germany and France, according to Table 3, deposits exhibit only a negative correlation with non-financial assets making them the only candidate with which the portfolio variance can be further reduced. In the euro area, the non-linear impact of the asset weights in the covariance term for deposits and non-financial assets requires to hold additionally non-financial assets. In Spain, next to deposits and non-financial assets, a small share is additionally held in insurance products. While deposits and insurance products exhibit a positive correlation and thereby lead, ceteris paribus, to a rise in the portfolio's variance, non-financial assets and insurance products are negatively correlated leading ceteris paribus to a reduction in the portfolio's variance. An identical link regarding

[16] When there are *n* assets, the variance-covariance matrix defined by equation (7) consist of n^2 elements, where *n* elements represent the *n* assets' variances, and $n^2 - n$ elements represent the assets' covariances. As a result, the portfolio variance, being a quadratic form and given by equation (6), consists of n^2 asset share weighted terms, where *n* terms comprise the asset share weighted variances, and $n^2 - n$ terms asset share weighted covariances. Hence, a rise in *n* reduces the influence of the assets' individual variances and, in a limiting case, causes the portfolio's variance to be determined (almost) only by the covariances. This result gives rise to the portfolio diversification effect, which says that the specific risk of the portfolio (represented by the assets' variances) can be reduced by adding additional assets to a portfolio, and that in a limiting case the portfolio variance just reflects market risk (reflected by the assets' covariances or correlations) which cannot be reduced any further. In the present case, there are 6 assets, i.e. the portfolio variance consists of 36 terms, where 6 terms comprise the assets' variances, and 30 terms include the assets' covariances.

the sign of correlations can be found in Italy between deposits, non-financial assets and debt securities, explaining why a small share is additionally held in debt securities.

The model 3 minimum variance portfolios resemble their model 2 counterparts a lot, as both deposits and non-financial assets represent the dominant asset holdings. However, as the share of non-financial assets is higher by definition, the overall portfolio variances are higher compared to model 2, and the only possibility to minimize the portfolio's variances is to hold the remaining share almost only in deposits. Except for Germany, all other countries hold additionally small shares in debt securities. Though debt securities and deposits are positively correlated in those countries, the negative correlations between deposits and non-financial assets (except in the euro area) and between non-financial assets and debt securities tend to decrease portfolio variance. The model 1 minimum variance portfolios are characterized like model 2 by a dominant share in deposits, where shares exceed 100% everywhere, and positive, but lower shares in non-financial assets. With the exception of investment fund shares in France, Italy, and the euro zone, equity in Germany, and debt securities in Spain and Italy, all other asset shares are negative, mainly due to their respective correlation structures.

As with the maximum return portfolios, adding additional constraints makes results less optimal, as the standard deviations of the minimum variance portfolios increase when moving from model 1 over model 2 to model 3. Graphically, this is reflected in an upward and rightward shift of the starting points of the efficient frontiers. In Spain, Italy, and the euro area, the model 1 and 2 starting points lie in the negative territory, as the minimum variance portfolios are dominated by deposits which exhibit negative returns.

Based on the portfolio's CVs, most of the minimum variance portfolios are inferior to the portfolios in 2018. According to Table 4, all model 1 and 2 minimum variance portfolios' performance is worse compared to the 2018 portfolios. Though both risk and expected returns drop in all countries, portfolio CVs rise in Germany and France as the reduction in returns to still positive values overcompensates the decline in risk. In all other countries, returns fall into negative territory causing negative portfolio CVs. Only model 3 minimum variance portfolios in Germany, France and the euro area lead to an improvement in performance, as the drop in risk is bigger than the reduction in expected returns; in all other countries, the opposite occurs leading to higher CV values.

A pure comparison of the possible changes in risk when shifting from 2018 portfolios to minimum variance portfolios indicates that the potential for risk reduction is highest for models 1 and 2, where standard deviation reductions range from -1.51 pp in Germany to -5.53 pp (model 1) and -5.41 pp (model 2) in Spain. According to model 3, the decline in risk is less, ranging from -0.31 pp in Italy to -0.68 pp in France. Summing up, a significant reduction in risk can be only achieved by a medium-run portfolio adjustment approach, but not by a more short-run oriented restructuring of the financial portfolio.

5.1.2 Optimal portfolio selection

5.1.2.1 The "classical" approach

In classical portfolio theory, the construction of the efficient frontier is followed by a maximation of expected utility with regards to the rate of return or the level of wealth, allowing for the determination of the "optimal portfolio" from all possible efficient frontier portfolios. The standard procedure to choose the optimal portfolio requires first to approximate an investor's expected utility regarding wealth or return by a corresponding (expected utility) function of mean and variance, which can be graphically illustrated as a set of indifference curves in the risk-return space. Markowitz (1959, 2012, 2014) and Levy and Markowitz (1979) have shown that the mean-variance approach to approximately maximize expected utility can be applied to a wide range of commonly used (concave) utility functions which represent risk-averse behaviour, such as log, power, square root or quadratic utility functions, and to theoretical and empirical return distributions which are not normal. Hence, for the maximation of expected utility, it is in many cases sufficient to know only the first two moments, namely the expected returns and their variances. Additional information like the underlying asset's periodic returns or higher moments like skewness or kurtosis are not needed. In a second step, these approximated mean-variance utility functions are maximized given the previously determined efficient frontier. Graphically, the optimal portfolio is determined as the point of tangency between the efficient frontier and an investor's indifference curve(s).

Some authors, as for example, Samuelson (1970), Cremers et al. (2003, 2005), and Adler and Kritzman (2007) have argued that the mean-variance or expected utility approximation approach can suffer from significant approximation error, as at least one of two (sufficient) conditions must be fulfilled to maximize expected utility optimally: First, the distribution of portfolio returns must be normal, and/or second, the utility function must be quadratic in portfolio returns resulting in an expected utility function of the form $E[U(r)] = E[r] - \psi\, Var[r]$, where $E[U(r)]$ denotes expected utility of the portfolio return r, $E[r]$ the expected portfolio return, $Var[r]$ the portfolio return's variance, and ψ the investor's risk aversion. If returns are normally distributed, the whole distribution of returns can be described by its expected value and variance. Furthermore, if returns should not be distributed normally, a quadratic utility function guarantees that investment decisions are not determined by metrics other than expected portfolio return and variance. The authors claim, however, that these two assumptions are rarely met under real-world conditions. First, many asset return distributions are not normal and higher moments like skewness and kurtosis may be of relevance. Second, quadratic utility does not seem to represent investor's real-world preferences towards risk and return correctly, as it implicitly and erroneously assumes that investors assess downside and upside risk equally, derive less utility from rising wealth if a certain wealth level is exceeded, and are subject to increasing absolute risk aversion. Hence, the more real-world conditions deviate from the assumptions of normality in return distributions and quadratic utility, the more the mean-variance approximation approach to the maximization of expected utility suffers from approximation error.

In order to eliminate the approximation error when maximizing expected utility, Cremers et al. (2005), Adler and Kritzman (2007) and others have suggested a full-scale optimization approach which uses empirical asset return distributions and allows for full flexibility as to the choice of the utility function. Though not being subject to approximation error when maximizing expected utility, the full-scale optimization approach is complex, requires a lot of computational power and time, and suffers from a lack of analytical solutions. Furthermore, Markowitz (2012, 2014) has stressed, in response to the criticism of the proponents of the full-scale optimization approach, that the approximation error of the mean-variance approach to approximately maximize expected utility is within reasonable limits for realistic real-world ranges of portfolio returns, even if return distributions are not normal and utility is not quadratic. Therefore, approximation errors which may be tolerable within certain boundaries combined with a much easier use in practice may explain why the mean-variance approach is still widely used among practitioners and in academia.

5.1.2.2 A "ceteris paribus" approach

As regards the asset returns' distribution, the problem of non-normality does not seem to apply to the present case as illustrated by the Jarque-Bera statistic in Table 2. However, the second prerequisite for optimal portfolio selection, i.e. detailed knowledge about investor preferences which are reflected in a specific type of utility function, seems not to be fulfilled in the present case. While it may be possible to define investor preferences for smaller and/or homogenous investor groups, it seems to be highly unlikely to do the same for an aggregated household sector in practice, mainly due to insurmountable information deficits and aggregation problems. Moreover, as the portfolio holdings data have shown in Figure 5, there may exist as well significant differences in household sectors' preferred asset mixes across countries. Consequently, it may seem analytically unsound and arbitrary to assume a national utility function being valid for all household units, and to assume identical utility functions across different countries.[17]

Due to the above mentioned difficulties, the approach chosen here does not aim to claim the determination of the optimal portfolio from a set of efficient portfolios for itself by maximizing expected utility based on a utility function, but rather to show first, "how far away" the actual 2018 portfolios are from a possible efficient portfolio in terms of an efficient risk-return mix, and second, to define two so-called "corner portfolios" describing risk-return combinations based on households' preferences as to

[17] If one classifies individual households alone according to the variables initial wealth (high vs. low) and risk aversion (risk averse vs. risk affine), which are the relevant variables for utility functions widely used, one can distinguish four possible groups whose preferences differ significantly, and thus may probably show up in quite different portfolio compositions. First, low-wealth/risk-averse households who prefer to hold, for example, deposits, insurance products or, if any, mutual funds. Second, high-wealth/risk-averse households who allocate their funds, for example, via family offices and prefer to hold mainly physical assets. Third, low-wealth/risk-affine households who speculate on stock markets by using debt. Fourth, high-wealth/risk-affine households who invest in hedge funds. As a result, summing up all individual households in one aggregate sector and trying to find a utility function which represents the preferences of the "average household" would require to identify each group, to define a utility function for each group, and to determine the relative weight of each group in the aggregate household sector, for example, by determining each group's share in total household sector wealth. Although this approach would be feasible from a model-theoretical perspective, it is practically impossible to implement due to numerous insurmountable information deficits.

risk and return chosen in 2018.[18] As all portfolios on the efficient frontiers determined by equation sets (8) to (10) represent valid reference points, rules had to be defined as to how portfolio allocations could be optimized, and thereby, which point(s) on the efficient frontier households should "target". Two assumptions were applied for the determination of those optimal portfolios. First, it was assumed that the current household portfolio allocation in 2018 can be used as an - albeit not optimal - indication of aggregated household preferences in terms of return and risk, because if households had not been satisfied with the current risk-return profile, portfolios would have been adjusted accordingly. Second, though households' actual portfolio choices may reflect more or less correct preferences, in most cases, the realized risk-return mixes are not fully efficient in mean-variance terms because of a lack of optimization tools and knowledge amongst ordinary households, i.e. the probability of reaching a point on the efficient frontier is not zero, but extremely small. Moreover, this assumption has been verified by the empirical results in Figure 7.

Based on these two assumptions, a simple "ceteris paribus analysis" was conducted to answer two questions. First, how would optimal portfolios look like if households were to realise the same expected return they realised in 2018, but minimized risk? Second, how would optimal portfolios look like, if households were to realise the same standard deviation they realised in 2018, but maximized the expected return? Consequently, the ceteris paribus approach is not able to determine one optimal portfolio, but two "corner portfolios" defining risk-return combinations based on households' preferences. However, as the corner portfolios are not determined via a full formal optimization procedure based on the efficient frontier and expected utility functions, it may occur that these portfolios do no longer lie on the efficient frontier, making it necessary to fall back on an alternative portfolio selection process (details see below).

In total, there are six corner portfolios for each country, as there are three efficient frontiers on each of which two optimal portfolios are determined. In Figure 7, the corner portfolios can be found as follows: Starting from the actual portfolio in 2018 and moving up "north", the intersections with the three efficient frontiers define all portfolios which take the standard deviation of the actual portfolio in 2018 as given and maximize the expected real portfolio return (labelled as models 1a, 2a, and 3a). By way of contrast, starting from the actual portfolio in 2018 and moving "west", the intersections with the three efficient frontiers define all portfolios which take the expected real portfolio return of the actual portfolio in 2018 as given and minimize the standard deviation (labelled as models 1b, 2b, and 3b). Table 5 reports the asset shares and the portfolio performance of the six corner portfolios and the actual portfolios in 2018.

[Place Table 5 here]

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[18] More specifically, the "classical" approach is determined to find the optimal portfolio from all efficient portfolios on the efficient frontier, and does not analyse (inefficient) portfolios which are located "below" the efficient frontier. The present approach, however, analyses actual, and potentially inefficient portfolios, and asks how these inefficient portfolios could be restructured to make them efficient in different ways.

The portfolios that maximize expected returns while risk is fixed to the 2018 standard deviation (models 1a, 2a, and 3a) are very similar to the maximum return portfolios reported in Table 4. As regards model 1a, the portfolio structures are just "less extreme" or "scaled down" variants of the respective maximum yield portfolios, mainly due to the fact that the model 1a portfolios generate less expected return than the maximum yield portfolios. All households engage in short-selling deposits and use the funds to invest significantly more than 100% of their wealth in insurance products exhibiting low CV values. The remaining asset holdings are identical to the maximum return portfolios as to the decision short sales versus holding positive shares, i.e. the signs of the holdings are identical; only the scales are different, with lower positive asset shares, and less negative asset holdings. Model 2a portfolios, whose returns are as well lower than the returns of the corresponding maximum return portfolios, are characterized as well by dominant shares in non-financial assets which are, however, lower than 100 %. The remaining funds are distributed mainly among assets with lowest CV values. Model 3a portfolios resemble both the respective maximum return portfolios and the model 2a portfolios with dominant shares in non-financial assets, and preferably investments in assets with second-highest yields and assets with low CV values. However, apart from Germany, model 3a portfolios are inefficient as they do not lie on the model 3 efficient frontier, indicated by the dashed model 3 lines in Figure 7. Thus, if households in Spain, France, Italy, and the euro area pursue to maximize portfolio returns, it would be optimal to choose the maximum return portfolios offering both a higher return and a lower standard deviation.[19]

When comparing the portfolio performance of models 1a, 2a, and 3a with the 2018 portfolio performance, one observes throughout an improvement by switching to the optimal portfolios. This result is not surprising as higher returns and constant standard deviations result in lower CV values. As mentioned before, under model 3a, households can improve portfolio performance further by choosing the maximum return portfolios. Respecting Germany, however, the model 3a performance, is better than the maximum return portfolio performance, due to a comparatively strong increase in risk when moving from the model 3a portfolio to the maximum return portfolio.

Turning to the potential for increasing real returns, adjusting the 2018 portfolios to the model 1a portfolio structures would lead to substantial increases ranging from +3.37 pp in Germany to +25.82 pp in Spain. According to model 2a, the range for possible return increases is much smaller and goes from +0.70 pp in Germany to +1.20 pp in Spain. The room for improvement for model 3a is even smaller and ranges from +0.44 pp in France, to +0.98 pp in Spain. By way of contrast, if households, apart from German households, choose the maximum return portfolios instead of the model 3a portfolios, the potential rise in return lies between +0.44 pp in France and +1.05 pp in Spain, being not very different from the model 3a results.

Summing up, the real-world potential for increasing portfolio returns, as determined by models 2a and 3a, seems to be quite limited, and actual household portfolios seem to be comparatively close to the

[19] In the case of France, only the standard deviation of the maximum return portfolio is higher while the returns are identical.

optimum. Moreover, as the portfolios of models 1a, 2a and 3a are similar to the respective maximum return portfolios, there are no significant differences respecting the potential for real return improvement.

If households want to adjust their portfolios in an optimal way in the medium-run compared to the 2018 holdings based on model 2a, the optimal strategy in all countries is to abandon deposits, equity, and investment fund shares completely, and to increase the share of non-financial assets. As for insurance products and debt securities, the shares should be either increased or decreased, mainly, but not always, on the basis of assets' CV values. If households follow a short-run strategy based on model 3a, the optimal strategy in all countries is to reduce the shares of deposits and investment fund shares (with the exception of Germany) to zero, and to reduce the share of equity. Respecting insurance products and debt securities, again national adjustments should be mainly, but not always, based on the respective CV values.

In contrast to the strong resemblance of the maximum return and the model 1a, 2a and 3a portfolios, there are significant differences between the minimum variance portfolios and the portfolios minimizing portfolio variance while keeping expected portfolio returns constant at their 2018 values (models 1b, 2b, and 3b). This discrepancy is conditioned by differences in the optimization problem. While models 1b, 2b, and 3b are solved by equation sets (8) to (10) while the target return constraints (i) are set to the 2018 expected return values, the minimum variance portfolios are computed by equation sets (8) to (10) without the target return constraints (i). The guiding principle for optimal portfolio construction for models 1b, 2b, and 3b is to select on the one hand assets with low CV and/or low risk values, and on the other hand higher yielding assets which can guarantee the 2018 target returns. As per model 1b, this trade-off causes all households to engage in short sales of deposits. Though deposits would be best to minimize risk, they earn a negative yield, and thus must be replaced with assets exhibiting a better risk-return-relationship. For that reason, all household hold significant shares in insurance products, as they offer everywhere both the second-lowest standard deviations and almost everywhere lowest positive CV values.

The next asset category held by all households are non-financial assets, offering CV and risk values which are slightly higher than insurance products. Respecting the other asset categories, the order of the assets according to ascending risk and CV values differ, which explains the sometimes considerable differences in asset holdings. For example, debt securities' CV values follow the CV values of insurance products in Germany, France, and the euro zone explaining the positive shares, while they exhibit less favourable values in Spain and Italy, causing short shales. Similar patterns can be observed for investment fund shares and equity. The construction of model 2b portfolios follows the same guiding principles. Expect for a tiny share in the euro area, households do not hold deposits due to their negative return. Moreover, households do not hold equity and investment fund shares owing to their adverse risk-return profile. By way of contrast, the largest shares, apart from France, are held in insurance products, followed by non-financial assets. In addition, French, Italian, and euro area households hold debt securities. When switching to model 3b, the risk-return trade-off becomes even more intense, as households are requested to hold a larger part of their wealth in non-financial assets than would be optimal according to models 1b and 2b if no constraints were applied. To compensate for the rise in risk, all households hold now low-risk deposits, and lower shares in insurance products compared to model 2b, where French and Italian households reduce their shares in insurance products even to zero. All households except in Germany, hold debt securities. High-risk equity investments and investment fund shares (with a tiny exception in Italy) are fully abandoned everywhere as in model 2b.

When switching from 2018 to model 1b, 2b, and 3b portfolios, portfolio performance increases everywhere, not surprisingly induced by reductions in portfolio standard deviations and constant expected returns which lead to a reduction in portfolios CV values. The potential for reductions in standard deviations compared to actual 2018 portfolio holdings is highest for model 1b, with a range from -0.96 pp in Germany up to -4.85 pp in Spain, followed by a range from -0.71 pp in the euro area to -3.66 pp in Spain for model 2b, and a range from -0.29 pp in Italy to -0.63 pp in France for model 3b. As in the case of expected returns, the potential for both realistic short-term (model 3b) and medium-term (model 2b) reductions in risk seem mostly to be quite limited, implying that households' 2018 portfolios are as well close to the optimum as regards risk.

If households want to re-allocate their portfolios optimally in the medium-run by shifting their 2018 portfolios to model 2b portfolios, they have to reduce deposits (with a tiny exception in the euro area), equity and investment fund holdings to zero, and lower their shares in non-financial assets considerably. The share in insurance products should be lowered in France, but increased significantly elsewhere. Debt securities' shares should be reduced to zero in Germany and Spain, but substantially increased in the other countries. If households wish to restructure their portfolios in the short run (model 3b), the share of deposits must be downsized in Germany and Spain, and increased elsewhere. Holdings of equity should all be completely diminished to zero. Investment fund shares should be decreased to a smaller positive share in Italy, but to zero in all other countries. The share of insurance products should be increased in Germany and reduced in all other jurisdictions, where shares in France, Italy and the euro area should fall to zero. The share of debt securities should be diminished in Germany to zero, but expanded in all other countries.

5.2 The "heuristic" approach

Most households do not possess the knowledge to solve complex portfolio optimization problems, and the few who could potentially do, perhaps lack resources, time, etc. to engage in an optimal portfolio restructuring process. Thus, households are likely to mostly rely on simple heuristic rules or rules of thumb. In the following, two common portfolio allocation strategies were explored with respect to their portfolio performances, namely the "1/n" and the "lifestyle fund" portfolio allocation strategy.[20] As these portfolio allocation strategies are mostly applied in the real world only to the allocation of financial assets, the present approach assumes as well that the shares of non-financial assets are fixed to the respective

[20] For more details, see, for example, Bernatzi and Thaler (2001, 2007), and Jacobs et al. (2014).

2018 values, and that only financial portfolios are restructured owing to the two heuristic rules. Accordingly, the two models should be interpreted as a further short-run portfolio allocation strategy.

5.2.1 Portfolio definitions

The “1/n” portfolio allocation strategy assumes that the share of non-financial assets is constant, and that the remaining financial wealth is split into *n* equal pieces, where each financial asset’s share in total financial wealth is 1/*n*. In the present approach, the shares of non-financial assets were set to the respective 2018 values, and the share of the remaining financial wealth which is composed of *n*=5 assets, was split into *n*=5 equal pieces, where each financial asset had a share of 1/*n*=1/5=20% in the financial portfolio. For example, referring to Table 5, the German household sector’s share in non-financial assets in 2018 was 58.04%, implying that 41.96% were held in financial assets. Splitting up the share of 41.96% into five equal pieces resulted in a share of 41.96%/5=8.39% for each financial asset. This method was applied accordingly to all other countries; the resulting portfolio structures are summarized in Table 5.

The “lifestyle fund” portfolio allocation strategy in its pure form refers predominantly to managed mutual fund shares combining a wider range of assets which are selected based on investors’ risk profiles. For example, one can find very often “conservative” (low risk), “moderate” (medium risk), and “aggressive” or “growth” funds (high risk). As it has been often argued that households in general are too risk averse, hold too many low-risk/low-yield assets like deposits or government bonds, and too few high-risk/high-yield assets like equity or equity-based mutual fund shares, the present approach asks how portfolios would perform if households adopted an “aggressive” portfolio strategy. For that reason, it was assumed that the share of non-financial assets was constant and equal to its 2018 values, and that the remaining financial wealth was split as follows: 60% mutual fund shares, 10% equity, 10% debt securities, 10% insurance products, and 10% deposits. This weighting scheme is in any case arbitrary, and different schemes would have been possible. The assumptions which led to the current allocation were as follows: First, for a fund be classified as “aggressive”, the share of securities with comparatively higher risk must be large. For that reason, the share in securities was set to 80%, and the share of low-risk financial assets (sum of deposits and insurance products) to 20%. Second, as real-world lifestyle funds do not hold individual securities, the share of mutual funds was assumed to be highest, followed by comparatively low shares in equity and debt securities. The portfolios determined in this way were labelled “opportunity portfolios” and are summarized in Table 5. For example, Italian households’ opportunity portfolio has a share of non-financial assets of 63.18%, and a share of financial assets of 36.82% which was split up into 36.82%·0.6=22.09% mutual fund shares, and equal shares of 36.82%·0.1=3.68% for equity, debt securities, insurance products and deposits.

5.2.2 Portfolio characteristics and performance

The heuristic portfolios’ expected returns were computed analogously to the mean-variance portfolios according to equation (5), where the individual assets’ expected returns were determined as before as historical arithmetic averages, and the asset weights were set exogenously to the values given in Table 5. The portfolio variances were computed owing to equation (6), where the same variance-covariance matrix (7) (see Table 3) was used as for the mean-variance portfolios in section 5.1. The portfolio characteristics are summarized in Table 5 and graphically illustrated in Figure 7.

Since the heuristic approaches are short-term portfolio strategies, the model 3 efficient frontiers illustrated in Figure 7 represent the optimal reference points when assessing portfolio performance. It would have been possible to apply the ceteris paribus approach from section 5.1.2.2 again to find optimal corner portfolios by using the two heuristic portfolios as starting points. This, however, does not coincide with households' real-world decision-making process, as the respective national starting points are the actual 2018 portfolios, providing theoretically four options to re-adjust portfolios. On the one hand, households can choose the two heuristic portfolios, and on the other hand, the model 3a and 3b corner portfolios. Therefore, each heuristic portfolios' performance is assessed in the following by comparing it to three alternative portfolios, namely the actual 2018 portfolio and the two mean-variance corner portfolios based on models 3a and 3b.

A first graphical inspection of Figure 7 reveals that both heuristic portfolios in Germany, Italy and the euro area, seen from the 2018 portfolios, lie in "north-eastern" direction indicating that both risk and return are higher compared to the 2018 portfolios. In Spain and France, by way of contrast, they lie very close to and distributed around the 2018 portfolios, signifying that both risk and return change only slightly, and that there is no clear trend regarding the change of risk and return compared to the 2018 values. Moreover, none of the portfolios is efficient.

Turning to the 1/*n* portfolio, a comparison with the 2018 portfolios shows that the heuristic portfolios have almost everywhere lower shares in deposits and insurance products, and higher shares in securities. The expected returns are higher, where changes range from +0.06 pp in France up to +0.41 pp in Germany. These improvements are, however, lower compared to the potential return changes when switching to the (inefficient) models 3a or to the (efficient) model 3 maximum return portfolios. Based on the actual 2018 portfolios, standard deviations, with the exception of Spain, are higher as well; changes range from +0.02 pp in France to +1.26 pp in Germany. Compared to efficient frontier models 3b, these changes are all worse, because risk was reduced everywhere when switching to the 3b portfolio structures. Regarding the change in portfolios' CV values compared to the actual 2018 portfolios, there are diverse developments. In Germany and the euro area, the 1/*n* portfolios' performances are worse, and in the remaining countries – albeit only slightly – better. Compared to the efficient frontier models 3a and 3b, the 1/*n* portfolio's performance is worse everywhere. Summing up, based on the actual 2018 portfolios, the potential for an increase in portfolio performance seems either non-existent or very small.

Compared to the portfolio structures in 2018, the opportunity portfolios exhibit everywhere much lower shares in deposits and insurance products, higher shares in debt securities and investment fund shares, and lower shares, except for Germany, in equities. Expected returns are slightly lower in France (-0.07

pp), but higher elsewhere and range from +0.24 pp in Spain to +0.72 pp in Germany. Compared to models 3a or the model 3 maximum return portfolios, however, these gains are almost everywhere smaller. Based on the 2018 portfolios, standard deviations drop slightly in France (-0.05 pp) and Spain (-0.18 pp), and rise in the other jurisdictions with a range from +0.07 pp in Italy to +1.92 pp in Germany. Compared to models 3b, these results are worse as well, as standard deviations were lowered everywhere. Compared to the 2018 portfolios, overall performance developments based on CV values are diverse. There is no change in France, and little improvements in Spain and Italy. Only in Germany and the euro area, opportunity portfolios exhibit a worse performance. Based on the model 3a portfolios, the opportunity portfolios' performance is worse everywhere. Compared to model 3b portfolios, the opportunity portfolio in Spain exhibits identical performance and there is a slight improvement in Italy. In all other countries, the model 3b portfolios are better. To conclude, based on actual 2018 portfolio structures, the potential for improvement is again quite limited.

If households really want to follow a heuristic rule, the question remains to be answered which of these two rules they should follow. Respecting the portfolio structure, except for investment fund shares, the 1/*n* portfolio exhibits larger portfolio shares for all financial assets. Expected returns in Germany, Italy, and the euro area are slightly higher under the opportunity portfolio, extending from +0.13 pp in the euro area to +0.31 pp in Germany; in Spain (+0.07 pp) and France (+0.12 pp), the 1/*n* portfolios generate slightly higher returns. Standard deviations in Germany (+0.66 pp) and the euro area (+0.02 pp) are slightly higher for the opportunity portfolio; in the other countries, the 1/*n* portfolios show less risk, ranging from -0.08 pp in France to -0.17 pp in Spain. Differences in the overall portfolio performance are very small, between 0.01 and 0.09 CV index points. In Germany and France, the 1/*n* portfolios are preferable, whereas the opportunity portfolios show a little advantage in the remaining jurisdictions.

6 Conclusion

Based on our newly developed data set, we were able to shed light on the current controversies regarding the development of real returns and the efficiency of household portfolios in the euro area in times of low interest rates.

Referring to the level debate and addressing research question one (how did real returns on total, financial and non-financial assets develop over time from 2000 to 2018?) two findings stand out. First, in all jurisdictions considered, real returns on total household wealth were mostly positive, but exhibit a declining trend which caused yields in 2018 to be still positive, but below their long-run averages. Second, real yields on total wealth are largely dominated and driven by non-financial assets' real yields both in terms of level and dynamics. In the last observation year 2018, returns were only positive due to non-financial assets' returns, as financial assets' real yields were negative everywhere.

Respecting research question two (does the evolution of average real returns in the "pre-crisis phase" (2000-2007) differ from that in the "crisis and low interest phase" (2008-2018)?) there are five additional findings to the level debate. First, in all countries except Germany, average real yields on total wealth declined significantly in the crisis and low interest phase. Second, this decline was caused to a considerable extent by a reduction of average real yields (price effect); in 50% of all possible cases, assets' real yields were subject to declines. In contrast, portfolio shifts (volume effect) played only a minor role. Third, though the reduction in real yields on real estate dominated the reduction in real yields on total wealth, real returns on non-financial assets were still the largest component of total asset returns. Fourth, the "expropriation of the saver argument" cannot be confirmed, as deposits' real yields were always insignificant for total returns despite recent negative returns in all countries except France. Fifth, the argument that overall real yields were boosted by positive revaluation effects of securities during the low interest phase cannot be supported as well, as first the contribution of securities to the real yield on total assets was always comparatively small due to low portfolio shares, and second, because securities' real returns and thereby their overall real yield contributions increased only slightly in 50% of all cases, and decreased in the remaining cases.

Turning to the efficiency debate and research question three (can portfolio allocations in 2018 be considered as efficient in terms of Markowitz's mean-variance approach, and in comparison to simple common heuristic portfolio allocation strategies?) three results are worth highlighting. First, the potential for increases in expected returns by short-run or medium-run portfolio adjustments based on the mean-variance approach are very limited, because actual household portfolios seem to be comparatively close to the optimum. If households were to optimize their portfolios both in the short run and in the medium run, they should mostly follow a strategy which is against the broad recommendations of financial advisors, namely to reduce the shares of equities and investment fund shares; in the medium run, they should additionally increase the share of non-financial assets. A clear improvement in overall portfolio performance based on CV values can be only achieved by choosing the corner portfolios, while the maximum expected return portfolios showed mixed results. Second, the potential for risk reduction based on the mean-variance approach seems as well to be mostly limited; only a shift to the minimum variance portfolio in the medium run results in a significant risk reduction. Hence, households' 2018 portfolios are as well close to the optimum as regards risk. A potential re-allocation of portfolios towards a minimization of risk would as well require the considerable reduction of equity and investment fund holdings; moreover, a medium run strategy should also involve a reduction of real estate. An improvement in overall portfolio performance can again be achieved by choosing the corner portfolios; a switch to the minimum variance portfolios would result mostly everywhere in a deterioration of portfolio performance. Third, the potential for increasing returns or reducing risk seems to be either non-existent or very limited should households follow heuristic portfolio allocation strategies. As to overall portfolio performance based on CV values, both improvements and deteriorations are possible. Our results therefore suggest that the claim that households are unable to sufficiently diversify their portfolios cannot be verified, at least not at the aggregate level.

7 References

Adler, T., and M. Kritzman, “Mean–variance versus full–scale optimisation: In and out of sample,” *Journal of Asset Management*, 7, 302–311, 2007.
Ampudia, M., and M. Ehrmann, “Macroeconomic Experiences and Risk Taking of Euro Area Households”, *European Economic Review*, 91, 146–156, 2017.
Andreasch, M., M. P. Radke and M. Rupprecht, “Rendite privater Haushalte nach Vermögensgruppen – Deutschland versus Österreich“, *Wirtschaftsdienst*, 100, 446–453, 2020.
Annuß, C., and M. Rupprecht, „Anlageverhalten privater Haushalte in Deutschland: Die Rolle der realen Rendite“, *DIW Quarterly Journal of Economic Research*, 85, 95–109, 2016.
Badarinza, C., J. Y. Campbell and T. Ramadorai, “International Comparative Household Finance”, *Annual Review of Economics*, 8, 111-144, 2016.
Benartzi, S., and R. Thaler, “Naive Diversification Strategies in Retirement Saving Plans”, *American Economic Review*, 91, 79–98, 2001.
———, “Heuristics and Biases in Retirement Savings Behavior”, *Journal of Economic Perspectives*, 21, 81–104, 2007.
Bindseil, U., C. Domnick and J. Zeuner, “Critique of accommodating central bank policies and the “expropriation of the saver” – a review”, *ECB Occasional Paper Series*, 161, 2015.
Campbell, J. Y., “Household Finance,” *Journal of Finance*, 61, 1553–1604, 2006.
Cremers, J. H., M. Kritzman and S. Page, “Portfolio formation with higher moments and plausible utility”, *Revere Street Working Paper Series, Financial Economics*, 272-12, 2003.
———, “Optimal Hedge Fund Allocations: Do Higher Moments Matter”, *The Journal of Portfolio Management*, 31, 70–81, 2005.
Deuflhard, F., G. Dimitris and I. Roman, “Financial Literacy and Savings Account Returns”, *Journal of the European Economic Association*, 17, 131-164, 2019.
Deutsche Bundesbank, “German households’ saving and investment behavior in light of the low-interest-rate environment”, *Monthly Report*, October 2015, 13-31, 2015.
European Commission, *European system of accounts – ESA 2010*, Publications Office of the European Union, Luxembourg, 2013.
European Central Bank, “New international standards in statistics – enhancements to methodology and data availability”, *ECB Monthly Bulletin*, August 2014, 83-97, 2014.
———, *Introductory statement to the press conference, 12 September 2019*, available at https://www.ecb.europa.eu/press/pressconf/2019/html/ecb.is190912~658eb51d68.en.html, Frankfurt. 2019.
———, *ECB announces €750 billion Pandemic Emergency Purchase Programme (PEPP), press release, 18 March 2020*, available at https://www.ecb.europa.eu/press/pr/date/2020/html/ecb.pr200318_1~3949d6f266.en.html, Frankfurt, 2020a.
———, *Monetary policy decisions, press release, 4 June 2020*, available at https://www.ecb.europa.eu/press/pr/date/2020/html/ecb.mp200604~a307d3429c.en.html, Frankfurt, 2020b.
———, *Monetary policy decisions, press release, 10 December 2020*, available at https://www.ecb.europa.eu/press/pr/date/2020/html/ecb.mp201210~8c2778b843.en.html, Frankfurt, 2020c.
French, K. and J. Poterba, “Investor Diversification and International Equity Markets”, *American Economic Review*, 81, 222-226, 1991.
Geiger, F., J. Muellbauer and M. Rupprecht, “The housing market, household portfolios and the German consumer”, *ECB Working Paper*, No. 1904, 2016.
Goetzmann, W. N. and A. Kumar, “Equity Portfolio Diversification,” *Review of Finance*, 12, 433-463, 2008.
Gomes, F., M. Haliassos and T. Ramadorai, “Household Finance”, *Journal of Economic Literature*, forthcoming, 2020.
Heise, M., “How the ECB Hurts Europe’s Savers”, *The Wall Street Journal, 2 February 2016*, available at https://www.wsj.com/articles/how-the-ecb-hurts-europes-savers-1454445450, 2016.
Household Finance and Consumption Network, “The household finance and consumption survey: Results from the 2017 wage”, *ECB Statistics Paper*, No. 36, 2020
Jacobs, H., S. Müller. and M. Weber, “How should individual investors diversify? An empirical evaluation of alternative asset allocation policies”, *Journal of Financial Markets*, 19, 62-85, 2014.
Jensen, M. C., “The Performance of Mutual Funds in the Period 1956-1964”, *Journal of Finance*, 23, 389-416, 1968.
Levy, H., and H. M. Markowitz, “Approximating expected utility by a function of mean and variance”, *American Economic Review,* 69, 308-317, 1979.
Markowitz, H. M., “Portfolio Selection”, Journal of Finance, 7, 77-91, 1952.
———, *Portfolio Selection: Efficient Diversification of Investments*, John Wiley & Sonss, New York, 1959.
———, “Mean-variance approximation to geometric mean”, *Annals of Financial Economics*, 7, 1-30, 2012.
———, “Mean-variance approximations to expected utility”, *European Journal of Operational Research*, 234, 346-355, 2014.
Oesterreichische Nationalbank, *Liquidität für Haushalte wichtiger als Rendite, press release, 22 October 2019*, available at https://www.oenb.at/Presse/20191022_1.html, Vienna, 2019.

Radke, M. P. and M. Rupprecht, „Was zehn Jahre Niedrigzinsen für die Sparer in Europa bedeuten", *Börsenzeitung, Nr. 204, 24. Oktober 2018,* available at: https://www.boersen-zeitung.de/ index.php?li=1&artid=2018204038&artsubm=&subm=), 2018.

———, "The Bill, Please! Households' Real Returns on Financial Assets Since the Introduction of the Euro", *Intereconomics, Review of European Economic Policy*, 54, 106-113, 2019.

Rupprecht, M., "Low Interest Rates and Household Portoflio Behaviour in Euro Area Countries", *Intereconomics, Review of European Economic Policy*, 53, 174-178, 2018.

———, "Income and Wealth of Euro Area Households in Times of Ultra-Loose Monetary Policy: Stylised facts from new national and financial accounts data", *Empirica*, 47, 281-302, 2020.

Samuelson, P., "The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments", *The Review of Economic Studies,* 37, 537–542, 1970.

Schröder, C., J. König, A. Fedorets, J. Goebel, M. M. Grabka, H. Lüthen, M. Metzing, F. Schikora and S. Liebig, "The economic research potentials of the German Socio-Economic Panel study", *German Economic Review*, 21, 313-333. 2020.

Sharpe, W. F., "Mutual Fund Performance", *Journal of Business*, 39, 119-138, 1966.

Stolper, O. A. and A. Walter, "Financial Literacy, Financial Advice and Financial Behaviour", *Journal of Business Economics*, 87, 581-643, 2017.

Tobin, J., "A general equilibrium approach to monetary theory", *Journal of Money, Credit and Banking*, 1, 15-29, 1969.

Tobin, J. and W. C. Brainard, "Pitfalls in financial model building", *American Economic Review*, 58, 99-122, 1968.

———, "Asset Markets and the Cost of Capital", in R. Nelson and B. Balassa, ed., *Economic Progress: Private Values and Public Policy (Essays in Honor of William Fellner)*, 235-262, North-Holland, Amsterdam, 1977.

Treynor, J. L., "How to Rate Management of Investment Funds", *Harvard Business Review*, 43, 63-75, 1965.

United Nations, *System of National Accounts 2008*, United Nations, New York, 2010.

Non-financial assets	**Financial assets**
Produced non-financial assets	*Monetary gold and special drawing rights (SDRs)*
Fixed assets	Currency and deposits
Dwellings	Debt securities
Other buildings and structures	*Loans*
Machinery and equipment	Equity and investment fund shares or units
Cultivated biological resources	Equity
Intellectual property products	Listed shares
Inventories	Unlisted shares
Valuables	*Other equity*
	Investment fund shares or units
Non-produced financial assets	Insurance, pension and standardised guarantee schemes
Natural resources	*Financial derivatives and employee stock options*
Land	*Other accounts receivable/payable*
Land underlying buildings and structures	
Other land	
Mineral and energy reserves	
Non-cultivated biological resources	
Water resources	
Other natural resources	
Contracts, leases and licences	
Purchases less sales of goodwill and marketing assets	

Figure 1. The asset side of a stylized household sector balance sheet according to ESA 2010

Note: All assets in italics were not included in the real return computation.

Source: Authors' illustration based on European Commission (2013).

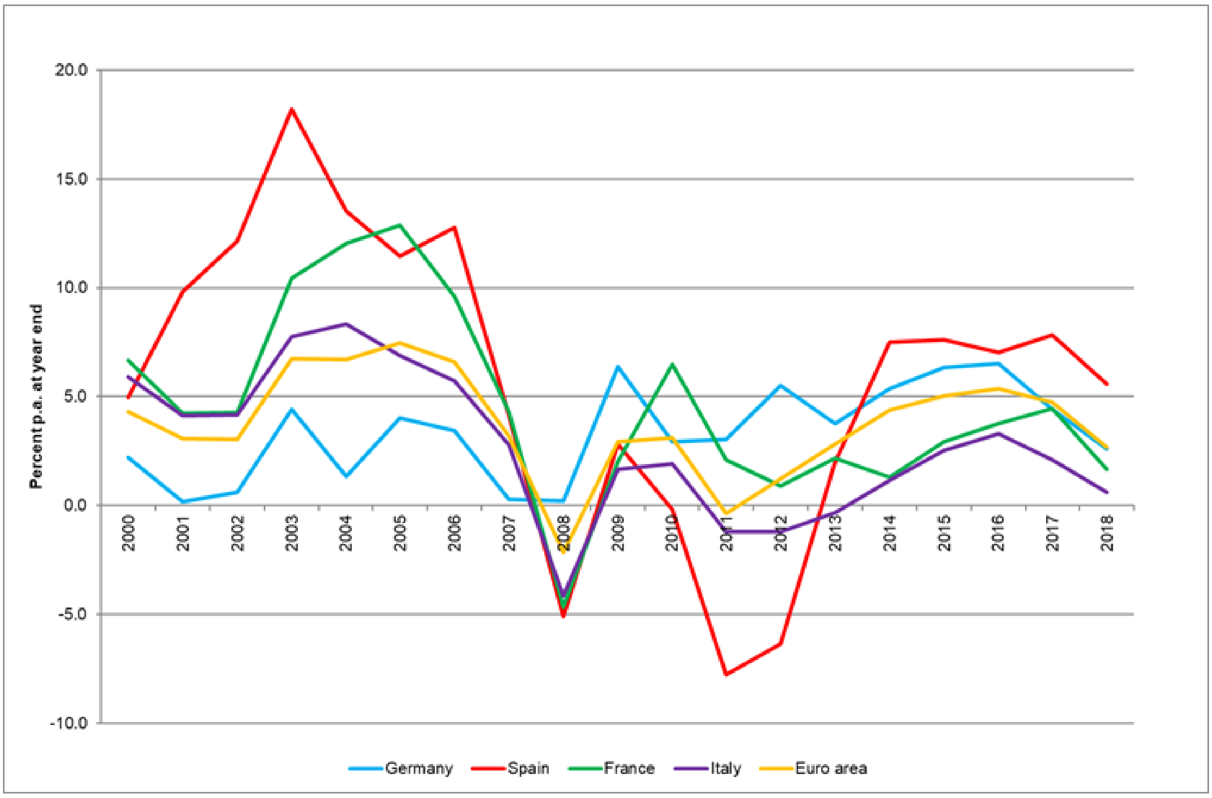

Figure 2. Real returns on euro area households' total wealth.

Source: Authors' calculations based on ECB, Eurostat, OECD, Thomson Reuters.

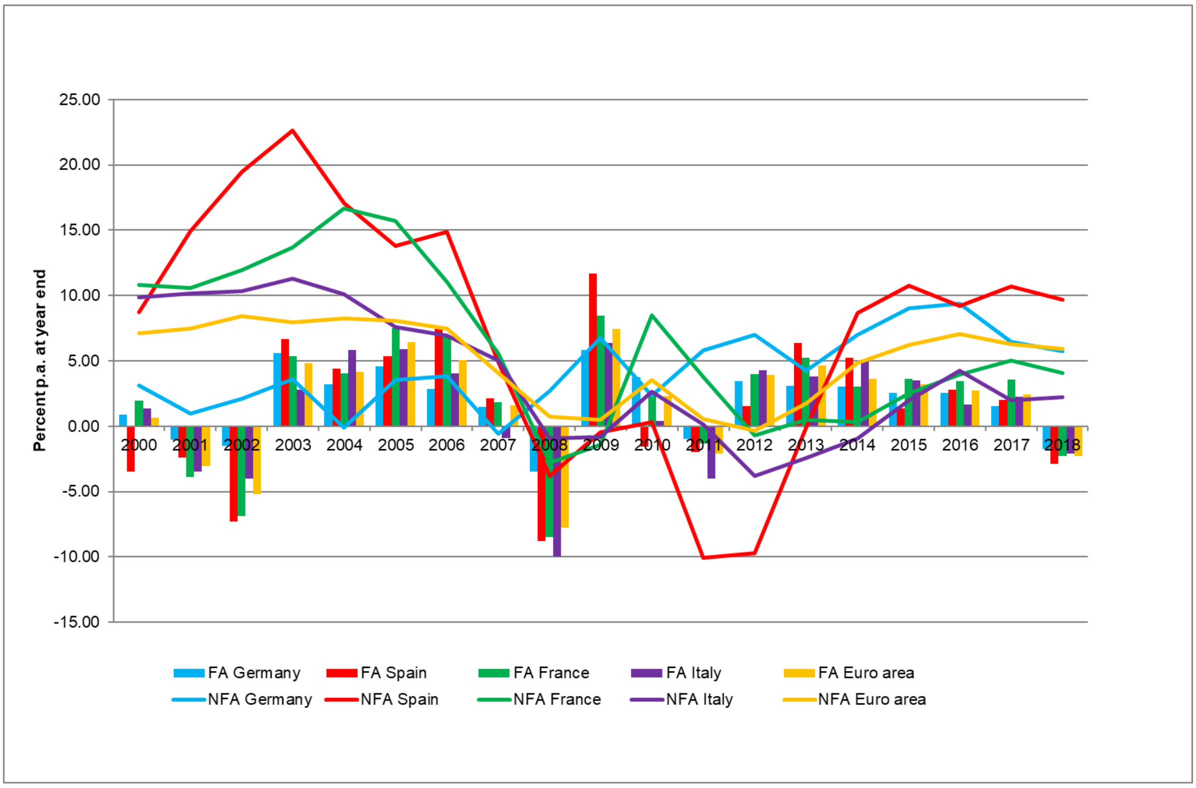

Figure 3. Real returns on euro area households' non-financial and financial assets.

Notes: 1. FA: Financial assets; NFA: Non-financial assets. 2. The respective national real returns on financial asset were calculated as weighted real returns on all national financial assets. Consequently, since national real returns on non-financial and financial assets have not been weighted in relation to national total assets, they do not add up to the respective national real returns on total assets.

Source: Authors' calculations based on ECB, Eurostat, OECD, Thomson Reuters.

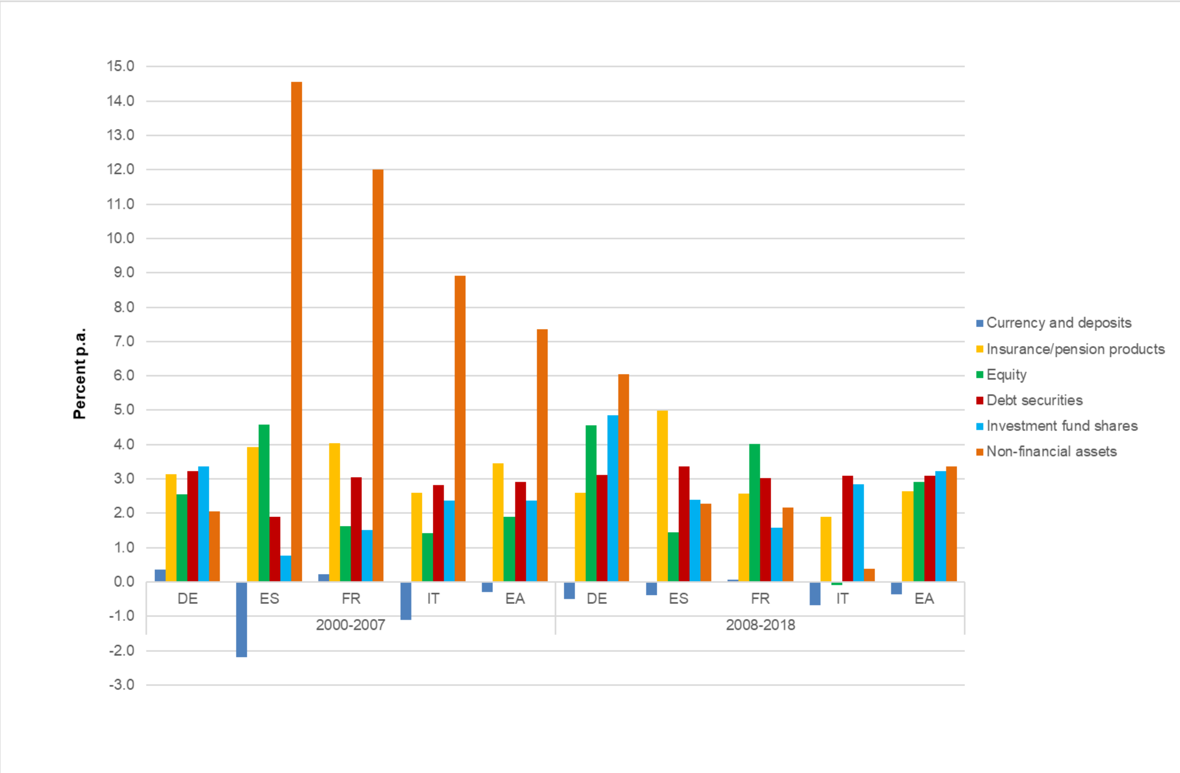

Figure 4. Average real returns on individual assets.

Note: DE: Germany; ES: Spain; FR: France; IT: Italy; EA: Euro area.

Source: Authors' calculations based on ECB, Eurostat, OECD, Thomson Reuters.

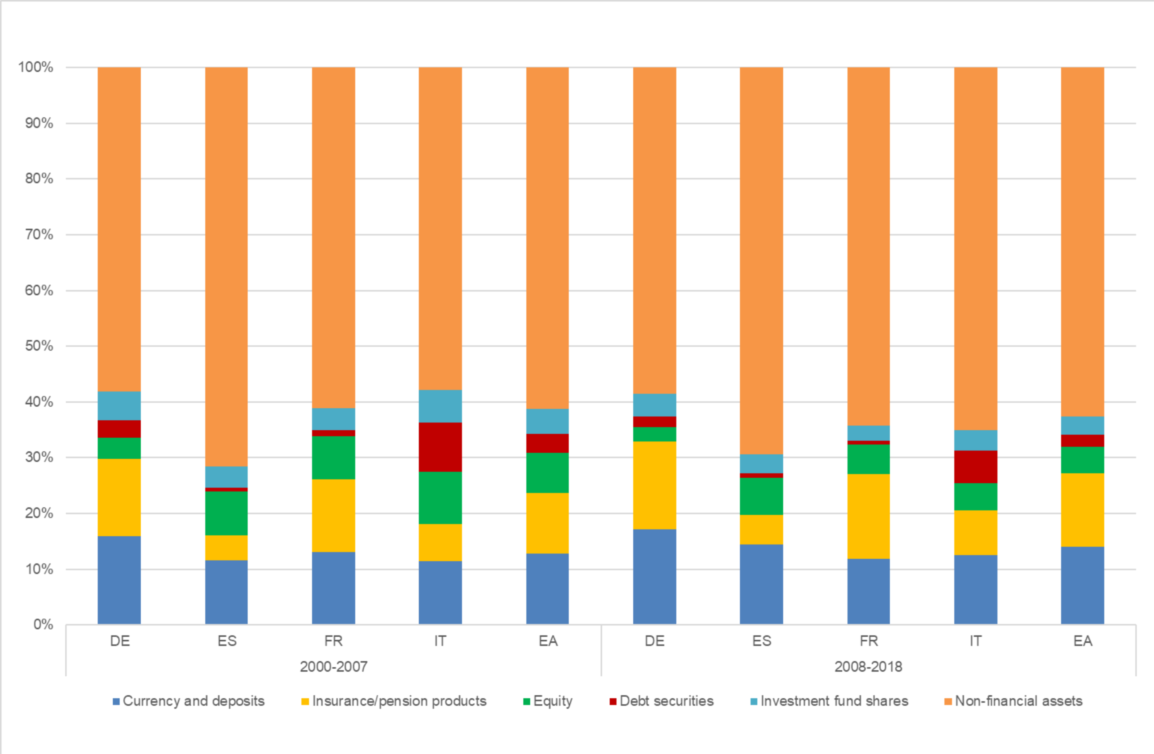

Figure 5. Euro area households' portfolio structure.

Note: DE: Germany; ES: Spain; FR: France; IT: Italy; EA: Euro area.

Source: Authors' calculations based on ECB, Eurostat, OECD, Thomson Reuters.

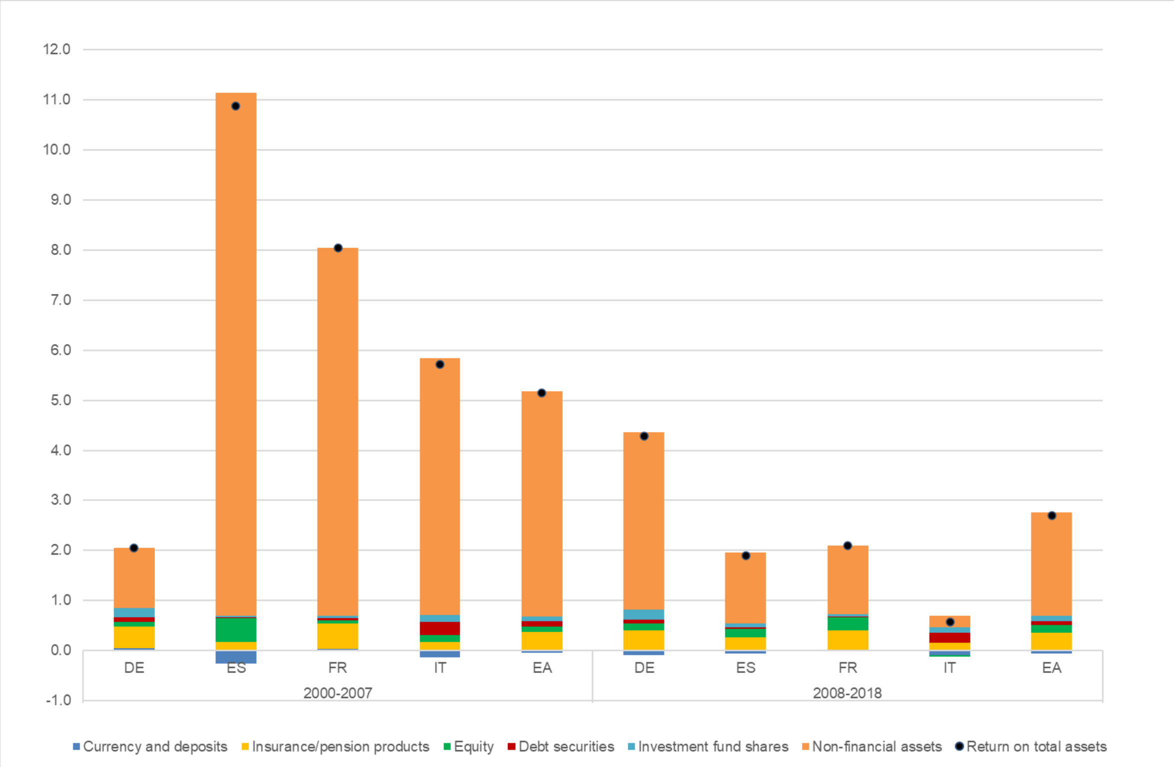

Figure 6. Contributions of individual assets to the real return on total assets.

Notes: 1. DE: Germany; ES: Spain; FR: France; IT: Italy; EA: Euro area. 2. The contribution of each asset to the real return on total assets represents the real return weighted by the respective portfolio share of total assets. The sum of the contributions of all assets is the total asset return.

Source: Authors' calculations based on ECB, Eurostat, OECD, Thomson Reuters.

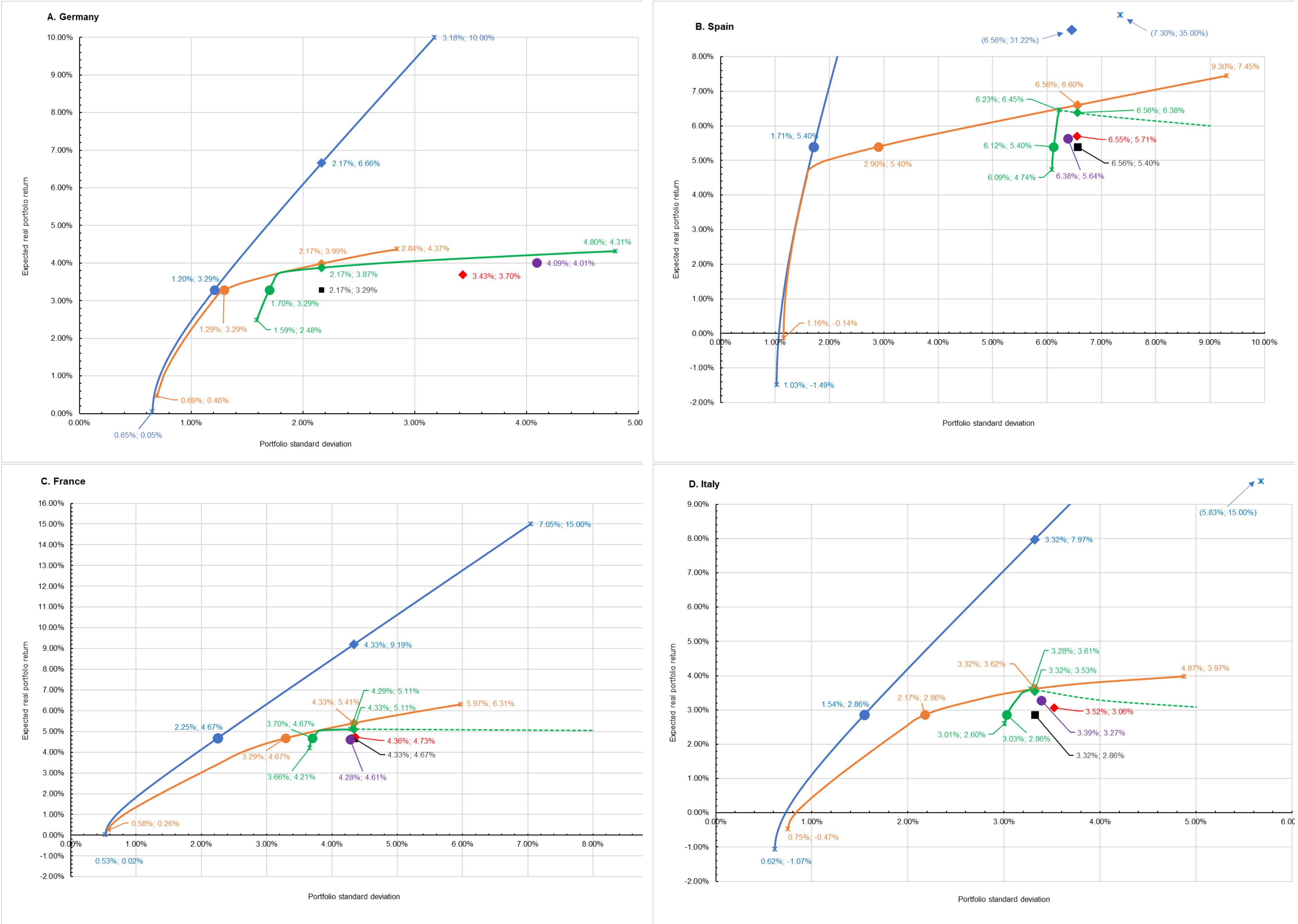

Figure 7. Efficient frontiers.

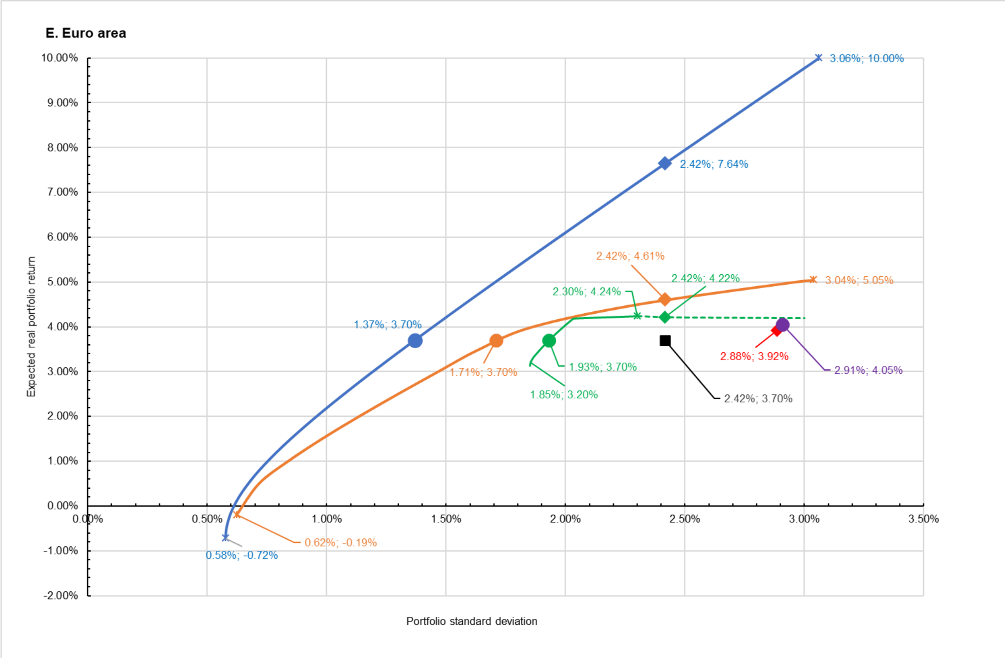

Figure 7 (continued). Efficient frontiers.

Note: The numbers attached to the data points first indicate the standard deviations followed by expected real returns of the respective selected portfolios.

Source: Authors' calculations.

TABLE 1
DESCRIPTIVE STATISTICS - REAL YIELDS ON TOTAL, FINANCIAL AND NON-FINANCIAL WEALTH

A. Germany

	Total Wealth			Financial wealth			Non-financial wealth		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	2.06	4.28	3.34	2.02	1.77	1.87	2.06	6.05	4.37
Median	1.75	4.41	3.44	2.17	2.55	2.55	2.60	6.47	3.86
Maximum	4.43	6.52	6.52	5.61	5.81	5.81	3.86	9.37	9.37
Minimum	0.16	0.19	0.16	-1.53	-3.48	-3.48	-0.59	2.34	-0.59
Std. Dev.	1.72	1.99	2.15	2.54	2.74	2.58	1.76	2.25	2.85
Skewness	0.23	-0.60	-0.08	-0.07	-0.64	-0.46	-0.45	-0.26	0.04
Kurtosis	1.44	2.53	1.85	1.81	2.48	2.34	1.60	2.26	2.18
Jarque-Bera	0.88	0.76	1.07	0.48	0.88	1.00	0.92	0.37	0.54
Probability	0.64	0.68	0.59	0.79	0.64	0.61	0.63	0.83	0.76
Observations	8	11	19	8	11	19	8	11	19

B. Spain

	Total Wealth			Financial wealth			Non-financial wealth		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	10.87	1.90	5.68	1.62	1.44	1.51	14.55	2.28	7.45
Median	11.80	2.81	7.04	3.29	1.55	2.01	14.89	0.29	9.19
Maximum	18.20	7.84	18.20	7.44	11.71	11.71	22.64	10.78	22.64
Minimum	4.14	-7.79	-7.79	-7.26	-8.81	-8.81	4.90	-10.10	-10.10
Std. Dev.	4.59	5.96	6.98	5.38	5.39	5.23	5.65	7.99	9.30
Skewness	-0.15	-0.53	-0.40	-0.48	0.03	-0.18	-0.36	-0.35	-0.42
Kurtosis	2.28	1.75	2.55	1.79	3.07	2.54	2.36	1.70	2.33
Jarque-Bera	0.20	1.23	0.66	0.80	0.00	0.27	0.32	1.00	0.90
Probability	0.90	0.54	0.72	0.67	1.00	0.87	0.85	0.61	0.64
Observations	8	11	19	8	11	19	8	11	19

C. France

	Total Wealth			Financial wealth			Non-financial wealth		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	8.05	2.10	4.61	2.10	1.99	2.03	12.00	2.17	6.31
Median	8.12	2.11	4.24	3.00	3.46	3.46	11.49	2.53	5.04
Maximum	12.87	6.50	12.87	7.48	8.44	8.44	16.64	8.49	16.64
Minimum	4.24	-4.67	-4.67	-6.88	-8.47	-8.47	5.58	-2.78	-2.78
Std. Dev.	3.63	2.77	4.30	5.11	4.51	4.63	3.46	3.31	5.97
Skewness	0.09	-1.04	0.25	-0.72	-1.07	-0.90	-0.42	0.27	0.20
Kurtosis	1.37	4.73	3.00	2.22	3.80	3.00	2.70	2.32	1.85
Jarque-Bera	0.90	3.33	0.20	0.90	2.38	2.54	0.27	0.34	1.18
Probability	0.64	0.19	0.91	0.64	0.30	0.28	0.87	0.84	0.55
Observations	8	11	19	8	11	19	8	11	19

D. Italy

	Total Wealth			Financial wealth			Non-financial wealth		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	5.71	0.58	2.74	1.44	1.02	1.20	8.91	0.38	3.97
Median	5.82	1.15	2.53	2.08	2.28	2.28	9.96	0.15	2.65
Maximum	8.33	3.29	8.33	5.88	6.37	6.37	11.28	4.24	11.28
Minimum	2.79	-4.14	-4.14	-4.00	-9.98	-9.98	5.04	-3.82	-3.82
Std. Dev.	1.93	2.15	3.28	3.91	4.77	4.32	2.14	2.43	4.87
Skewness	-0.10	-0.87	-0.09	-0.26	-1.16	-0.95	-0.74	-0.15	0.11
Kurtosis	1.80	3.08	2.49	1.61	3.53	3.34	2.21	2.07	1.67
Jarque-Bera	0.49	1.40	0.23	0.73	2.58	2.95	0.94	0.44	1.43
Probability	0.78	0.50	0.89	0.69	0.28	0.23	0.63	0.80	0.49
Observations	8	11	19	8	11	19	8	11	19

TABLE 1 (CONTINUED)
DESCRIPTIVE STATISTICS - REAL YIELDS ON TOTAL, FINANCIAL AND NON-FINANCIAL WEALTH

E. Euro area

	Total Wealth			**Financial wealth**			**Non-financial wealth**		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	5.14	2.70	3.73	1.81	1.65	1.72	7.35	3.37	5.05
Median	5.44	2.91	3.20	2.88	2.73	2.73	7.70	3.53	6.24
Maximum	7.48	5.35	7.48	6.43	7.45	7.45	8.44	7.07	8.44
Minimum	3.05	-2.17	-2.17	-5.19	-7.75	-7.75	4.07	-0.33	-0.33
Std. Dev.	1.91	2.35	2.45	4.15	4.17	4.04	1.40	2.81	3.04
Skewness	-0.05	-0.84	-0.61	-0.61	-1.01	-0.85	-1.84	0.00	-0.59
Kurtosis	1.20	2.76	3.20	2.00	3.46	2.90	5.07	1.33	1.83
Jarque-Bera	1.08	1.31	1.19	0.83	1.96	2.29	5.93	1.27	2.18
Probability	0.58	0.52	0.55	0.66	0.38	0.32	0.05	0.53	0.34
Observations	8	11	19	8	11	19	8	11	19

Notes:
1. All statistics either quoted in or based on percentages.
2. The respective national real returns on financial wealth were calculated as weighted real returns on all
2. Jarque-Bera test statistic: H0: Normal distribution; H1: Non-normal distribution. A small probability leads to

Source: Authors' calculations based on ECB, Eurostat, OECD, Thomson Reuters.

TABLE 2
DESCRIPTIVE STATISTICS - REAL YIELDS ON INDIVIDUAL ASSETS

A. Germany

	Currency and deposits			Debt securities			Equity			Investment fund shares			Insurance/pension products			Non-financial assets		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	0.36	-0.49	-0.13	3.22	3.12	3.16	2.55	4.57	3.72	3.37	4.86	4.23	3.15	2.59	2.83	2.06	6.05	4.37
Median	0.25	-0.27	0.02	3.34	3.07	3.12	9.69	9.54	9.54	2.61	6.50	6.50	2.91	2.59	2.59	2.60	6.47	3.86
Maximum	0.96	1.15	1.15	7.80	10.17	10.17	28.51	23.49	28.51	13.87	18.19	18.19	6.03	4.93	6.03	3.86	9.37	9.37
Minimum	-0.17	-1.71	-1.71	-2.03	-2.09	-2.09	-40.69	-42.73	-42.73	-7.95	-18.82	-18.82	1.61	0.47	0.47	-0.59	2.34	-0.59
Std. Dev.	0.41	0.85	0.81	3.27	4.10	3.67	23.67	20.46	21.25	8.94	10.48	9.63	1.42	1.27	1.32	1.76	2.25	2.85
Skewness	0.34	0.31	-0.40	-0.35	0.48	0.26	-0.68	-1.18	-0.94	-0.01	-1.02	-0.68	0.98	0.06	0.54	-0.45	-0.26	0.04
Kurtosis	1.77	2.33	2.34	2.12	1.89	2.02	2.31	3.54	2.90	1.27	3.38	2.71	3.10	2.52	3.30	1.60	2.26	2.18
Jarque-Bera	0.65	0.38	0.85	0.42	0.99	0.98	0.77	2.68	2.82	1.00	1.96	1.52	1.28	0.11	1.00	0.92	0.37	0.54
Probability	0.72	0.83	0.65	0.81	0.61	0.61	0.68	0.26	0.24	0.61	0.38	0.47	0.53	0.95	0.61	0.63	0.83	0.76
Observations	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19

B. Spain

	Currency and deposits			Debt securities			Equity			Investment fund shares			Insurance/pension products			Non-financial assets		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	-2.18	-0.37	-1.13	1.90	3.37	2.75	4.59	1.45	2.77	0.77	2.39	1.71	3.93	4.98	4.54	14.55	2.28	7.45
Median	-2.20	-0.56	-1.40	2.74	1.92	2.17	10.78	3.19	4.25	0.05	2.19	2.02	4.33	4.45	4.45	14.89	0.29	9.19
Maximum	-0.67	1.61	1.61	6.10	10.80	10.80	25.36	39.55	39.55	3.65	8.06	8.06	5.67	7.82	7.82	22.64	10.78	22.64
Minimum	-3.10	-1.89	-3.10	-3.93	-1.46	-3.93	-30.97	-39.51	-39.51	-2.69	-3.26	-3.26	1.22	3.19	1.22	4.90	-10.10	-10.10
Std. Dev.	0.73	1.20	1.36	3.23	4.21	3.81	20.30	20.70	20.02	2.22	3.71	3.20	1.55	1.68	1.67	5.65	7.99	9.30
Skewness	0.89	0.32	0.55	-0.61	0.41	0.31	-0.58	-0.08	-0.29	0.05	0.16	0.46	-0.66	0.42	0.10	-0.36	-0.35	-0.42
Kurtosis	3.55	1.72	2.28	2.42	1.80	2.43	2.04	3.15	2.66	1.89	1.93	2.44	2.20	1.65	2.59	2.36	1.70	2.33
Jarque-Bera	1.17	0.94	1.37	0.60	0.96	0.56	0.76	0.02	0.35	0.42	0.58	0.91	0.80	1.15	0.17	0.32	1.00	0.90
Probability	0.56	0.62	0.50	0.74	0.62	0.76	0.68	0.99	0.84	0.81	0.75	0.64	0.67	0.56	0.92	0.85	0.61	0.64
Observations	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19

TABLE 2 (CONTINUED)
DESCRIPTIVE STATISTICS - REAL YIELDS ON INDIVIDUAL ASSETS

C. France

	Currency and deposits			Debt securities			Equity			Investment fund shares			Insurance/pension products			Non-financial assets		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	0.22	0.08	0.14	3.05	3.03	3.04	1.62	4.01	3.00	1.51	1.57	1.55	4.04	2.57	3.19	12.00	2.17	6.31
Median	0.23	-0.13	0.18	3.60	2.39	2.44	4.36	9.19	7.80	2.00	2.63	2.63	5.90	3.28	3.98	11.49	2.53	5.04
Maximum	0.72	1.40	1.40	7.34	10.96	10.96	25.03	28.77	28.77	4.91	8.83	8.83	7.40	7.29	7.40	16.64	8.49	16.64
Minimum	-0.15	-1.24	-1.24	-2.26	-1.93	-2.26	-34.02	-41.37	-41.37	-3.85	-9.12	-9.12	-2.94	-5.65	-5.65	5.58	-2.78	-2.78
Std. Dev.	0.27	0.76	0.60	3.12	4.18	3.67	19.66	19.48	19.04	3.48	4.73	4.14	3.78	3.35	3.51	3.46	3.31	5.97
Skewness	0.44	0.11	-0.11	-0.41	0.77	0.52	-0.70	-1.10	-0.93	-0.29	-0.85	-0.75	-1.03	-1.31	-1.03	-0.42	0.27	0.20
Kurtosis	2.74	2.28	3.29	2.20	2.25	2.41	2.39	3.72	3.12	1.49	3.62	3.54	2.45	4.57	3.42	2.70	2.32	1.85
Jarque-Bera	0.28	0.26	0.11	0.44	1.35	1.13	0.77	2.47	2.74	0.87	1.51	2.01	1.53	4.29	3.53	0.27	0.34	1.18
Probability	0.87	0.88	0.95	0.80	0.51	0.57	0.68	0.29	0.25	0.65	0.47	0.37	0.47	0.12	0.17	0.87	0.84	0.55
Observations	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19

D. Italy

	Currency and deposits			Debt securities			Equity			Investment fund shares			Insurance/pension products			Non-financial assets		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	-1.11	-0.68	-0.86	2.81	3.10	2.98	1.41	-0.10	0.54	2.37	2.84	2.64	2.60	1.89	2.19	8.91	0.38	3.97
Median	-1.15	-0.79	-0.93	3.61	2.13	2.23	6.05	7.59	7.59	1.97	3.71	2.10	2.72	3.23	2.75	9.96	0.15	2.65
Maximum	-0.35	0.53	0.53	6.82	14.34	14.34	17.82	23.46	23.46	6.34	9.88	9.88	3.98	5.06	5.06	11.28	4.24	11.28
Minimum	-1.68	-2.12	-2.12	-2.68	-5.29	-5.29	-25.58	-46.97	-46.97	-1.58	-5.81	-5.81	0.60	-4.38	-4.38	5.04	-3.82	-3.82
Std. Dev.	0.42	0.98	0.81	3.26	5.75	4.74	17.28	19.74	18.25	3.01	5.04	4.20	1.11	2.75	2.19	2.14	2.43	4.87
Skewness	0.39	-0.18	0.31	-0.46	0.67	0.64	-0.56	-1.20	-1.02	0.09	-0.22	-0.11	-0.61	-1.11	-1.53	-0.74	-0.15	0.11
Kurtosis	2.53	1.69	2.18	2.03	2.71	3.36	1.78	3.89	3.44	1.55	2.03	2.34	2.39	3.40	5.32	2.21	2.07	1.67
Jarque-Bera	0.28	0.85	0.84	0.60	0.87	1.40	0.91	2.99	3.44	0.71	0.52	0.38	0.62	2.34	11.67	0.94	0.44	1.43
Probability	0.87	0.66	0.66	0.74	0.65	0.50	0.63	0.22	0.18	0.70	0.77	0.83	0.73	0.31	0.00	0.63	0.80	0.49
Observations	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19

TABLE 2 (CONTINUED)
DESCRIPTIVE STATISTICS - REAL YIELDS ON INDIVIDUAL ASSETS

E. Euro area

	Currency and deposits			**Debt securities**			**Equity**			**Investment fund shares**			**Insurance/pension products**			**Non-financial assets**		
	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018	2000-2007	2008-2018	2000-2018
Mean	-0.30	-0.37	-0.34	2.91	3.10	3.02	1.90	2.91	2.49	2.38	3.22	2.87	3.44	2.63	2.97	7.35	3.37	5.05
Median	-0.37	-0.39	-0.39	3.77	2.36	2.63	7.70	6.19	6.19	2.00	4.41	4.06	3.86	3.45	3.47	7.70	3.53	6.24
Maximum	0.15	1.03	1.03	7.10	12.71	12.71	23.61	28.02	28.02	7.78	11.99	11.99	5.47	6.01	6.01	8.44	7.07	8.44
Minimum	-0.63	-1.35	-1.35	-2.55	-3.36	-3.36	-34.68	-45.34	-45.34	-3.88	-11.08	-11.08	1.02	-2.28	-2.28	4.07	-0.33	-0.33
Std. Dev.	0.26	0.81	0.63	3.23	4.99	4.23	20.22	20.58	19.86	4.92	6.68	5.86	1.35	2.18	1.88	1.40	2.81	3.04
Skewness	0.75	0.33	0.28	-0.48	0.71	0.59	-0.74	-1.14	-0.98	0.02	-0.80	-0.58	-0.42	-0.86	-1.06	-1.84	0.00	-0.59
Kurtosis	2.40	1.76	2.61	2.12	2.50	2.90	2.28	3.78	3.18	1.29	2.94	2.74	2.65	3.59	4.49	5.07	1.33	1.83
Jarque-Bera	0.86	0.90	0.37	0.56	1.05	1.10	0.91	2.65	3.05	0.98	1.17	1.10	0.28	1.52	5.32	5.93	1.27	2.18
Probability	0.65	0.64	0.83	0.76	0.59	0.58	0.64	0.27	0.22	0.61	0.56	0.58	0.87	0.47	0.07	0.05	0.53	0.34
Observations	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19	8	11	19

Notes:
1. All statistics either quoted in or based on percentages.
2. Jarque-Bera test statistic: H0: Normal distribution; H1: Non-normal distribution. A small probability leads to a rejection of H0.

Source: Authors' calculations based on ECB, Eurostat, OECD, Thomson Reuters.

TABLE 3
VARIANCE-COVARIANCE MATRICES AND CORRELATIONS

A. Germany

Covariance / **Correlation**	Currency and deposits	Debt securities	Equity	Investment fund shares	Insurance/ pension products	Non-financial assets
Currency and deposits	0.66					
	1.00					
Debt securities	1.09	13.50				
	0.37	1.00				
Equity	0.77	-2.10	451.49			
	0.04	-0.03	1.00			
Investment fund shares	1.07	9.74	183.74	92.64		
	0.14	0.28	0.90	1.00		
Insurance products	0.73	2.32	4.88	4.23	1.75	
	0.68	0.48	0.17	0.33	1.00	
Non-financial assets	-0.68	0.28	14.32	8.15	0.20	8.11
	-0.29	0.03	0.24	0.30	0.05	1.00

B. Spain

Covariance / **Correlation**	Currency and deposits	Debt securities	Equity	Investment fund shares	Insurance/ pension products	Non-financial assets
Currency and deposits	1.84					
	1.00					
Debt securities	1.83	14.48				
	0.35	1.00				
Equity	9.69	10.57	400.82			
	0.36	0.14	1.00			
Investment fund shares	2.38	8.02	37.73	10.23		
	0.55	0.66	0.59	1.00		
Insurance products	1.42	3.09	-1.24	2.00	2.78	
	0.63	0.49	-0.04	0.37	1.00	
Non-financial assets	-5.16	-5.62	28.64	-2.67	-1.30	86.57
	-0.41	-0.16	0.15	-0.09	-0.08	1.00

TABLE 3 (CONTINUED)
VARIANCE-COVARIANCE MATRICES AND CORRELATIONS

C. France

Covariance **Correlation**	Currency and deposits	Debt securities	Equity	Investment fund shares	Insurance/ pension products	Non-financial assets
Currency and deposits	0.35					
	1.00					
Debt securities	0.53	13.47				
	0.24	1.00				
Equity	4.29	2.26	362.71			
	0.38	0.03	1.00			
Investment fund shares	0.95	3.56	72.38	17.14		
	0.39	0.23	0.92	1.00		
Insurance products	0.91	0.16	59.83	12.88	12.34	
	0.43	0.01	0.89	0.89	1.00	
Non-financial assets	-0.35	-2.35	5.17	3.78	6.31	35.65
	-0.10	-0.11	0.05	0.15	0.30	1.00

D. Italy

Covariance **Correlation**	Currency and deposits	Debt securities	Equity	Investment fund shares	Insurance/ pension products	Non-financial assets
Currency and deposits	0.66					
	1.00					
Debt securities	0.11	22.51				
	0.03	1.00				
Equity	6.09	16.68	333.20			
	0.41	0.19	1.00			
Investment fund shares	1.04	13.14	59.06	17.68		
	0.30	0.66	0.77	1.00		
Insurance products	1.15	2.39	31.99	6.52	4.82	
	0.64	0.23	0.80	0.71	1.00	
Non-financial assets	-0.71	-3.81	-2.11	-2.17	1.95	23.76
	-0.18	-0.16	-0.02	-0.11	0.18	1.00

TABLE 3 (CONTINUED)
VARIANCE-COVARIANCE MATRICES AND CORRELATIONS

E. Euro area

Covariance **Correlation**	Currency and deposits	Debt securities	Equity	Investment fund shares	Insurance/ pension products	Non-financial assets
Currency and deposits	0.40					
	1.00					
Debt securities	0.70	17.91				
	0.26	1.00				
Equity	3.82	8.79	394.60			
	0.31	0.10	1.00			
Investment fund shares	1.32	10.45	103.06	34.35		
	0.36	0.42	0.89	1.00		
Insurance products	0.71	1.52	29.27	8.75	3.52	
	0.60	0.19	0.79	0.80	1.00	
Non-financial assets	0.14	-1.84	0.37	0.12	1.37	9.23
	0.07	-0.14	0.01	0.01	0.24	1.00

Notes:
1. Sample: 2000 to 2018.
2. Degree-of-freedom corrected covariances; covariances in squared percentage returns.

Source: Authors' calculations.

TABLE 4
MIMIMUM VARIANCE AND MAXIMUM EXPECTED REAL RETURN PORTFOLIOS

A. Germany

				Portfolio 2018	Optimal mean-variance portfolios					
					Model 1		Model 2		Model 3	
					Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return
Asset categories	Expected real return	Standard deviation	Coefficient of variation	Portfolio shares						
Currency and deposits	-0.13	0.81	-6.15	17.74	101.00	-228.30	86.77	0.00	41.96	0.00
Insurance/pension products	2.83	1.32	0.47	16.03	-13.54	292.46	0.00	0.00	0.00	0.00
Equity	3.72	21.25	5.71	2.66	0.57	6.02	0.00	0.00	0.00	0.00
Debt securities	3.16	3.67	1.16	1.07	-1.15	10.08	0.00	0.00	0.00	0.00
Investment fund shares	4.23	9.63	2.27	4.46	-2.47	-22.61	0.00	0.00	0.00	41.96
Non-financial assets	4.37	2.85	0.65	58.04	15.59	42.35	13.23	100.00	58.04	58.04
Portfolio performance										
Expected real portfolio return				3.29	0.05	10.00	0.46	4.37	2.48	4.31
Portfolio standard deviation				2.17	0.65	3.18	0.69	2.84	1.59	4.80
Coefficient of variation				0.66	14.17	0.32	1.50	0.65	0.64	1.11

TABLE 4 (CONTINUED)
MIMIMUM VARIANCE AND MAXIMUM EXPECTED REAL RETURN PORTFOLIOS

B. Spain

	Expected real return	Standard deviation	Coefficient of variation	Portfolio 2018	Optimal mean-variance portfolios					
					Model 1		Model 2		Model 3	
					Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return
Asset categories				Portfolio shares						
Currency and deposits	-1.13	1.36	-1.20	14.55	110.80	-547.57	85.99	0.00	30.21	0.00
Insurance/pension products	4.54	1.67	0.37	5.90	-19.54	673.50	7.23	0.00	0.00	32.72
Equity	2.77	20.02	7.22	6.94	-3.17	16.52	0.00	0.00	0.00	0.00
Debt securities	2.75	3.81	1.38	0.28	3.34	-62.71	0.00	0.00	2.51	0.00
Investment fund shares	1.71	3.20	1.87	5.05	-0.22	35.25	0.00	0.00	0.00	0.00
Non-financial assets	7.45	9.30	1.25	67.28	8.79	-14.99	6.78	100.00	67.28	67.28
Portfolio performance										
Expected real portfolio return				5.40	-1.49	35.00	-0.14	7.45	4.74	6.45
Portfolio standard deviation				6.56	1.03	7.30	1.16	9.30	6.09	6.23
Coefficient of variation				1.22	-0.69	0.21	-8.17	1.25	1.29	0.97

TABLE 4 (CONTINUED)
MIMIMUM VARIANCE AND MAXIMUM EXPECTED REAL RETURN PORTFOLIOS

C. France

				Portfolio 2018	Optimal mean-variance portfolios					
					Model 1		Model 2		Model 3	
					Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return
Asset categories	Expected real return	Standard deviation	Coefficient of variation	Portfolio shares						
Currency and deposits	0.14	0.60	4.34	12.82	101.18	-274.42	98.08	0.00	29.09	0.00
Insurance/pension products	3.19	3.51	1.10	16.50	-7.07	451.34	0.00	0.00	0.00	38.23
Equity	3.00	19.04	6.35	6.32	-1.79	-29.81	0.00	0.00	0.00	0.00
Debt securities	3.04	3.67	1.21	0.38	-3.58	140.63	0.00	0.00	9.14	0.00
Investment fund shares	1.55	4.14	2.67	2.20	9.19	-198.67	0.00	0.00	0.00	0.00
Non-financial assets	6.31	5.97	0.95	61.77	2.08	10.93	1.92	100.00	61.77	61.77
Portfolio performance										
Expected real portfolio return				4.67	0.02	15.00	0.26	6.31	4.21	5.11
Portfolio standard deviation				4.33	0.53	7.05	0.58	5.97	3.66	4.29
Coefficient of variation				0.93	21.46	0.47	2.28	0.95	0.87	0.84

TABLE 4 (CONTINUED)
MIMIMUM VARIANCE AND MAXIMUM EXPECTED REAL RETURN PORTFOLIOS

D. Italy

				Portfolio 2018	Optimal mean-variance portfolios					
					Model 1		Model 2		Model 3	
					Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return
Asset categories	Expected real return	Standard deviation	Coefficient of variation	Portfolio shares						
Currency and deposits	-0.86	0.81	-0.94	14.00	114.12	-392.82	91.41	0.00	27.39	0.00
Insurance/pension products	2.19	2.19	1.00	10.08	-33.04	507.72	0.00	0.00	0.00	0.00
Equity	0.54	18.25	34.02	4.88	-0.59	-48.76	0.00	0.00	0.00	0.00
Debt securities	2.98	4.74	1.59	2.95	0.56	-6.19	3.01	0.00	9.42	36.82
Investment fund shares	2.64	4.20	1.59	4.90	10.25	48.39	0.00	0.00	0.00	0.00
Non-financial assets	3.97	4.87	1.23	63.18	8.69	-8.34	5.58	100.00	63.18	63.18
Portfolio performance										
Expected real portfolio return				2.86	-1.07	15.00	-0.47	3.97	2.56	3.61
Portfolio standard deviation				3.32	0.62	5.83	0.75	4.87	3.01	3.28
Coefficient of variation				1.16	-0.57	0.39	-1.58	1.23	1.18	0.91

TABLE 4 (CONTINUED)
MIMIMUM VARIANCE AND MAXIMUM EXPECTED REAL RETURN PORTFOLIOS

E. Euro area

				Portfolio 2018	Optimal mean-variance portfolios					
					Model 1		Model 2		Model 3	
					Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return	Minimum variance	Maximum expected real return
Asset categories	Expected real return	Standard deviation	Coefficient of variation	Portfolio shares						
Currency and deposits	-0.34	0.63	-1.85	15.03	114.08	-194.23	97.28	0.00	33.22	0.00
Insurance/pension products	2.97	1.88	0.63	14.68	-19.06	283.55	0.00	0.00	0.00	0.00
Equity	2.49	19.86	7.99	5.05	-0.65	-10.82	0.00	0.00	0.00	0.00
Debt securities	3.02	4.23	1.40	1.03	-2.70	29.19	0.00	0.00	6.36	39.57
Investment fund shares	2.87	5.86	2.04	3.79	4.19	-31.50	0.00	0.00	0.00	0.00
Non-financial assets	5.05	3.04	0.60	60.43	4.13	23.82	2.72	100.00	60.43	60.43
Portfolio performance										
Expected real portfolio return				3.70	-0.72	10.00	-0.19	5.05	3.13	4.24
Portfolio standard deviation				2.42	0.58	3.06	0.62	3.04	1.85	2.30
Coefficient of variation				0.65	-0.80	0.31	-3.23	0.60	0.59	0.54

Notes:

1. All numbers, except for the coefficient of variation being dimensionless, in percentages.
2. Time range for computation of individual assets' expeced real returns and standard deviations: 2000-2018. Individual assets' expected real returns computed as arithmetic averages.

3. Coefficient of variation computed as ratio of standard deviation to expected real return.

Source: Authors' calculations.

TABLE 5

MEAN-VARIANCE "CORNER" PORTFOLIOS AND HEURISTIC PORTFOLIOS

A. Germany

				Portfolio 2018	**Mean-variance "corner" portfolios**						**Heuristic portfolios**	
					Model 1		**Model 2**		**Model 3**			
					(1a)	**(1b)**	**(2a)**	**(2b)**	**(3a)**	**(3b)**		
					Standard deviation 2018 + expected real return maximized	Expected real return 2018 + Standard deviation minimized	Standard deviation 2018 + expected real return maximized	Expected real return 2018 + Standard deviation minimized	Standard deviation 2018 + expected real return maximized	Expected real return 2018 + Standard deviation minimized	1/n portfolio	Opportunity portfolio
Asset categories	Expected real return[4)]	Standard deviation[5)]	Coefficient of variation	Portfolio shares								
Currency and deposits	-0.13	0.81	-6.15	17.74	-117.76	-6.27	0.00	0.00	0.00	14.69	8.39	4.20
Insurance/pension products	2.83	1.32	0.47	16.03	189.74	86.14	17.67	70.04	16.08	27.27	8.39	4.20
Equity	3.72	21.25	5.71	2.66	4.19	2.35	0.00	0.00	0.00	0.00	8.39	4.20
Debt securities	3.16	3.67	1.16	1.07	6.31	2.51	8.95	0.00	20.36	0.00	8.39	4.20
Investment fund shares	4.23	9.63	2.27	4.46	-15.85	-9.03	0.00	0.00	5.52	0.00	8.39	25.18
Non-financial assets	4.37	2.85	0.65	58.04	33.37	24.31	73.38	29.96	58.04	58.04	58.04	58.04
Portfolio performance												
Expected real portfolio return				3.29	6.66	3.29	3.99	3.29	3.87	3.29	3.70	4.01
Portfolio standard deviation				2.17	2.17	1.20	2.17	1.29	2.17	1.70	3.43	4.09
Coefficient of variation				0.66	0.33	0.37	0.54	0.39	0.56	0.52	0.93	1.02

TABLE 5 (CONTINUED)
MEAN-VARIANCE "CORNER" PORTFOLIOS AND HEURISTIC PORTFOLIOS

B. Spain

				Portfolio 2018	Mean-variance "corner" portfolios						Heuristic portfolios	
					Model 1		Model 2		Model 3			
					(1a) Standard deviation 2018 + expected real return maximized	(1b) Expected real return 2018 + Standard deviation minimized	(2a) Standard deviation 2018 + expected real return maximized	(2b) Expected real return 2018 + Standard deviation minimized	(3a) Standard deviation 2018 + expected real return maximized	(3b) Expected real return 2018 + Standard deviation minimized	1/n portfolio	Opportunity portfolio
Asset categories	Expected real return[4)]	Standard deviation[5)]	Coefficient of variation	Portfolio shares								
Currency and deposits	-1.13	1.36	-1.20	14.55	-479.40	-13.46	0.00	0.00	0.00	13.86	6.54	3.27
Insurance/pension products	4.54	1.67	0.37	5.90	601.73	111.26	29.21	70.45	26.22	1.52	6.54	3.27
Equity	2.77	20.02	7.22	6.94	14.48	0.55	0.00	0.00	6.51	0.00	6.54	3.27
Debt securities	2.75	3.81	1.38	0.28	-55.86	-9.12	0.00	0.00	0.00	17.34	6.54	3.27
Investment fund shares	1.71	3.20	1.87	5.05	31.57	6.47	0.00	0.00	0.00	0.00	6.54	19.63
Non-financial assets	7.45	9.30	1.25	67.28	-12.53	4.30	70.79	29.55	67.28	67.28	67.28	67.28
Portfolio performance												
Expected real portfolio return				5.40	31.22	5.40	6.60	5.40	6.38	5.40	5.71	5.64
Portfolio standard deviation				6.56	6.56	1.71	6.56	2.90	6.56	6.12	6.55	6.38
Coefficient of variation				1.22	0.21	0.32	0.99	0.54	1.03	1.13	1.15	1.13

TABLE 5 (CONTINUED)
MEAN-VARIANCE "CORNER" PORTFOLIOS AND HEURISTIC PORTFOLIOS

C. France

				Portfolio 2018	Mean-variance "corner" portfolios						Heuristic portfolios	
					Model 1		Model 2		Model 3			
					(1a) Standard deviation 2018 + expected real return maximized	(1b) Expected real return 2018 + Standard deviation minimized	(2a) Standard deviation 2018 + expected real return maximized	(2b) Expected real return 2018 + Standard deviation minimized	(3a) Standard deviation 2018 + expected real return maximized	(3b) Expected real return 2018 + Standard deviation minimized	1/n portfolio	Opportunity portfolio
Asset categories	Expected real return[4]	Standard deviation[5]	Coefficient of variation	Portfolio shares								
Currency and deposits	0.14	0.60	4.34	12.82	-128.76	-15.46	0.00	0.00	0.00	13.16	7.65	3.82
Insurance/pension products	3.19	3.51	1.10	16.50	273.56	135.28	0.00	11.98	37.16	0.00	7.65	3.82
Equity	3.00	19.04	6.35	6.32	-18.95	-10.49	0.00	0.00	1.07	0.00	7.65	3.82
Debt securities	3.04	3.67	1.21	0.38	84.70	41.20	27.57	38.47	0.00	25.07	7.65	3.82
Investment fund shares	1.55	4.14	2.67	2.20	-118.05	-55.36	0.00	0.00	0.00	0.00	7.65	22.94
Non-financial assets	6.31	5.97	0.95	61.77	7.50	4.83	72.43	49.55	61.77	61.77	61.77	61.77
Portfolio performance												
Expected real portfolio return				4.67	9.19	4.67	5.41	4.67	5.11	4.67	4.73	4.61
Portfolio standard deviation				4.33	4.33	2.25	4.33	3.29	4.33	3.70	4.36	4.28
Coefficient of variation				0.93	0.47	0.48	0.80	0.70	0.85	0.79	0.92	0.93

TABLE 5 (CONTINUED)
MEAN-VARIANCE "CORNER" PORTFOLIOS AND HEURISTIC PORTFOLIOS

D. Italy

				Portfolio 2018	Mean-variance "corner" portfolios						Heuristic portfolios	
					Model 1		Model 2		Model 3			
					(1a) Standard deviation 2018 + expected real return maximized	(1b) Expected real return 2018 + Standard deviation minimized	(2a) Standard deviation 2018 + expected real return maximized	(2b) Expected real return 2018 + Standard deviation minimized	(3a) Standard deviation 2018 + expected real return maximized	(3b) Expected real return 2018 + Standard deviation minimized	1/n portfolio	Opportunity portfolio
Asset categories	Expected real return[4)]	Standard deviation[5)]	Coefficient of variation	Portfolio shares								
Currency and deposits	-0.86	0.81	-0.94	14.00	-171.24	-9.81	0.00	0.00	0.00	19.45	7.36	3.68
Insurance/pension products	2.19	2.19	1.00	10.08	271.36	99.16	0.00	50.85	0.00	0.00	7.36	3.68
Equity	0.54	18.25	34.02	4.88	-27.71	-12.37	0.00	0.00	3.14	0.00	7.36	3.68
Debt securities	2.98	4.74	1.59	2.95	-3.24	-1.09	35.28	21.34	33.68	15.44	7.36	3.68
Investment fund shares	2.64	4.20	1.59	4.90	31.73	19.58	0.00	0.00	0.00	1.93	7.36	22.09
Non-financial assets	3.97	4.87	1.23	63.18	-0.90	4.53	64.72	27.81	63.18	63.18	63.18	63.18
Portfolio performance												
Expected real portfolio return				2.86	7.97	2.86	3.62	2.86	3.53	2.86	3.06	3.27
Portfolio standard deviation				3.32	3.32	1.54	3.32	2.17	3.32	3.03	3.52	3.39
Coefficient of variation				1.16	0.42	0.54	0.92	0.76	0.94	1.06	1.15	1.03

TABLE 5 (CONTINUED)
MEAN-VARIANCE "CORNER" PORTFOLIOS AND HEURISTIC PORTFOLIOS

E. Euro area

				Portfolio 2018	Mean-variance "corner" portfolios						Heuristic portfolios	
					Model 1		Model 2		Model 3			
					(1a)	(1b)	(2a)	(2b)	(3a)	(3b)		
					Standard deviation 2018 + expected real return maximized	Expected real return 2018 + Standard deviation minimized	Standard deviation 2018 + expected real return maximized	Expected real return 2018 + Standard deviation minimized	Standard deviation 2018 + expected real return maximized	Expected real return 2018 + Standard deviation minimized	1/n portfolio	Opportunity portfolio
Asset categories	Expected real return[4)]	Standard deviation[5)]	Coefficient of variation	Portfolio shares								
Currency and deposits	-0.34	0.63	-1.85	15.03	-126.48	-13.07	0.00	0.40	0.00	16.06	7.91	3.96
Insurance/pension products	2.97	1.88	0.63	14.68	217.03	105.74	4.78	50.89	0.00	10.31	7.91	3.96
Equity	2.49	19.86	7.99	5.05	-8.59	-4.84	0.00	0.00	4.33	0.00	7.91	3.96
Debt securities	3.02	4.23	1.40	1.03	22.18	10.45	16.73	13.32	35.24	13.20	7.91	3.96
Investment fund shares	2.87	5.86	2.04	3.79	-23.66	-10.53	0.00	0.00	0.00	0.00	7.91	23.74
Non-financial assets	5.05	3.04	0.60	60.43	19.50	12.25	78.49	35.39	60.43	60.43	60.43	60.43
Portfolio performance												
Expected real portfolio return				3.70	7.64	3.70	4.61	3.70	4.22	3.70	3.92	4.05
Portfolio standard deviation				2.42	2.42	1.37	2.42	1.71	2.42	1.93	2.88	2.91
Coefficient of variation				0.65	0.32	0.37	0.52	0.46	0.57	0.52	0.74	0.72

Notes:

1. All numbers, except for the coefficient of variation being dimensionless, in percentages.
2. Time range for computation of individual assets' expeced real returns and standard deviations: 2000-2018. Individual assets' expected real returns computed as the arithmetic averages.
3. Coefficient of variation computed as ratio of standard deviation to expected real return.
4. Model 3a portfolios are not efficient in the case of Spain, France, Italy, and the euro area.

Source: Authors' calculations.